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(*b*, Warsaw, Poland, 2 February 1896; *d*. Warsaw, 18 June 1980)

mathematics.

Kuratowski's father was a well-known warsaw lawyer. After completing his secondary education in Poland, Kuratowski enrolled as an engineering student at the University of Glasgow in 1913. He was spending his summer vacation of 1914 at home when war broke out, so he had to remain in Poland. In 1915 he began to study mathematics at the Universit of Warsaw, which had reopened after almost half a century of inactivity. He was a student of Stefan Mazurkiewicz and Zygmunt Janiszewski in the Seminar on Topology (initiated in 1916), and of the philosopher and logician Jan Lukasiewicz. Kuratowski graduated from the university in 1919. He obtained his doctorate in 1921 under the supervision of Waclaw Sierpinski, with a dissertation on fundamental questions in set theory that contributed to international recognition of the embryonic Polish school of mathematics.

In 1927 Kuratowski was named to the chair of mathematics at Lwów Technical University. In 1933 he became professor at Warsaw University, where he was in charge of a wide range of academic and administrative functions inside and outside of Poland. He was secretary of the Mathematical Commission of the Council of Exact and Natural Sciences, Which formulated the organizational plan of the Polish school of mathematics from 1936 to 1939. The outbreak of the war and the dramatic events of the following years did not put an end to mathematical education, which the scientific elite carried out through the clandestine university network. Kuratowski was active in the restructuring of mathematics in Poland after the liberation in February 1945.

Kuratowski is prominent in the history of mathematics in Poland as a result of his original contributions and his intense activity in mathematical education. A member of the editorial board of *Fundamenta mathematicae* from 1928, Kuratowski replaced Sierpinski as its editor in chief (1952) and held the post until his death. He was one of the founders and editor of the series Monografie Matematyczne (1932), which published the works of well-known Polish mathematicians. Kuratowski was vice president of the Polish Academy of Sciences and founded and directed its Institute of Mathematics. He was also vice president of the International Mathematical Union and a foreign member of the Academy of Sciences of the U.S. S.R., of the Royal Society of Edinburgh, of the Accademia Nazionale dei Lincei, and of the academies of Palermo, Hungary, Austria, the German Democratic Republic, and Argentina.

Kuratowski's mathematical activity focused on the properties and applications of topological spaces. His first contribution to general topology was the axiomatization of the closure operator (1922). He used Boolean algebra to characterize the topology of an abstract space independently of the notion of points. Subsequent research showed that, together with Felix Hausdorff's definition of a topological space in terms of neighborhoods, the closure operator yielded more fertile results than the axiomatic theories based on Maurice Frechet's convergence (1906) and Frigyes Riesz's points of accumulation(1907).

Another field that interested Kuratowski was compactness, which, along with the metrization of topological spaces, was a pillar of mathematics in the 1920's. Prior to the appearance of the pioneering work of Pavel Aleksandrov and Pavel Urysohn (1923), many workers in this field used the properties of Emile Borel and Henri Lebesgue, of Bernard Bolzano and Karl Weierstrass, and of Georg Cantor, with little discretion. Kuratowski and Sierpinski were the first to publish a comprehensive study of the Borel-Lebesgue property (1921). In this context they made a remarkable presentation of Lindelof spaces. Kuratowski and his colleagues in the Seminar of Mathematics at Warsaw made important contributions in the field of metric spaces, the spaces of greatest interest in that period. The first volume of his Topologie (1933) was the first complete work on metric spaces to appear in several decades.

In the theory of connectedness, the efforts of Bronislaw Knaster and Kuratowski to organize the ideas of contemporary mathematicians culminated in an important contribution (1921) that revealed the conditions for connectedness of a subspace and the union of a family of subspaces of a topological space. However, the most remarkable feature of this work was theingenious construction of a set known as the Knaster-Kuratowski fan, a subspace of the plane, obtained from the Cantor set by the category method, that has a central point that, if removed, causes the fan to become completely disconnected. The appearance of the Knaster-Kuratowski fan and its singular characteristics gave a new impulse to the research in connectedness and dimension.

The topology of the continuum was one of the original areas of activity of the Polish mathematicians. Kuratowski, who made significant contributions in this field, dealt with the problem of the indecomposable continuum and the common frontier. He showed that such a frontier is itself an indecomposable continuum or the union of two indecomposable continua (1924).

Knaster proved the second of these propositions (1925). The implicit use of minimal principles in the study of irreducible continua and the need for a theory to deduce a method to eliminate the transfinite ordinals from the topological demonstrations led Kuratowski to formulate the Kuratowski-Zorn lemma, one of the fundamental notions in set theory. Before Max Zorn formulated this lemma (1935), Kuratowski used the axiom of choice to establish a minimal principle in a paper that proposed to generalize certain results of Janiszewski (1910) and L. E. J. Brouwer (1911) with regard to an irreducible continuum containing two given points (1922). The appearance of such a general method represented a historic moment of reaction to the generalized and even artificial use of the well-ordered sets and the transfinites of Cantor. From this time on it became customary to use the transfinites only when it was absolutely necessary. A detailed account of kuratowski's impact in this area is in Kuratowski and Andrzej Mostowski's *Set Theory* (1968).

Another concern of the Polish school of mathematics was measure theory. Are there nonmeasurable sets in the theory of a completely additive measure? The study of this question led to the Banach-Tarski paradox (1929), Stefan Banach and Kuratowski (1929) answered in the affirmative, provided the continuum hypothesis is assumed. In his general formulation of the problem, Banach raised questions about the cardinality of a set on which a measure was desired (1930). Stimulated by Kuratowski, his student Stanislaw Ulam provided the solution in the same year. This was a beautiful example of scientific collaboration and understanding, and of the ability to organize and encourage creative activity at its height.

Even though at first the theory of dimension was considered to be a part of point-set topology, the situation changed radically after the publication of the work of Aleksandrov (1926), in which the dimension of a metric space was characterized in terms of the dimension of its polyhedra. The work of Kuratowski in this field reveals his talent and his ability to adapt to new theories. Extending the principal results of Aleksandrov, Kuratowski (1933) devised a method to characterize the dimension of a metric space by means of barycentric mapping into the nerve of an arbitrary finite cover. The generality of this method permitted the extension of various spaces to normal spaces. In addition to the generalization of the theorem of Georg Nöbeling and Lev S. Pontriagin, Kuratowski, working alone or with Karl Menger and Edward Otto, among others, derived many additional results in the theory of dimension.

Mention also should be made of the theory of projective sets and analytic sets, in which Kuratowski extended the results in Euclidean spaces to Polish spaces (complete and separable); the theory of graphs, in which he obtained a rather difficult characterization of planar graphs; and the structure and classification of linear spaces from the point of view of topological range or type of dimension.

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