Laurent, Pierre Alphonse | Encyclopedia.com

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(*b*. Paris, France, 18 July 1813; *d*. Paris, 2 September 1854),

mathematics, optics.

In 1832, after studying for two years at the École Polytechnique, Laurent graduated among the highest in his class, receiving the rank of a second lieutenant in the engineering corps. When he left the École d'application at Metz he was sent to Algeria, where he took part in the Tlemeen and Tafna expeditions. He returned to France and participated in the study leading to the enlargement of the port of Le Havre. Laurent spent about six years in Le Havre directing the difficult hydraulic construction projects. His superiors considered him a promising officer; they admired his sure judgment and his extensive practical training.

In the midst of these technical operations Laurent composed his first scientific memoirs. Around 1843 he sent to the Académie des Sciences a "Mémoire sur le calcul des variations." The Academy had set the following problem as the subject of the Grand Prize in the mathematical sciences for 1842: Find the limiting equations that must be joined to the indefinite equations in order to determine completely the maxima and minima of multiple integrals. The prize was won by Pierre Frédéric Sarrus, then dean of the Faculty of Sciences of Strasbourg. A memoir by Delaunay was accorded an honorable mention. Laurent submitted his memoir to the Academy after the close of the competition but before the judges announced their decision. His entry presented great similarities to Sarrus's work. Although some of Laurent's methods could be considered more inductive than rigorous, Cauchy, in his report of 20 May, concluded that the piece should be approved by the Academy and inserted in the *Recueil des savant's étrangers*. Laurent's work was never published, however while Delaunay's memoir appeared in the *Journal de l'Ecole polytechnique* in 1843, and Sarrus's in the *Recueil des savants étrangers* in 1846.

A similar fate befell Laurent's "Extension du théorème de M. Cauchy relatif à la convergence du développement d'une fonction suivant les puissance ascendants de la variable x." The content of this paper is known only through Cauchy's report to the Academy in 1843. Characteristically, Cauchy spoke more about himself than about the author. He stated his own theorem first:

Let *x* designate a real or imaginary variable; a real or imaginary function of x will be developable in a convergent series ordered according to the ascending powers of this variable, while the modulus of the variable will preserve a value less than the smallest of the values for which the function or its derivative ceases to be finite or continuoues.

While carefully examining the first demonstration of this theorem, Laurent recognized that Cauchy's analysis could lead to a more general theorem, which Cauchy formulated in his report in the following way:

Let *x* designate a real or imaginary variable; a real or imaginary function of *x* can be represented by the sum of two convergent series, one ordered according to the integral and ascending powers of *x*, and the other according to the integral and descending powers of *x*; and the modulus of *x* will take on a value in an interval within which the function or its derivative does not cease to be finite and continuous.

Cauchy thought Laurent's memoir merited approval by the Academy and inclusion in the *Recueil des savants étrangers*, but it too was not published. One can gain an idea of Laurent's methods by his study published posthumously in 1863.

Meanwhile Laurent abandoned research in pure mathematics and concentrated on the theory of light waves. The majority of his investigations in this area appeared in notes published in the *Comptes rendus hebdomadaires des séances de l'Académie des science*. Laurent summarized the principal ideas of his research in a letter to Arago. He declared that the theory of polarization was still at the point where Fresnel had left it. He criticized Cauchy's method of finding differential equations to explain this group of phenomena and asserted that the equations were purely empirical. He rejected the use of single material points in determining the equations of motion of light, and employed instead a system combining the spheroids and a system of material points. Cauchy responded vigorously to Laurent's claim that he had thereby proved that the molecules of bodies have finite dimensions.

When Jacobi, a correspondent of the Academy since 1830, was elected a foreign associate member in 1846, Cauchy nominated Laurent for Jacobi's former position, but he was not elected. A short time later Lauret was promoted to major and called to Paris to join the committee on fortifications. While carrying out his new duties, he continued his scientific research. He died in 1854 at the age of forty-two, leaving a wife and three children.

His widow arranged for two more of his memoirs to be presented to the Academy of science. One, on optics, *Examen de la théorie de la lumière dans le système des ondes,* was never published, despite Cauchy's recommendation that it be printed in the *Recueil des savants étrangers*. The other did not appear until 1863, when it was published in the *Journal de l'Ecole polytechnique*. Designating the modulus of a complex number as *X* and its argument as *p*, Laurent proposed to study the continuous integrals of the equation

where *F* is a function of the form subject to the condition $F(x,\pi) = F(x, -\pi)$ and which, together with its first order partial derivatives, remains finite and continuous for all x and all *p* relative to the points of the plane included between two closed curves *A* and *B*, each of which encircles the origin of the system of coordinates. If *C* is a curve encircling the origin, contained between *A* and *B* and respected by the polar equation

$X= P(p)$, with $P(\pi) = P(-\pi)$,

Laurent demonstrated that the integral

is independent of the curve *C*, that is, of the function *P*. He thus showed that the first variation of *I* is identically zero when *C* undergoes an infinitely small variation. He arrived at the same result, moreover, by calculating a double integral following a procedure devised by Cauchy. Laurent deduced his theorem from it by a method analogous to the one Cauchy had employed to establish his own theorem. He showed that if *p*1 and *p*2 are the radii of two circles centered at the origin and tangent respectively to the curves *A* and *B*, then at every polar coordinate point (X, p) situated in the annulus delimited by these circles:

$F(X, p)$

In the memoir he applied these results to the problem of the equilibrium of temperatures in a body and to the phenomenon of elasticity.

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Jean Itard