Lipschitz, Rudolf Otto Sigismund | Encyclopedia.com

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(*b*. near Königsberg, Germany [now Kaliningrad, R.S.F.S.R.], 14 May 1832;*d*.Bonn, Germany, 7 October 1903),

mathematics.

Lipschitz was born on his father’s estate, Bönkein. At the age of fifteen he began the study of mathematics at the University of Königsberg, where Franz Neumann was teaching. He then went to Dirichlet in Berlin, and he always considered himself Dirichlet’s student. After interrupting his studies for a year because of illness, he received his doctorate from the University of Berlin on 9 August 1853. After a period of taining and teaching at the Gymnasiums in Königsberg and Elbing, Lipschitz became a *Privatdozent* in mathematics at the University of Berlin in 1857. In the same year he married Ida Pascha, the daughter of a neighboring landowner. In 1862 he became an associate professor at Breslau, and in 1864 a full professor at Bonn, where he was examiner for the dissertation of nineteen-year-old Felix Klein in 1868. He rejected an offer to succeed Clebsch at Göttingen in 1873. Lipschitz was a corresponding member of the academies of Paris, Berlin, Göttingen, and Rome. He loved music, especially classical music.

Lipschitz distinguished himself through the unusual breadth of his research. He carried out many important and fruitful investigations in number theory, in the theory of Bessel functions and of Fourier series, in ordinary and partial differential equations, and in analytical mechanics and potential theory. Of special note are his extensive investigations concerning *n*-dimensional differential forms and the related questions of the calculus of variations, geometry, and mechanics. This work—in which he drew upon the developments that Riemann had presented in his famous lecture on the basic hypotheses underlying geometry—contributed to the creation of a new branch of mathematics.

Lipschitz was also very interested in the fundamental questions of mathematical research and of mathematical instruction in the universities. He gathered together his studies on these topics in the two-volume *Grundtagen der Analysis*. Until then a work of this kind had never appeared in German, although such books existed in French. The work begins with the theory of the rational integers and goes on to differential equations and function theory. The foundation of mathematics is also considered in terms of its applications.

In basic analysis Lipschitz furnished a condition, named for him, which is today as important for proofs of existence and uniqueness as for approximation theory and constructive function theory: If *f* is a function defined in the interval $\langle a, b\rangle$, then *f* may be said to satisfy a Lipschitz condition with the exponent α and the coefficient *M* if for any two values *x, y* in $\langle a, b\rangle$, the condition

$$|f(y) - f(x)| \leq M \mid y - x \mid^{\alpha}, \alpha > 0,$$

is satisfied.

In his algebraic number theory investigations of sums of arbitrarily many squares, Lipschitz obtained certain symbolic expressions from real transformations and derived computational rules for them. In this manner he obtained a hypercomplex system that is today termed a Lipschitz algebra. In the case of sums of two squares, his symbolic expressions go over into the numbers of the Gaussian number field; and in that of three squares, into the Hamiltonian quaternions. Related studies were carried out by H. Grassmann.

Lipschitz’s most important achievements are contained in his investigations of forms on *n* differentials which he published, starting in 1869, in numerous articles, especially in the *Journal für die reine and angewandte Mathematik*. In this area he was one of the direct followers of Riemann, who in his lecture of 1854, before the Göttingen philosophical faculty, had formulated the principal problems of differential geometry in higher-dimension manifolds and had begun the study of the possible metric structures of an *n*-dimensional manifold. This lecture, which was not exclusively for mathematicians and which presented only the basic ideas, was published in 1868, following Riemann’s death. A year later Lipschitz published his first work on this subject. With E. B. Christoffel, he was one of the first to employ cogredient differentiation; and in the process he created an easily used computational method. He showed that the vanishing of a certain expression is a necessary and sufficient condition for a Riemannian manifold to be Euclidean. The expression in question is a fourth-degree curvature quantity. Riemann knew this and had argued it in a work submitted in 1861 to a competition held by the French Academy of Sciences. Riemann did not win the prize, however; and the essay was not published until 1876, when it appeared in his collected works.

Lipschitz was especially successful in his investigations of the properties of Riemannian submanifolds V_m of dimension m in a Riemannian manifold V_n of dimension n. He showed flexure invariants for V_m in V_n, proved theorems concerning curvature, and investigated minimal submanifolds V_m in V_n. He was also responsible for the chief theorem concerning the mean curvature vector that yields the condition for a minimal submanifold: A submanifold V_m of V_n is a minimal submanifold if and only if the mean curvature vector vanishes at every point. Thus a manifold V_m is said to be a minimal manifold in V_n if, the boundary being fixed, the variation of the content of each subset bounded by a closed V_{m-1} vanishes. The definition of the mean curvature vector of a submanifold V_m is based on the fundamental concept of the curvature vector of a nonisotropic curve in V_m. This vector can be described with the help of Chris-toffel's three index symbols. Its length is the first curvature of the curve and vanishes for geodesics inV_m.

Lipschitz's investigations were continued by G. Ricci. The latter's absolute differential calculus was employed, beginning in 1913 by Einstein, who in turn stimulated interest and further research in the differential geometry of higher-dimension manifolds.

BIBLIOGRAPHY

Lipschitz's books are *Grundlagen der Analysis,* 2 vols. (Bonn, 1877-1880); and *Untersuchungen über die Summenvon Quadraten* (Bonn, 1886). He also published in many German and foreign journals, especially the *Journal für die reine und angewandte Mathematik* (from 1869). There is a bibliography in Poggendorff, IV, 897.

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