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(*b*. Rochester, England, 9 June 1885; *d*. Cambridge, England, 6 September 1977)

mathematics.

Littlewood was the eldest son of Edward Thornton Littlewood and Sylvia Ackland. His father, a graduate of Peterhouse, Cambridge, took his family to South Africa in 1892 when he became headmaster of a school at Wynberg, near Cape Town. John spent the years 1900 to 1903 at St. Paul's School in London, where his mathematics master was Francis Sowerby Macaulay, himself a creative mathematician.

Littlewood went to Cambridge as a scholar at Trinity College in October 1903. At this time the physical sciences were in a very strong position in Cambridge. As a result mathematics was looked upon as ancillary to physical science, which meant that the emphasis was on special functions and differential equations, where the treatment was far from rigorous. Moreover, great emphasis was put on manipulative skill. All this was not to Littlewood's taste: "I wasted my time except for rare interludes for the first two academic years." He did, however, value lectures given by Alfred North Whitehead on the foundations of mechanics.

Littlewood's research began during the long vacation of 1906. His tutor and director of studies was E. W. Barnes, who later left Cambridge for a career in the Anglican church and became better known as bishop of Birmingham. In 1907 Littlewood accepted the post of lecturer at Manchester University. He returned to Cambridge in 1910 and succeeded Whitehead as college lecturer at Trinity College.

It would be wrong to give the impression that Littlewood was concerned solely with mathematics; he had very wide interests. He was strong, somewhat shorter than average, and a very able athlete. He rowed for his college in Cambridge and later was very active in rock climbing and skiing. He also had a strong interest in music and was a good raconteur. However, his absorbing passion was math ematics, and although he was by no means averse to applied mathematics (indeed, he made important contributions to ballistics; *Collected Papers*, p. 21), his real interest was in analysis. Thus any account of Littlewood must be largely occupied with his contributions to pure mathematics. The mathemat ically inclined reader will find an excellent account in J. C. Burkill et al. (1978).

At the time Littlewood began his research, Barnes had studied entire functions of nonzero order, but his methods did not extend to functions of zero order, so he suggested that Littlewood might work on those functions. Littlewood later said, "I rather luckily struck oil at once (by switching to more elementary methods) and after that never looked back." In fact the "switch" was a big leap forward. He was able to establish for general functions a relation between the maximum and minimum moduli of these functions on large circles extending to infinity. Barnes had worked with special functions. There was a curious sequel that reveals much about analysis in Cambridge at the time. Barnes now suggested a problem on the zero of a certain analytic function. This was the notorious Riemann zeta function, which is very important in prime number theory. In 1859 Georg Friedrich Riemann had conjectured that all its complex zeros have real part 1/2. This is the famous Riemann hypothesis, which has never been proved. That Barnes should suggest this problem even to a very brilliant pupil shows that he could have had no idea of what was involved. Although Littlewood failed to prove the Riemann hypothesis, his investigations bore fruit. His work on the Riemann zeta function had great permanent value and led to his maxim "Never be afraid to tackle a difficult problem, however difficult it may appear. You may not solve it, but it could lead you on to something else."

Littlewood's second achievement about this time was the discovery of his famous Tauberian theorem (*Collected Papers*, p. 757). Sometime before, an earlier but less deep Tauberian theorem had been proved by G. H. Hardy, and the common interest of these two mathematicians in Tauberian theory was an important factor in initiating their lifelong collaboration. Hardy and Littlewood had very different personalities—indeed, they had little in common apart from the fact that they were both mathematicians working in Cambridge. Their joint work was collected in the papers of Hardy, and some three-quarters of the papers in the first three volumes of his papers were written jointly with Littlewood. During some of the most fruitful years, Littlewood was in Cambridge and Hardy in Oxford; they worked by correspondence. However, after Hardy's return to Cambridge, they still preferred to work in this way even though they were both at Trinity College This remarkable collaboration between two equally outstanding scientists was probably the greatest ever between two mathematicians. Harald Bohr states that it was not started without misgivings, It was important to them that their collaboration not cramp either of their styles or encroach on their freedom;

Therefore as a safety measure. . . they amused themselves by formulating some so-called axioms. . . The first of them said that, when one wrote to the other it was completely indifferent whether what they wrote was right or wrong. As Hardy put it, otherwise they could not write completely as they pleased, but would have to feel a certain responsibility thereby. The second axiom was to the effect that, when one received a letter from the other he was under no obligation whatsoever to read it, let alone to answer it, . . . it might be that the recipient…would prefer not to work at that particular time, or perhaps he was just interested in other problems. The third axiom was to the effect that, although it did not really matter if they both simultaneously thought about the same detail, still it was preferable that they should not do so. And finally, the fourth and perhaps the most important axiom, stated that it was quite indifferent if one of them had not contributed the least bit to the contents of a paper under their common names; otherwise…now one and then the other, would oppose being named as coauthor. (*Collected Works*, I, p. xxviii)

The contribution of Hardy and Littlewood to analysis was enormous. It extended over a vast range including Diophantine approximation, additive number theory, Waring's problem, the Riemann zeta function, prime number theory, inequalities, and Fourier series, In many cases their results are the best known to date, and work is still being done on many of their problems. Much of this is dealt with adequately by Titchmarsh et al., so we mention only the work on the rearrangement of functions and the Hardy-Littlewood maximal function, which has become fundamental in harmonic analysis (Stein and Weiss, p. 53).

Littlewood's work on the Riemann zeta function was outstanding and of great permanent value. An account of it and the subsequent developments is given by Montgomery in the Royal Society biographical memoir. Littlewood did not approve the Riemann hypothesis and became increasingly skeptical about it. Indeed, he privately expressed the opinion that it was false, but that the first zero in the critical script not on the critical line would be so far removed as to be beyond computation even by the most sophisticated methods, thereby rendering the problem unsolvable in the foreseeable future, This view was partly the result of his work on the functions $\pi(x)$ and li(x) discussed by Montgomery,

Littlewood collaborated with other mathematicians besides Hardy. He and Harald Bohr prepared a book on the Riemann zeta function, but when it was completed, they were too exhausted to send it to the publisher, The manuscript was passed on to Ingham and Edward Charles Titchmarsh, and later incorporated in their larger works. Littlewood collaborated with Dame Mary Cartwright on differential equations, with A. C. Offord on random equations and entire functions, and with R. E. A. C. Paley on Fourier analysis. The differential equations considered in the work with Cartwright arose in the study of electric circuits. A brief account with further developments is given by Peter Swinnerton-Dyer in Littlewood's *Collected Papers* (p. 295) and by Cartwright (1974). Following the work with Offord, there is now a considerable literature on zeros of polynomials and related matters. There is a brief account of the developments by Bollobas in Littlewood's *Collected Papers* (pp. 1343, 1421) and further development by Kleitman.

Littlewood's work with Paley represents one of the most far-reaching advances in Fourier analysis. In terms of its one-dimensional Fourier series, they define the dyactic decomposition of a function and, by employing the Poisson integral, a certain nonlinear operator they called the *g*-function. The Littlewood-Paley theory was based on complex variable methods and thus was limited to one dimension. It was later realized that an *n*-dimensional result could be de fuced from the one-dimensional theory, and this led in turn to some of the most exciting developments in analysis, There is a brief account by Brannan in Littlewood's *Collected Papers* (p. 664) and also by E. M. Stein in *Singular Integrals* (pp, 81–94) and *Topics in Harmonic Analysis*.

At the age of eighty-two Littlewood gave a lecture at Rockefeller University entitled "The Mathematician's Art of Work." After reading this and *AMathematician's Miscellany*, it seems fair to say that few mathematicians have told us so much about themselves and their style of work as has Littlewood.

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