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(*fl*. China, *ca*, a.d. 250)

mathematics.

Nothing is known about the life of Liu Hui, except that he flourished in the kingdom of Wei toward the end of the Three Kingdoms period (a.d. 221–265). His mathematical writings, on the other hand, are well known; his commentary on the *Chiu-chang suan-shu* ("Nine Chapters on the Mathematical Art") has exerted a profound influence on Chinese mathematics for well over 1,000 years. He wrote another important, but much shorter, work: the *Hai-tao suan-ching* ("Sea Island Mathematical Manual").

Some scholars believe that the *Chiu-chang suan-shu*, also called the *Chiu-chang suan-ching*("Mathematical Manual in Nine Chapters"), was already in existence in China during the third century b.c Ch'ien Paotsung, in his *Chung-kuo suan-hsüeh-shih*, and Chang Yin-lin (*Yenching Hsüeh Pao*, **2** [1927], 301) have noted that the titles of certain officials mentioned in the problems date from Ch'in and earlier (third and early second centuries b.c.). There are also references which must indicate a taxation system of 203 b.c. According to Liu Hui's preface, the book was burned during the time of Emperor Ch'in Shih-huang (221–209 b.c.); but remnants of it were later recovered and put in order. In the following two centuries, commentaries on this book were written by Chang Ts'ang (*fl*. 165–142 b.c) and Keng Shou-ch'ang (*fl*. 75–49 b.c.). In a study by Ch'ien Pao-tsung (1963) it is suggested, from internal textual evidence, that the *Chiu-chang suan-shu* was written between 50 b.c. and a.d. 100 and that it is doubtful whether Chang Ts'ang and Keng Shou-ch'ang had anything to do with the book. Yet Li Yen and Tu Shih-jan, both colleagues of Ch'ien Pao-tsung, still believed Liu Hui's preface when they wrote about the *Chiu-chang suan-shu* in the same year.

During the seventh century both the *Chiu-chang suan-shu* and the *Hai-tao suan-ching* (a.d. 263) were included in *Suan-ching shih-shu* ("Ten Mathematical Manuals," a.d. 656), to which the T'ang mathematician and astronomer Li Shun-feng (602–670) added his annotations and commentaries. These works then became standard texts for students of mathematics; official regulations prescribed that three years be devoted to the works of Liu Hui. Liu Hui's works also found their way to Japan with these the mathematical manuals. When schools were established in Japan in 702 and mathematics was taught, both the *Chiu-chang suan-shu* and the *Hai-tao suan-ching* were among the prescribed texts.

According to Ch'eng Ta-wei's mathematical treatise, the *Suan-fa t'ung-tsung* ("Systematic Treatise on Arithmetic"; 1592), both the *Chiu-chang suan-shu* and the *Hai-tao suan-ching* were first printed officially in 1084. There was another printed version of them by Pao Huan-chih in 1213. In the early fifteenth century they were included, although considerably rearranged, in the vast Ming encyclopedia, the *Yung-lo ta-tien* (1403–1407). In the second part of the eighteenth century Tai Chen (1724–1777) reconstructed these two texts after having extracted them piecemeal from the *Yung-lo to-tilen*. They were subsequently included by K'ung Chi-han (1739–1787) in his *Wei-po-hesieh ts'ung-shu* (1773). Three years later ch'ü Tseng-fa printed them separately with preface by Tai Chen.

Other reproductions based on Tai Chen's reconstruction in the *Wei-po-hsieh ts'ung-shu* are found in the *Suan-ching shih-shu* ("Ten Mathematical Manuals") of Mei Ch'i-chao (1862 and in the *Wan-yu-wen-K'u* (1929–1933) and *Ssu-pu ts'ung-k'an* series (1920–1922; both of the Commercial Press, Shanghai). Two nineteenth-century scholars, Chung Hsiang and Li Huang, discovered that certain passages in the text had been rendered incomprehensible by Tai Chen's attempt to improve on the original text of the *Chiu-chang suan-shu*. A fragment of the early thirteenth century edition of the *Chiu-chang suan-shu*. consisting of only five chapters, was found during the seventeenth century in Nanking, in the private library of Huang Yü-chi (1629–1691). This copy was seen by the famous Ch'ing scholar Mei Wen-ting (1633–1721) in 1678, and it later came into the possession of K'ung Chi-han (1739–1784) and then Chang Tun-jen (1754–1834); finally it was acquired by the Shanghai Library, where it is now kept. In 1684, Mao I (1640-after 1710) made a handwritten copy of the original text found in the library of Huang Yü-chi. This copy was later acquired by the emperor during the Ch'ien-lung reign (1736–1795). In 1932 it was reproduced in the *T'ien-lu-lin-lang ts'ung-shu* series.

In 1261 Yang Hui wrote the *Hsiang-csieh chiu-chang suan-fa* ("Detailed Analysis of the Mathematical Rules in the Nine Chapters") to elucidate the problems in the *Chiu-chang suan-shu*. Ch'ien Pao-tsung in 1963 collated the text of the *Chiu-chang suan-shu* from Tai Chen's version, the fragments of the late Sung edition as reproduced in the *T'ien-lu-lin-lang ts'ung-shu* series, and Yang Hui's *Hsiang-chieh chiu-chang suan-fa*.

As for the *Hai-tao suan-ching*, only the reconstructed version by Tai Chen remains. It was reproduced in the *Wu-ying-tien* palace edition (before 1794), the “Ten Mathematical Manuals” in K’ung Chi-han’s *Wei-po-hsieh ts’ung-shu*, and the appendix to Chü Tseng-fa’s *Chiu-chang suan-shu*.

The *Chiu-chang suan-shu* was intended as a practical handbook, a kind of aide-mémoire for architects, engineers, officials, and tradesmen. This is the reason for the presence of so many problems on building canals and dikes, city walls, taxation, barter, public services, etc. It consists of nine chapters, with a total of 246 problems. The chapters may be outlined as follows:

(1) *Fang-t’ien*(“Land Surveying”) contains the rules for finding the areas of triangles, trapezoids, rectangles, circles, sectors of circles, and annuli. It gives rules for addition, subtraction, multiplication and division of fractions. There is an interesting but Inaccurate formula for the area of the segment of a where the chord c and the sagitta s are known, in the form $s(c + s)/2$. This expression later appeared during the ninth century in Mahāvīra’s *Ganitasārasangraha*.

Of special interest is the value of the ratio of the circumference of a circle to its diameter that Liu Hui used. The ancient value of π used in China was 3, but since the first century Chinese mathematicians had been searching for a more accurate value. Liu Hsin (*d*. a.d. 23) used 3.1547, while Chang Hen (78–139) gave $\sqrt{10}$ and 92/29. Wang Fan (219–257) found 142/45, and then Liu Hui gave 3.14. The most important names in this connection are, however, those of Tsu Ch’ung-chih (430–501), a brilliant mathematician, astronomer, and engineer of the Liu Sung and Ch’i dynasties, and his son, Tsu Cheng-chih. Tsu Ch’ung-chih gave two values for π first an “inaccurate” one (*yo lü*), equal to 22/7, given earlier by Archimedes, and then a “more accurate” one (*(mi lu)*), 355/113 (3.1415929). He even looked for further approximations and found that π lies between 3.1415926 and 3.1415927. His method was probably described in the *Chui Shu,* which he and his son wrote but is now lost. Tsu Ch’ung-chih’s value of 355/113 for π disappeared for many centuries in China until it was again taken up by Chao Yu-ch’in (*(fl, ca. 1300)*). Liu Hui obtained the accurate value 3.14 by taking the ratio of the perimeter of a regular polygon of ninety-six sides to the diameter of a circle enclosing this polygon. Let us begin with a regular hexagon of side L_6. The ratio of the perimeter of the hexagon to the diameter of the circle enclosing it is 3. If we change the hexagon to a regular polygon of twelve sides, as shown in Figure 1—noting that $L_6 = r$, the radius of the circumscribed circle—then the side of the twelve-sided polygon is given by

Hence, if $L_n$is known, then L_{2n} can be found from the expression

Taking $r = 1$, the following values can be found: $L_6 = 1$; $L_{12} = 0.517638$; $L_{24} = 0.261052$; $L_{48} = 0.130806$; $L_{96} = 0.065438$.

The perimeter of a regular polygon of $n = 96$ and $r = 1$ is $96 \times 0.065438 = 6.282048$. Hence $\pi = 6.282048/2 = 3.141024$, or approximately 3.14. Liu Hui also used a polygon of 3,072 sides and obtained his best value, 3.14159.

(2)*Su-mi* (“Millet and Rice”) deals with percentages and proportions. Indeterminate equations are avoided in the last nine problems in this chapter by the use of proportions.

(3)*Ts’ui-fen*(“Distribution by Progression”) concerns distribution of properties among partners according to given rates. It also includes problems in taxation of goods of different qualities, and others in arithmetical and geometrical progressions, all solved by use of proportions.

(4)*Shao-kuang* (“Diminishing Breadth”) involves finding the sides of a rectangle when the area and one of the sides are given, the circumference of a circle

when its area is known, the side of a cube given its volume, and the diameter of a sphere of known volume. The use of the least common multiple in fractions is shown. It is interesting that unit fractions are used, for example, in problem 11 in this chapter. The given width of a rectangular form is expressed as

$1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + 1/8 + 1/9 + 1/10 + 1/11 + 1/12.$

The problems in this chapteralso lead to the extraction of square roots and cube roots; problem 13, for example, involves finding the square root of 25,281. According to the method given in the *Chiu-chang suan-shu,* this number, known as the *shih* (dividend), is first placed in the second row from the top of the counting board. Next, one counting rod, called the preliminary *chieh-suan,* is put on the bottom row of the counting board in the farthest right-hand digit column. This rod is moved to the left, two places at a time, as for as it can go without overshooting the farthest left digit of the number in the *shih* row. With its new place value this rod is called the *chieh-sucn*. It is shown in Figure 2*a*.

The first figure of the root is found to lie between 100 and 200. Then 1 is taken as the first figure of the root and is placed on the top row in the hundreds column. The top row is called *fang*. The *chieh-suan* is multiplied by the first figure of the root. The product, called *fa*, is placed in the third row. The *shih* (25,281) less the *fa* (10,000) leaves the “first remainder” (15,281), which is written on the second row, as shown in Figure 2*b*. After the division has been made, the *fa* is doubled to form the *ting-fa*. This is moved one digit to the right, while the *chieh-suan* is shifted two digits to the right, as shown in Figure 2*c*.

The second figure, selected by trial and error, is found to lie between 5 and 6. The tens’ digit is therefore taken to be 5 and will be placed in its appropriate position on the top row in Figure 2*e*. The *chieh-suan* (which is now 100) is multiplied by this second figure and the product is added to the *ting-fa*, which becomes 2,500. The *ting-fa* multiplied by 5 is subtracted from the first remainder, which gives a remainder of 2,781 (15,281 — 2,500 × 5 = 2,781), as shown in Figure 2*d*. The *ting-fa* is next shifted one digit to the right and the *chieh-suan* two places (see Figure 2*e*). The third figure, again selected by trial and error, is found to be 9. This unit digit is placed in its appropriate position on the top row. The *Chieh-suan*, which is now 1, is multiplied by this third figure and the product is added to the *ting-fa*, which becomes 259. The second remainder is divided by the *ting-fa*, which leaves a remainder of zero (2,781 ÷ 259 = 9 +0). Hence the answer is 159 (see Figure 2*f*).

(5) *Shang-kung* ("Consultations on Engineering Works") gives the volumes of such solid figures as the prism, the pyramid, the tetrahedron, the wedge, the cylinder, the cone, and the frustum of a cone:

(a) Volume of square prism = square of side of base times height.

(b) Volume of cylinder =1/12 square of circumference of circle times height (where π is taken to be approximately 3).

(c) Volume of truncated square pyramid = 1/3 the height times the sum of the squares of the sides of the upper and lower squares and the product of the sides of the upper and lower squares.

(d) Volume of square pyramid = 1/3 the height times the square of the side of the base.

(e) Volume of frustum of a circular cone = 1/36 the height times the sum of the squares of the circumferences of the upper and lower circular faces and the product of these two circumferences (where π is taken to be approximately 3).

(f) Volume of circular cone = 1/36 the height times the square of the circumference of the base (where π is taken to be approximately 3).

(g) Volume of a right triangular prism = 1/2 the product of the width, the length, and the height.

(h) Volume of a rectangular pyramid = 1/3 the product of the width and length of the base and the height.

(i) Volume of tetrahedron with two opposite edges perpendicular to each other = 1/6 the product of the two perpendicular opposite edges and the perpendicular common to these two edges.

(6) *Chün-shu*("Impartial Taxation") concerns problems of pursuit and alligation, especially in connection with the time required for taxpayers to get their grain contributions from their native towns to the capital. It also deals with problems of ratios in connection with the allocation of tax burdens according to the population. Problem 12 in this chapter says:

A good runner can go 100 paces while a bad runner goes 60 paces. The bad runner has gone a distance of 100 paces before the good runner starts pursuing him. In how many paces will the good runner catch up? [Answer: 250 paces.]

(7) *Ying pu-tsu* or *ying-nü* ("Excess and Deficiency"). *Ying*, referring to the full moon, and *pu-tsu* or *nü* to the new moon, mean "too much" and "too little," respectively. This section deals with a Chinese algebraic invention used mainly for solving problems of the type $ax + b = 0$ in a rather roundabout manner. The method came to be known in Europe as the rule of false position. In this method two guesses, x_1 and x_2 are made, giving rise to values c_1 and c_2, respectively, either greater or less than 0. From these we have the following equations:

Multiplying (1) by x_2 and (2) by x_1, we have

From (1) and (2),

Hence

Problem 1 in this chapter says:

In a situation where certain things are purchased jointly, if each person pays 8 [units of money], the surplus is 3 [units], and if each person pays 7, the deficiency is 4. Find the number of persons and the price of the things brought. [Answer: 7 persons and 53 units of money,]

According to the method of excess and deficiency, the rates (that is, the "guesses" 8 and 7) are first set on the counting board with the excess (3) and deficiency (—4) placed below them. The rates are then cross multiplied by the excess and deficiency, and the products are added to form the dividend. Then the excess and deficiency are added together to form the divisor. The quotient gives the correct amount of money payable by each person. To get the number of persons, add the excess and deficiency and divide the sum by the difference between the two rates. In other words, x and a are obtained using equations (5) and (4) above.

Sometimes a straightforward problem may be transformed into one involving the use of the rule of false position. Problem 18 in the same chapter says:

There are 9 [equal] pieces of gold and 11 [equal] pieces of silver. The two lots weigh the same. One piece is taken from each lot and put in the other. The lot containing mainly gold is now found to weigh less than the lot containing mainly silver by 13 ounces. Find the weight of each piece of gold and silver.

Here two guesses are made for the weight of gold. The method says that if each piece of gold weighs 3 pounds, then each piece of silver would weigh 2 5/11 pounds, giving a deficiency of 49/11 ounces; and if each piece of gold weighs 2 pounds, then each piece of silver would weigh 1 7/11 pounds, giving an excess of 15/11 ounces. Following this, the rule of false position is applied.

(8) *Fang-ch'eng* ("Calculation by Tabulation") is concerned with simultaneous linear equations, using both positive and negative numbers. Problem 18 in this chapter involves five unknowns but gives only four equations, thus heralding the indeterminate equation. The process of solving simultaneous linear equations given here is the same as the modern procedure for solving the simultaneous system

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_2y + c_3z = d_3$,

except that the coefficients and constants are arranged in vertical columns instead of being written horizontally:

$a_1a_2a_3$

$b_1b_2b_3$

$c_1c_2c_3$

$d_1d_2d_3$.

In this chapter Liu Hui also explains the algebraic addition and subtraction of positive and negative numbers. (Liu Hui denoted positive numbers and negative numbers by red and black calculating rods, respectively.)

(9) *Kou-ku* ("Right Angles") deals with the application of the Pythagorean theorem. Some of its problems are as follows:

A cylindrical piece of wood with a cross-section diameter of 2 feet, 5 inches, is to be cut into a piece of plank 7 inches thick. What is the width? [problem 4] There is a tree 20 feet high and 3 feet in circumference.A creeper winds round the tree seven times and just reaches the top. Find the length of the vine, [problem 5] There is a pond 7 feet square with a reed growing at the center and measuring I foot above the water. The reed just reaches the bank at the water level when drawn toward it. Find the depth of the water and the length of the reed. [problem 6]

There is a bamboo 10 feet high. When bent, the upper end touches the ground 3 feet away from the stem. Find the height of the break, [problem 13]

It is interesting that a problem similar to 13 appeared in Brahmagupta's work in the seventh century.

Problem 20 has aroused even greater interest:

There is a square town of unknown dimension. A gate is at the middle of each side. Twenty paces out of the north gate is a tree. If one walks 14 paces from the south gate, turns west, and takes 1,775 paces, the tree will just come into view. Find the length of the side of the town.

The book indicates that the answer can be obtained by evolving the root of the quadratic equation.

$x^2 + (14 + 20)x = 2(1775 \times 20)$.

The method of solving this equation is not described. Mikami suggests that it is highly probable that the root extraction was carried out with an additional term in the first-degree coefficient in the unknown and that this additional term was called *tsung*, but in his literal translation of some parts of the text concerning root extractions he does not notice that the successive steps correspond closely to those in Horner's method. Ch'ien Pao-tsung and Li Yen have both tried to compare the method described in the *Chiu-chang suan-shu* with that of Horner, but they have not clarified the textual obscurities. Wang Ling and Needham say that it is possible to show that if the text of the *Chiu-chang suan-shu* is very carefully followed, the essentials of the methods used by the Chinese for solving numerical equations of the second and higher degrees, similar to that developed by Horner in 1819, are present in a work that may be dated in the first century b.c.

The *Hai-tao suan-ching,* originally known by the name *Ch'ung* ch'a ("Method of Double Differences"), was appended to the *Chiu-chang suan-shu* as its tenth chapter. It was separated from the main text during the seventh century, when the "Ten Mathematical Manuals" were chosen, and was given the title *Hai-tao suan-cluig*. According to Mikami, the term ch'ung ch'a was intended to mean double, or repeated, application of proportions of the sides of right triangles. The name *Hai-tao* probably came from the first problem of the book, which deals with an island in the sea. Consisting of only nine problems, the book is equivalent to less than one chapter of the *Chiu-chang suan-shu*.

In its preface Liu Hui describes the classical Chinese method of determining the distance from the sun to the flat earth by means of double triangulation. According to this method, two vertical poles eight feet high were erected at the same level along the same meridian, one at the ancient Chou capital of Yan-ch'eng and the other 10,000 *li* (1 ,*li* = 1,800 feet) to the north. The lengths of the shadows cast by the sun at midday of the summer solstice were measured, and from these the distance of sun could be derived. Liu Hui then shows how the same method can be applied to more everyday examples. Problem 1 says:

A sea island is viewed from a distance. Two poles, each 30 feet high, are erected on the same level 1,000 *pu* [1 *pu* = 6 ft.] apart so that the pole at the rear is in a straight line with the island and the other pole. If one moves 123 *pu* back from the nearer pole, the top of the is just visible through the end of the pole if he views it from ground level. Should he move back 127 *pu* from the other pole, the top of the island is just visible through the end of the pole if viewed from ground level. Find the elevation of the island and its distance from the [nearer] pole. [Answer: The elevation of the island is 4 *li*, 55 *pu*. The distance to the [nearer] pole is 102 *li*, 150 *pu* (300 *pu* = 1 *li*).]

The rule for solving this problem is given as follows:

Multiply the height of the pole by the distance between the poles and divide the product by the difference between the distances that one has to walk back from the poles in order to view the highest point on the island. Adding the height of the pole to the quotient gives the elevation of the island. To find the distance from the nearer pole to the island, multiply the distance walked back from that pole by the distance between the poles. Dividing the product by the difference between the distances that one has to walk back from the poles gives that distance.

Problem 7 is of special interest:

A person is looking into an abyss with a piece of white rock at the bottom. From the shore a crossbar is turned to lie on the side that is normally upright [so that its base is vertical]. If the base is 3 feet and one looks at the surface of the water [directly above the rock] from the tip of the base, the line of sight meets the height of the crossbar at a distance of 4 feet, 5 inches; and when one looks at the rock, the line of sight meets the height of the crossbar at a distance of 2 feet, 4 inches. A similar crossbar is set up 4 feet above the first. If one looks from the tip of the base, the line of sight to the water surface [directly above the rock] would meet the height of the crossbar at a distance of 4 feet; and if one looks at the rock, it will be 2 feet, 2 inches. Find the depth of the water.

In Figure 3, if P is the water surface above the white rock, R, and BC and FG are the two crossbars, then $BC = FG = 3$ feet; $GC = 4$ feet; $AC = 4$ feet, 5 inches; $DC = 2$ feet, 4 inches; $EG = 4$ feet; and $HG = 2$ feet, 2 inches. The depth of the water, PR, is sought. To obtain the answer, Liu Hui gives the following rule:

Liu Hui has not taken into account here the refractive index of water. The rule given is an extension of that used in solving problem 4, which uses the same method for determining the depth of a valley:

A person is looking at a deep valley. From the edge of the valley a crossbar is turned to lie on the side that is normally upright [so that its base is vertical]. The base

is 6 feet long. If one looks at the bottom of the valley from the edge of the base, the line of sight meets the vertical side at a distance of 9 feet, 1 inch. Another crossbar is set 30 feet directly above the first. If the bottom of the valley is observed from the edge of the base, the line of sight will meet the vertical side at a distance of 8 feet, 5 inches. Find the depth of the valley.

If we refer again to Figure 3, ignoring the broken lines, we have $CB = GF = 6$ feet; $CG = 30$ feet; $AC = 9$ feet, 1 inch; $EG = 8$ feet, 5 inches; and CQ is the depth. From similar triangles ABC and PBQ,

$$QB \cdot AC = PQ \cdot CB;$$

and from similar triangles EFG and PFQ,

$QF \cdot EG = PQ \cdot GF$.

Since $CB = GF$, and $QF = QB = BF$,

$QB \cdot AC = (QB + BF)EG$,

$QB(AC - EG) = BF \cdot EG = GC \cdot EG$,

that is,

$(CQ + CB)(AC - EG) = GC \cdot EG$.

Hence,

In problem 7 one also obtain the distance from the bank to the bottom of the abyss (CS in Figure 3) from the expression

PR is derived from the difference between CS and CQ.

As for the other problems, problem 2 concerns finding the height of a tree on a hill; problem 3 deals with the size of a distant walled city; problem 5 shows how to measure the height of a tower on a plain as seen from a hill; problem 6 gives a method for finding the width of a gulf seen from a distance on land; problem 8 is a case of finding the width of a river seen from a hill; and problem 9 seeks the size of a city seen the a mountain.

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