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mathematics.

Luzin was the son of a trade official, Nikolai Mitrofanovich Luzin, and Olga Nikolaievna. He completed his secondary schooling at Tomsk in 1901 and entered the mathematics division of the Physics-Mathematics Faculty of Moscow University. Here he became an active member of the circle of science students headed by N. E. Zhukovsky and studied mathematics under the guidance of D. F, Egorov. He spent the winter and spring of 1906 in Paris, where he attended lectures at the Sorbonne and the College de France. At the end of 1906 Luzin completed the course at Moscow University and remained there to prepare for a professorship. In 1910, after passing the examinations for the master's degree, he was appointed assistant professor at Moscow University but did not take up his duties there; he was sent to Gottingen and Paris for further study in those fields that had interested him for several years—the theory of functions of a real variable, integration theory, and the theory of trigonometric series. Luzin's first memoirs, published in 1911-1913 in *Matematicheskii sboniik* and the *Comptes rendus* of the Paris Academy of Sciences, immediately attracted the attention of the scientific world, in the spring of 1914, upon his return to Moscow, Luzin started lecturing, one of his courses being on function theory; he also organized a special research seminar on the subject. In 1915 he submitted a monograph entitled *Integral i trigonometrichesky ryad* ("Integrals and the Trigonometric Series"), published in *Matematicheskii sbornik*, and defended it at the Physics-Mathematics Faculty as his master's thesis. On the basis of its outstanding merits, the scientific council awarded Luzin the doctorate in pure mathematics, and in 1917 he became a professor.

During the period 1914-1924 Luzin, a brilliant lecturer and scientific organizer, was the center of a Moscow school of function theory which greatly influenced the subsequent development of mathematics, both in the Soviet Union and abroad. Such outstanding mathematicians as P. S. Alexandrov, A. Y. Khinchin, D. E. Menshov, M. Y. Suslin, A. N. Kolmogorov, N. K. Bari, and P. S. Novikov were his pupils.

Luzin was a member of the Moscow Mathematical Society, of the Moscow Society of Explorers of Nature, and of the Cracow Academy of Sciences; an honorary member of the Calcutta Mathematical Society and of the Belgian Mathematical Society; and was elected vice-president of the International Mathematical Congress held at Bologna in 1928. In 1927 Luzin was elected a corresponding member of the Soviet Academy of Sciences, and in 1929 he became an academician.

After 1930 Luzin devoted less time to teaching and worked mainly at the Soviet Academy of Sciences: the Mathematical Institute (1929-1936; 1941-1950) and the Institute of Automatics and Telemechanics (1936-1950).

Luzin's mathematical creativity relates mostly to the theory of functions of a real variable in its two branches, metric and descriptive. Insofar as the first is concerned, Luzin's thesis (Moscow, 1915) is paramount, since it contains important results on the structure of measurable sets and functions, on primitive functions, on convergence of trigonometric series, and on representability of functions by trigonometric series. Cited in the thesis are examples of power series with coefficients tending to zero and nevertheless diverging everywhere on the boundary of the circle of convergence (1912), as well as a trigonometric series with coefficients tending to zero that nevertheless diverges almost everywhere (1912). These examples stimulated many later investigations. Luzin later showed that every measurable function can, in a certain sense, be represented as a trigonometric series that may be summable almost everywhere to that function by the methods of Poisson and Riemann.

Another important result obtained by Luzin on absolute convergence resides in the fact that when a trigonometric series converges absolutely at two points, the distance between which is incommensurable with respect toπ this series converges absolutely over an everywhere dense set and therefore will either converge everywhere or diverge everywhere.

The investigation of the structure of measurable sets and functions led Luzin to prove the so-called *C* property: that is, that every measurable function which is almost everywhere finite over a given segment can be made continuous over the segment by varying its value on a set of arbitrarily small measure (1912). In Luzin's hands this theorem was a most important means of investigation; by resorting to it he managed to solve completely a number of cardinal problems: finding a primitive function, the represenlability of a function as a trigonometric series, and finding a harmonic function that is holomorphic inside a circle and takes given values on the circumference.

In the theory of integration, Luzin solved the problem of how to distinguish the Lebesgue integral or the Denjoy integral from the other primitives of the given function. For this purpose he introduced a new concept of complete variation of a function for a perfect set The metric theory of functions was applied by Luzin to the study of boundary properties of analytical functions.

After 1915 Luzin turned to the descriptive theory of functions. Having investigated the so-called B sets studied by Borel, Baire, and Lebesgue, Luzin raised questions concerning the power of B sets and the construction of sets that are not *B* sets without resorting to Zermelo's axiom, to which many mathematicians objected. Both of the problems were solved by Luzin's pupils; M. Y. Suslin constructed a more extended class of sets than that of *B* sets, called analytical or A sets. Luzin introduced new definitions of both *A* sets and *B* sets, as well as the "sieving process," which became very important in the theory of *A* sets. Through the use of this process, Luzin also showed that a segment may be represented as a sum of pairwise disjoint *B* sets. It is to date the strongest result in the theory of sets that is not based upon Zermelo's axiom. Luzin also created a theory of "projective" sets (1925), which are obtained from *B* sets by successively performing the operation of projection and taking a complement. Through the introduction of this concept, some difficulties of a mathematical and logical nature came to light and were studied by his successors.

Luzin also provided decisive results in the solution of the problem of bending on the main base (1938) that dated back to K. M. Peterson, and the evaluation of the convergence of S. A. Chaplygin's method of approximate solution of differential equations (1932). He also wrote brilliant articles on Euler and Newton (1933, 1943). Luzin's many manuals and textbooks on mathematical analysis and the theory of functions of a real variable went through many editions and are still used by students in the Soviet Union.

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