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(*b*. Castagneta, near Bergamo, Italy, 13 May 1750; *d*. Paris, France, 14 July 1800)

mathematics.

Mascheroni was the son of Paolo Mascheroni dell'Olmo, a prosperous landowner, and Maria Ciribelli. He was ordained a priest at seventeen and at twenty was teaching rhetoric and then, from 1778, Physics and mathematics at the seminary of Bergamo. His *Nuove ricerche su l'equilibrio delle vòlte* (1785) led to his appointment as professor of algebra and geometry at the University of Pavia in 1786. In 1789 and 1793 he was rector of the university and, from 1788 to 1791, was head of the Accademia degli Affidati. Mascheroni was a member of the Academy of Padua, of the Royal Academy of Mantua, and of the Società Italiana delle Scienze. In his *Adnotationes ad calculum integrale Euleri* (1790) he calculated Euler's constant, sometimes called the Euler-Mascheroni constant, to thirty-two decimal places; the figure was corrected from the twentieth decimal place by Johann von Soldner in 1809.

In 1797 Mascheroni was appointed deputy to the legislative body in Milan. Sent to Paris by a commission to study the new system of money and of weights and measures, he published his findings in 1798 but was prevented from returning home by the Austrian occupation of Milan in 1799. Also a poet, Mascheroni dedicated his *Geometria del compasso* (1797) to Napoleon in verse; his celebrated *Invito a Lesbia Cidonia* (1793) glorifies the athenaeum of Pavia. He died after a brief illness, apparently from the complications of a cold. The poet Monti mourned his death in the *Mascheroniana*.

Mascheroni's *Nuove ricerche* is a well-composed work on statics, and the *Adnotationes* shows a profound understanding of Euler's calculus. He is best known, however, for his *Geometria del compasso*, in which he shows that all plane construction problems that can be solved with ruler and compass can also be solved with compass alone. It is understood that the given and unknowns are points; in particular, a straight line is considered known if two points of it are known.

In the preface Mascheroni recounts the genesis of his work. He was moved initially by a desire to make an original contribution to elementary geometry. It occurred to him that ruler and compass could perhaps be separated, as water can be separated into two gases; but he was also assailed by doubts and fears often attendant upon research. He then chanced to reread an article on the way Graham and Bird had divided their great astronomical quadrant, and he realized that the division had been made by compass alone, although, to be sure, by trial and error. This encouraged him, and he continued his work with two purposes in mind: to give a theoretical solution to the problem of constructions with compass alone and to offer practical constructions that might be of help in making precision instruments. The second concern is shown in the brief solutions of many specific problems and in a chapter on approximate solutions.

The theoretical solution (see especially §191) depends on the solution of the following problems: (1) to bisect a given circular arc of given center;(2) to add and subtract given segments; (3) to find the fourth proportional to three given segments; (4) to find the intersection of two given lines; and (5) to find the intersection of a given line and given circle.

In 1906 August Adler applied the theory of inversion to the Mascheroni constructions. Since this theory places lines and circles on an equal footing, it sheds light on Mascheroni's problem; but the solution via inversion is not as elegant—and certainly not as simple or as brief—as Mascheroni's.

Mascheroni's theory is but a chapter in the long history of geometrical constructions by specified means. The limitation to ruler and compass occurs in book I of Euclid's *Elements*—at least the first three postulates have been called the postulates of construction; and there are even reasons to suppose that Euclid's so-called axiomatic procedure is really only an axiomatization of the Euclidean constructions.

Euclid,of course,had inherited a tradition of restricting construction to ruler and compass. Oenopides is credited by Proclus with the construction for dropping a perpendicular (*Elements* 1.12) and with the method of transferring an angle (1.23). The tradition itself appears to be of religious origin (see Seidenberg, 1959, 1962).

About 980, the Arab mathematician Abu'I-Wafā' proposed using a ruler and a compass of fixed opening, and in the sixteenth century da Vinci, Dürer, Cardano, Tartaglia, and Ferrari were also concerned with this restriction. In 1672 Georg Mohr showed that all the construction problems of the first six books of the *Elements* can be done with compass alone. Lambert in 1774 discussed the problem "Given a parallelogram, construct, with ruler only, a parallel to a given line."

Poncelet, who mentions Mascheroni in this connection, showed in 1822 that in the presence of a given circle with given center, all the Euclidean constructions can be carried out with ruler alone. This has also been credited to Jacob Steiner, although he had heard of Poncelet's result, or "conjecture," as he called it. Poncelet and others also studied constructions with ruler alone; abstractly, his result is related to the axiomatic introduction of coordinates in the projective plane. Johannes Trolle Hjelmslev and others have studied the analogue of the Mascheroni constructions in non-Euclidean geometry.

The question has recently been posed whether the notion of two points being a unit apart could serve as the sole primitive notion in Euclidean plane geometry. An affirmative answer was given, based on a device of Peaucellier's for converting circular motion into rectilineal motion.

In 1928 Mohr's,*Euclides danicus*, which had fallen into obscurity, was republished with a preface by Hjelmslev, according to whom Mascheroni's result had been known and systematically expounded 125 years earlier by Mohr. The justice of this judgment and the question of the independence of Mascheroni's work will now be examined.

The term "independent invention" is used in two different but often confused senses. Anthropologists use it in reference to the appearance of identical, or similar, complex phenomena in different cultures. A controversy rages, the opposing positions of which, perhaps stripped of necessary qualifications, can be put thus: According to the "independent inventionists," the appearance of identical social phenomena in different cultures (especially in New World and Old World cultures) is evidence for the view that the human mind works similarly under similar circumstances; for the "diffusionists," it is evidence of a historical connection, but not necessarily a direct one.

The historian, dealing with a single community, uses the term in a different sense. When he says two inventions are independent, he means that each was made without direct reliance on the other. Simultaneous and independent solutions of outstanding problems that are widely published in the scholarly press are no more surprising than the simultaneous solutions by schoolboys of an assigned problem; and the simultaneous development in similar directions of a common fund of knowledge can also be expected. Even so, examples of independent identical innovations are rare and difficult to establish.

Although five centuries separate Abu'l-Wafā' and Leonardo, presumably no one will doubt that the Italians got the compass problem of a single opening from the Arabs (or, possibly, that both got it from a third source).

When the works of Mascheroni and Mohr are compared, it is apparent that the main ideas of their solutions of individual problems are in most cases quite different. In particular, this can be said for the bisection of a given segment. Moreover, the problem of bisection of a given segment. Moreover, the problem of bisection plays no role in Mascheroni's general solution, whereas it is central in Mohr's constructions. Still more significantly, the general problem is not formulated in Mohr's book. Thus, any suggestion of Mascheroni's direct reliance on Mohr would be quite inappropriate. Of course, the possibility cannot be excluded that Mascheroni, who explicitly denied knowledge that anyone had previously treated the matter, had heard of a partial formulation of the problem.

It appears that Hjelmslev's judgment is not entirely accurate. Mohr's book is quite remarkable and contains the basis for a simple proof of Mascheroni's result, but there is no evidence within the book itself that Mohr formulated the problem of constructions with compass alone in complete generality.

BIBLIOGRAPHY

I. Original Works. Mascheroni's mathematical works are *Nuove ricerche su l'equilibrio delle vòlte* (Bergamo, 1785); *Adnotationes ad calculum integrale Euleri* (Pavia, 1790); and *Geometria del compasso* (Pavia, 1797). A non mathematical work is *Invito a Lesbia Cidonia* (Pavia,1793).

II. Secondary Literature. Biographical details are presented in A. Fiamazzo, *Nuovo contributo alla biografia di L. Mascheroni*, 2 vols. (Bergamo, 1904); J. W. L. Glaisher, "History of Euler's Constant," in *Messenger of Mathematics*, **1** (1872), 25–30; G. Loria and C. Alasia, "Bibliographie de Mascheroni," in *Intermédiaire des mathématiciens*, **19** (1912), 92–94; and G. Natali, "Mascheroni," in *Enciclopedia italiana*, XXII (Rome, 1934), 496.

Adler's application of the theory of inversion to Mascheroni's constructions is presented in his *Theorie der geometrischen Konstruktionen* (Leipzig, 1906).

Support for the view that Euclid's so-called axiomatic procedure is merely an axiomatization of Euclidean constructions may be found in A. Seidenberg, "Peg and Cord in Ancient Greek Geometry," in *Scripta mathematica*, **24** (1959), 107–122; and "The Ritual Origin of Geometry," in *Archive for History of Exact Sciences*, **1** (1962), 488–527. Opposition to the above view is presented in T. L. Heath's ed. of Euclid's *Elements*, I, 124; and A. D. Steele, "über die Rolle von Zirkel und Lineal in der griechischen Mathematik," in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B, **3** (1936), 287–369. W. M. Kutta, "Zur Geschichte der Geometrie mit constanter Zirkelïffnung," in *Nova acta Academiae Caesarae Leopoldino Carolinae*, **71** (1898), 71–101, discusses the use of a ruler and a compass of fixed opening.

Georg Mohr’s *Euclides danicus* was translated into German by J. Pál and provided with a foreword by J. Hjelmslev (Copenhagen, 1928). Lambert’s work is referred to in R. C. Archibald, “Outline of the History of Mathematics,” in *American Mathematical Monthly*, **56,** no. 1, supp. (1949), note 277, 98.

Poncelet’s and Steiner’s contributions can be found in Poncelet’s *Traité des propriétés projectives des figures*, 2nd ed. (Paris, 1865), I, 181–184, 413–414; and in Steiner’s *Geometrical Constructions With a Ruler*, translated by M. E. Stark and edited by R. C. Archibald (New York, 1950), p. 10.

Analogues of Mascheroni’s work in non-Euclidean geometry are discussed in J. Hjelmslev, “Om et af den danske Matematiker Georg Mohr udgivet skrift *Euclides Danicus*,” in *Matematisk tidsskrift*, B (1928), 1–7; and in articles by A. S. Smogorzhevsky, V. F. Rogachenko, and K. K. Mokrishchev that are reviewed in *Mathematical Reviews*, **14** (1953), 576, 1007; **15** (1954), 148; and **17** (1956), 885, 998.

The question of whether the notion of two points being a unit apart can be the sole primitive notion in Euclidean plane geometry is discussed in R. M. Robinson, “Binary Relations as Primitive Notions in Elementary Geometry,” in Leon Henkin, Patrick Suppes, and Alfred Tarski, eds., *The Axiomatic Method With Special Reference to Geometry and Physics* (Amsterdam, 1959), 68–85.

A. Seidenberg