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(*b*. Schulpforla, near Naumburg, Germany, 17 November 1790; *d*. Leipzig, Germany, 26 September 1868)

mathematics, astronomy.

Möbius was the only child of Johann Heinrieh Möbius, a dancing teacher in Schulpforta until his death in 1793, and the former Johanne Catharine Christiane Keil, a descendant of Luther. His father's unmarried brother succeeded him as dancing teacher and as provider for the family until his own death in 1804. Möbius was taught at home until his thirteenth year, by which time he had already shown an interest in mathematics. He pursued formal education from 1803 to 1809 in Schulpforta, where he studied mathematics under Johann Gottlieb Schmidt. In 1809 he entered Leipzig University with the intention of studying law, but his early love for mathematics soon came to dominance. Consequently he studied mathematics under Moritz von Prassc, physics with Ludwig Wilhelm Gilbert, and astronomy with Mollweide, whose assistant he became.

Having been selected for a traveling fellowship, he left Leipzig in May 1813, a few months before the Battle of Leipzig, and went to Göttingen, where he spent two semesters studying theoretical astronomy with Gauss. He then proceeded to Halle for studies in mathematics with Johann Friedrich Pfaff. When in 1814 Prasse died, Mollweide succeeded him as mathematics professor, thereby opening up the position in astronomy at Leipzig. The position was given to Möbius, who received his doctorate from Leipzig in 1814 and qualified for instruction in early 1815 with his *De peculiaribus quibusdam aequationum trigonometricarum affectionibus*. In the same year he published his doctoral thesis entitled *De comptitand is occultationibus fixarum per planetas*. In spring 1816 he became extraordinary professor of astronomy at Leipzig and also observer at the observatory. In preparation for these duties he visited a number of the leading German observatories and eventually made recommendations for the refurbishing and reconstruction of the observatory at Leipzig; these were carried out by 1821. Other instruments were added later, including a six-foot Fraunhofer refractor.

In 1820 Möbius' mother, who had come to live with him, died. Shortly thereafter he married Dorothea Christiane Johanna Rathe, whose subsequent blindness did not prevent her from raising a daughter, Emilie, and two sons, Theodor and Paul Heinrich, both of whom became distinguished literary scholars. The former is best known for his research on Scandinavian and Icelandic literature; the latter is sometimes confused with Paul Julius Möbius the neurologist, who was Möbius' grandson.

Although Möbius was offered attractive positions as an astronomer at Greifswald in 1816 and as a mathematician at Dorpat in 1819, he refused them both to remain at Leipzig. In 1829 he became a corresponding member of the Berlin Academy of Sciences, but it was not until 1844, after he had been invited to succeed J. F. Fries at Jena, that Leipzig promoted him to ordinary professor of astronomy and higher mechanics. The slowness of his promotion and his modest salary have been attributed to his quiet and reserved manner, while his refusal to leave Leipzig stemmed from his love for his native Saxony and the quality of Leipzig University. In 1848 Möbius became director of the observatory, and d'Arrest became the observer and eventually his son-in-law. Möbius rarely traveled, and in general his life centered around his study, the observatory, and his family.

His writings were fully developed and original; he was not widely read in the mathematical literature of his day and consequently found at times that others had previously discovered ideas presented in his writings. Also his investigations were frequently aimed not so much at finding new results, but rather at developing more ellective and simpler means for treating existing areas. In 1868, not long after having celebrated his fiftieth year of teaching at Leipzig, Möbius died; his wife's death had come nine years earlier.

Möbius" scientific contributions may be divided into two areas—astronomy and mathematics. Like his contemporaries Gauss and W. R. Hamilton, Möbius was employed as an astronomer but made his most important contributions to mathematics.

His early publications were in astronomy; two short papers on Juno and Pallas were followed by the separate publication in 1815 of his doctoral dissertation on occultation phenomena (see above) and in 1816 by his *De minima variatkme azimuthi stellarum circulos parallelos uniformiter describentium commentatio*. By 1823 his observational activities had borne fruit to the extent that he published his only work of that sort, his *Beobachtungen auf der Königlichen Universitäts-Sternwarte zu Leipzig*. He published a few observational papers in later decades and in the 1830's made measurements on terrestrial magnetism. He also published two popular treatises on the path of Halley's comet (1835) and on the fundamental laws of astronomy (1836), the latter having gone through many editions. His greatest contribution to astronomy was his *Die Elemente der Mechanik des Himmels* (1843), wherein he gave a thorough mathematical treatment of celestial mechanics without the use of higher mathematics. Although astronomical amateurs could therefore read the book, it nevertheless contained results important to professionals. Moreover he introduced (for the first time, he thought) the use of vectorial addition and subtraction to represent velocities and forces and effectively showed the computational usefulness of that very ancient mathematical device, the epicycle.

When Mollweide died in 1825, Möbius hoped to follow his example by exchanging his own position in astronomy for that in mathematics, but in 1826 M. W. Drobisch was selected. In the following year Möbius published his greatest work, which later became a mathematical classic. Möbius' *Der barycentrische Culcul: Ein neues Hülfsmittel zur analytischen Behandlung der Geometrie* (1827) was not only his most important mathematical publication, but also the source of much of his later work. He had come upon the fundamental ideas for his barycentric calculus in 1818 and by 1821 decided that they merited book-length treatment. In an appendix to his 1823 astronomical treatise, he had given a first discussion of his new method. As he stated in the foreword to his 1827 treatise, the concept of the centroid had been recognized by Archimedes as a useful tool for geometrical investigations.

Möbius proceeded from the well-known law of mechanics, that a combination of weights positioned at various points can be replaced by a single weight of magnitude equal to the sum of the individual weights and positioned at the center of gravity of the combination. Thus Möbius constructed a mathematical system, the fundamental entities of which were points, to each of which a weight or numerical coefficient was assigned. The position of any point could be expressed in this system by varying the numerical coefficients of any four or more non-coplaner points. Thus Möbius used an equation such as $aA+bB+cC=D$, where a, b, c are numerical coefficients (positive or negative), and A, B, C,D are points, to express the fact that if A, B, C are not collinear, then D must lie in the plane of A, B, C. Möbius went on in his treatise to apply this method with noteworthy success to many important geometrical problems. Since barycentric coordinates are a form of homogeneous coordinates, their creatoris recognized with Feuerbach and Plücker, whose publications were independent and nearly simultaneous, as a discoverer of homogeneous coordinates.

Moreover Mouml;uml;bius developed important results in projective and affine geometry and also was among the first fully to appreciate the principle of duality and to give a thorough treatment of the cross ratio. He was the first mathematician to make use of a system wherein geometrical entities, such as lines, plane figures, and solids, were consistently treated as spatially oriented and to which a positive or negative sign could be affixed. Moreover he presented in this work the construction now known as the Möbius net. Finally at one point in the treatise he commented that two equal and similar solid figures, which are however mirror images of each other, could be made to coincide, if one were "able to let one system make a half revolution in a space of four dimensions. But since such a space cannot be conceived, this coincidence is impossible in this case" (*Werke*, I, 172).

Nearly all of Möbius' subsequent mathematical publications appeared in Crelle's *Journal für die reine und angewandte Mathematik* and from 1846 in either the *Abhandlungen* . . . or the *Berichte der Königlichen Sächsischen Gesellschaft der Wissenschaften zu Leipzig*. Some of these merit special attention. An 1828 paper discussed two tetrahedrons which mutually circumscribe and inscribe each other; such tetrahedrons are now known as Möbius tetrahedrons. Two dioptrical papers appeared in 1830 wherein Möbius used continued fractions to develop his results; another optical paper appeared in 1855 based on the concept of collineation. The Möbius function in number theory was presented in an 1832 paper, but most of his energies during the 1830's went into a series of papers on statics, which culminated in his 1837 two-volume *Lehrbuch der Statik*, wherein he treated the subject, following Poinsot, through combining individual forces with couples of forces and introduced the concept of a null system.

It is frequently stated that in 1840 Möbius posed for the first time the four-color conjecture, that is, that four colors are sufficient for the unambiguous construction of any map, no matter how complex, on a plane surface. This attribution is, however, incorrect; its source lies in the correct statement that in 1840 Möbius presented a lecture in which he posed the problem of how a kingdom might be divided into five regions in such a way that every region would border on each of the four other regions. In 1846 Möbius published a treatment of spherical trigonometry based on his barycentric calculus and in 1852 a paper on lines of the third order. His 1855 "Theorie der Kreisverwandschaft in rein geometrischer Darstellung" is the culmination of a number of studies on circular transformations, which are now frequently called Möbius transformations.

Möbius had been visited in 1844 by a high school teacher, Hermann Grassmann, whose now famous *Ausdehnungslehre* of 1844 contained among other things results similar to Möbius' point system of analysis. Grassmann requested Möbius to review the book, but Möbius failed to appreciate it, as did many others. When Grassmann in 1846 won the prize in a mathematical contest, which he had entered at Möbius' suggestion, Möbius did agree to write a commentary on the prize-winning essay. This 1847 work was the only significant published analysis of Grassmann's ideas until the late 1860's, when their significance was realized. Möbius was stimulated in the early 1860's to write his own treatise, "Ueber geometrische Addition und Multiplication," but this was not published until nineteen years after his death.

MÖbius is now most frequently remembered for his discovery of the one-sided surface called the Möbius strip, which is formed by taking a rectangular strip of paper and connecting its ends after giving it a half twist (*Werke*, II, 484–485). The Paris Academy had offered a prize for research on the geometrical theory of polyhedrons, and in 1858 Möbius began to prepare an essay on this subject. The results of his essay were for the most part given in two important papers: his "Theorie der elementaren Verwandtschaft" of 1863 and his "Ueber die Bestimmung des Inhaltes eines Polyëders" of 1865. The latter contains his discovery of the "Möbius strip" and proof that there are polyhedrons to which no volume can be assigned. Curt Reinhardt has shown from an examination of Möbius' notebooks that he discovered this surface around September 1858

(*Werke*, II, 517–521); this date is significant, since it is now known that Johann Benedict Listing discovered the same surface in July 1858 and published his discovery in 1861. Listing and Möbius, who worked independently of each other, should thus share the credit for this discovery.

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