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(*b*. Beaune, France, 9 May 1746; *d*. Paris, France, 28 July 1818)

geometry, calculus, chemistry, theory of machines.

Monge revived the study of certain branches of geometry, and his work was the starting point for the remarkable flowering of that subject during the nineteenth century. Beyond that, his investigations extended to other fields of mathematical analysis, in particular to the theory of partial differential equations, and to problems of physics, chemistry, and technology. A celebrated professor and peerless *cef d'́cole* Monge assumed important administrative and political responsibilities during the Revolution and the Empire. He was thus one of the most original mathematicians of his age, while his civic activities represented the main concerns of the Revolution more fully than did those of any other among contemporary French scientists of comparable stature.

The elder son of Jacques Monge, a merchant originally of Haute-Savoie, and the former Jeanne Rousseaux, of Burgundian origin, Monge was a brilliant student at the Oratorian *coĺge* in Beaunc. From 1762 to 1764, he completed his education at the College de la Trinít in Lyons, where he was placed in charge of a course in physics. After returning to Beaune in the summer of 1764. he sketched a plan of his native city. The high quality of his work attracted the aitendon of an officer at the ́ole Royale du Ǵnie at Ḿircs, and this event determined the course of his career,

Created in 1748. the École Royale du Ǵnie at Ḿzíres had great prestige, merited by the quality of the scientific and practical training that it offered. Admitted to the school at the beginning of 1765 in the very modest position of draftsman and technician, Monge was limited to preparing plans of fortifications and to making architectural models, tasks he found somewhat disappointing. But barely a year after his arrival he had an opportunity to display his mathematical abilities. The result was the start of a career worthy of his talents.

Monge was requested to solve a practical exercise in defilading—specifically, to establish a plan for a fortification capable of shielding a position from both the view and the firepower of the enemy no matter what his location. For the very complicated method previously employed he substituted a rapid graphical procedure inspired by the methods of what was soon to become descriptive geometry. This success led to his becoming ŕptiteur to the professor of mathematics, Charles Bossut. In January 1769 Monge succeeded the latter, even though he did not hold the rank of professor. The following year he succeeded the Abb́ Nollet as instructor of experimental physics at the school. In this double assignment, devoted partially to practical ends, Monge showed himself to be an able mathematician and physicist, a talented draftsman, a skilled experimenter, and a first-class teacher. The influence he exerted until he left the school at the end of 1784 helped to initiate several brilliant careers of future engineering officers and to give the engineering corps as a whole a solid technical training and a marked appreciation for science. The administrators of the school recognized his ability and, after obtaining for him the official title of "royal professor of mathematics and physics" (1775), steadily increased his salary.

Parallel to this brilliant professional career, Monge very early commenced his personal work. His youthful investigations (1766–1772) were quite varied but exhibit several characteristics that marked his entire output: an acute sense of geometric reality; an interest in practical problems; great analytical ability; and the simultaneous examination of several aspects of a single problem: analytic, geometric, and practical.

This was the period in which Monge developed descriptive geometry. He systematized its basic principles and applied it to various graphical problems studied at the ́cole du Ǵnie—problems taken, for example, from fortification, architecture, and scaffolding. That Monge left only a few documents bearing on this work is not surprising, since he was essentially coordinating and rationalizing earlier knowledge, rather than producing really original material. Elements of descriptive geometry appeared very early in his teaching—to the degree that his familiarity with the graphical procedures currently in use and with the various branches of geometry allowed him to make the necessary synthesis. The documents fiom this period record the many investigations inspired by his readings in the rich collections of the library of the École du Genie. This research dealt with topics in infinitesimal calculus, infinitesimal geometry, analytic geometry, and the calculus of variations. His first important original work was "Menioire stir les d́veiopṕes, les rayons de courbure et diffcrents genres d inflexions des courbcs á double courbure' He published an extract from it in June 1769 in the *Journal encycloṕmatiques*, and in October 1770 he finished a more complete version that he read before the Academie des Sciences in August 1771; the latter, however, was not published until 1785 *Ḿmoires de math́matiques et de physique pŕsents á 'Academic … par divers scavanns*, **10** , 511–550), By then some of the most important ideas in the memoir no longer seemed so original, because Monge had employed them in other works published in the intervening years. Nevertheless, this memoir is of exceptional interest, for it presents

most of the new conceptions that Monge developed in his later works, as well as his very personal method of exposition, which combined pure geometry, analytic geometry, and infinitesimal calculus.

Wishing to make himself known and to have his work discussed, Monge sought out d Alembert and Condorcet at the beginning of 1771. On the hitter's advice, he later in the same year presented before the Piiris Academy four memoirs corresponding to the main areas of his research. The first, which was not published, dealt with a problem to which he never returned; the extension of the calculus of variations to the study of extrema of double integrals. The second was the memoir on infinitesimal geometry mentioned above. The fourth treated a problem in combinatorial analysis related to a card trick.

In the third memoir Monge entered a field of study that was to hold his interest for many years: the theory of partial differential equations. In particular he, undertook the parallel examination of certain equations of this type and of the families of corresponding surfaces. The geometric construction of a particular, solution of the equations under consideration allowed, him to determine the general nature of the arbitrary, function involved in the solutions of a partial differential equation. Moreover, this finding enabled him to take a position on a question then being disputed by d Alembert Euler and Daniel Bernoulli. Monge, developed the ideas set forth in this memoir in two, others sent to the Academy in 1772. The work, presented in these papers was extended in four, publications dating from 1776; two of these, appeared in the *Ḿmoires* of the Academy of Turin, and two in the *Ḿmoires* of the Paris Academy. In, another paper (1774) Monge discussed the nature of, the arbitrary functions involved in the integrals of, finite difference equations. He also considered the, equation of vibrating strings, a topic he later investigated more fully.

In May 1772 the Academy of Sciences elected, Monge to be Bossut's correspondent, At this time he became friendly with Condorcet and Vandermonde. The latter's influence was probably responsible for, two unpublished memoirs Monge wrote during this, period, on the theory of determinants and on the, knight's moves on a chessboard.

In 1775 Monge returned to infinitesimal geometry. Working on the theory of developable surfaces outlined by Euler in 1772, he applied it to the problem, of shadows and penumbrae and treated several, problems concerning ruled surfaces. A memoir composed in 1776 on Condorcet's prompting (and, reworked in 1781 on the basis of a more thorough, understanding) is of major importance, although, not for its contributions to the practical problem of, cuts and fills that served as its point of departure. Its great interest lies in its introduction of lines of, curvature and congruences of straight lines.

Although in 1776 Monge was still interested in, Lagrange's memoir on singular integrals, his predilection for mathematics was meanwhile slowly, yielding to a preference for physics and chemistry. In 1774, while traveling in the Pyrenees, he had, collaborated with the chemist Jean d'Arcet in making, altitude measurements with the aid of a barometer. Having some instruments at his disposal in Ḿzíres and working with Vandcrmonde and Lavoisier during, his stays in Paris, Monge carried out experiments, on expansion, solution, the effects of a vacuum, and, other phenomena; acquired an extensive knowledge, of contemporary physics; and participated in the, elaboration of certain theories, including the theory, of caloric and triboelectricity.

In 1777 Monge married Catherine Huart. They had, three daughters, the two elder of whom married two, former members of the National Convention, N.L-JL, Marey and J. Eschasśriaux: the two present branches of Mongers descendants are their issue.

During the period 1777–1780 Monge was interested, primarily in physics and chemistry and arranged for, a well-equipped chemistry laboratory to be set up at, the ́eole du Genie, Moreover, having for some time, been responsible for supervising the operation of a, forge belonging to his wife, he had become interested, in metallurgy.

His election to the Academy of Sciences as *adjoint ǵoḿire* in June 1780 altered Monge's life, obliging, him to stay in Paris on a regular basis. Thus for some, years he divided his time between the capital and Ḿzíres, In Paris he participated in the Academy's, projects and presented memoirs on physics, chemistry, and mathematics. He also substituted for Bossut in, the latter's course in hydrodynamics (created by, A.-R.-J, Turgot in 1775) and in this capacity trained, young disciples such as S. F. Lacroix and M. R. de Prony. At Ḿzíres, where he arranged for a, substitute to give some of his courses—although he, kept his title and salary—Monge conducted research, in chemistry. In June – July 1783 he synthesized water. He then turned his attention to collecting stores of, hydrogen and to the outer coverings of balloons. Finally, with J. F. Clouet he succeeded in liquefying, sulfur dioxide.

In October 1783 Monge was named examiner of, naval cadets, replacing Bevout. He attempted to, reconcile his existing obligations with the long, absences required by this new post, but it proved, to be impossible. In December 1784 he had to give, up his professorship at Ḿíres, thus leaving the, school at which he had spent twenty of the most, fruitful years of his career. From 1784 to 1792 Monge, divided his time between his tours of inspection of, naval schools and his stays in Paris, where he continued to participate in the activities of the Academy, and to conduct research in mathematics, physics, and chemistry. A list of the subjects of his communications to the Academy attests to their variety: the, composition of nitrous acid, the generation of curved surfaces, finite difference equations, and partial, differential equations (1785); double refraction and, the structure of Iceland spar, the composition of, iron, steel, and cast iron, and the action of electric, sparks on carbon dioxide gas (1786); capillary, phenomena (1787); and the causes of certain meteorological phenomena; and a study in physiological, optics (1789).

Meanwhile, with other members of the Academy, Monge assisted Lavoisier in certain experiments. For example, in February 1785 he participated in, the analysis and synthesis of water. In fact, he was, one of the first to accept Lavoisier's new chemical, theory. After having collaborated with Vandermonde, and Berthollet on a memoir on "iron considered in, its different metallurgical states" (1786), he participated in several investigations of metallurgy in, France. In 1788 he joined in the refutation, instigated, by Lavoisier, of a treatise by the Irish chemist Kirvan, who was a partisan of the phlogiston theory. That, Monge was among the founders of the *Annales de, chimie* testifies to his standing in chemistry. During, this period Monger's position as naval examiner, obliged him to write a course in mathematics to, replace B́zout's. Only one volume was published, *Traiťĺmentaire de statique* (1788).

When the Revolution began in 1789, Monge was, among the most widely known of French scientists. A very active member of the Academy of Sciences, he had established a reputation in mathematics, physics, and chemistry. As an examiner of naval, cadets he directed a branch of France's military, schools, which were then virtually the only institutions, offering a scientific education of any merit. This, position also placed him in contact, in each port, he visited, with bureaucracy that was soon to come, under his administration. It also enabled him to visit, iron mines, foundries, and factories, and thus to, become an expert on metallurgical and technological, questions. Furthermore, the important reform of, teaching in the naval schools that he had effected, in 1786 prepared him for the efforts to renew scientific, and technical education that he undertook during the, Revolution.

Although Monge was a resolute supporter of the, Revolution from the outset, his political role remained, discreet until August 1792. He joined several revolutionary societies and clubs but devoted most of, his time to tours of inspection as examiner of naval, cadets and to his functions as a member of the, Academy, particularly to the work of the Academy's, Commission on Weights and Measures.

After the fall of the monarchy on 10 August 1792, a government was created to carry on the very difficult struggle imposed on the young republic by, adherents of the *ancien ŕgime*. On the designation of, the Legislative Assembly, Monge accepted the post, of minister of the navy, which he held for eight, months. Although not outstanding, his work showed, his desire to coordinate all efforts to assure the nation's, survival and independence. His politics, however, were judged by some to be too moderate; and attacked, from several sides and exhausted by the incessant, struggle he had to wage, he resigned on 10 April 1793. Henceforth he never played more than a minor, political role. A confirmed republican, he associated, with Jacobins such as Pache and Hassenfratz; but, he never allied himself with any faction or participated, in any concrete political action. On the other hand, he was an ardent patriot, who placed all his energy, talent, and experience in the service of the nation, and he played a very important role in developing the, manufacture of arms and munitions, and in establishing a new system of scientific and technical, education.

Monge resumed his former activities for a short, time; but after the suppression of the Academy of, Sciences on 8 August 1793 his work came under the, direct control of the political authorities, especially, of the Committee of Public Safety. From the beginning, of September 1793 until October 1794, he took part, in the work of the Committee on Arms. He wrote, with Vandermonde and Berthollet, a work on the, manufacture of forge and case-hardened steels, drew, up numerous orders concerning arms manufacture, for Lazare Carnot and C. L. Prieur, supervised Paris, arms workshops, assembled technical literature on, the making of cannons, gave "revolutionary courses" on this latter subject (February-March 1794), and, wrote an important work on it. He also was involved, in the extracting and refining of saltpeter and the, construction and operation of the great powderworks, of Paris. In addition, he participated in the development of military balloons.

Monge also engaged in tasks of a different sort. After the suppression of the Academy he joined the, Socíť Philomatique; participated in the work of the, Temporary Commission on Weights and Measures, which continued the projects of the Academy's, commission; and took part in the activities of the, Commission on the Arts, which was responsible for, preserving the nation's artistic and cultural heritage. He was also active in the projects for educational, reform then under discussion. His experience at the École de Ḿzières and in the naval schools explains, the special interest that the renewal of scientific and, technical instruction held for him. At the elementary, level, he prepared for the department of Paris a plan for schools for artisans and workers that the, Convention adopted on 15 September 1793 but, rejected the next day. At a more advanced level, he, was convinced of the value of creating a single national, school for training civil and military engineers. Consequently, when he was appointed by the Convention (11 March 1794) to the commission responsible for establishing an École Centrale des Travaux, Publics, he played an active role in its work. The, memoir that Fourcroy prepared in September 1794 to, guide the first steps of the future establishment ("D́veloppements sur l'enseignement …") shows, the influence of Monge's thinking, which derived, from his experience at Ḿzières. Appointed instructor, of descriptive geometry on 9 November 1794, Monge, supervised the operation of the training school of, the future *chefs de brigade*, or foremen, taught descriptive geometry in "revolutionary courses" designed to, complete the training of the future students, and was, one of the most active members of the governing, council. After a two-month delay caused by political, difficulties, the school—soon to be called the École, Polytechnique—began to function normally in June 1795. Monge's lectures, devoted to the principles and, applications of infinitesimal geometry, were printed, on unbound sheets; these constituted a preliminary, edition of his *Application de l'analyse à la ǵoḿtrie*.

Monge was also one of the professors at the, ephemeral École Normale de l'An III. From 20 January to 20 May 1795 this school brought together in, Paris 1,200 students, who were to be trained to teach, in the secondary schools then being planned. The, lectures he gave, assisted by his former student S.-F. Faeroix and by J. Fournier, constituted the first, public course in

descriptive geometry. Like those of, the other professors, the lectures were taken down by, stenographers and published in installments in the *Journal des śances deścoles normales*.

Monge, who regretted the suppression of the, Academy of Sciences, actively participated in the, meetings held from December 1795 to March 1796 to prepare its rebirth as the first section of the Institut, National, created by the Convention on 26 October 1795. But just when Monge's activities seemed to be, returning to normal, events intervened that prevented, this from happening.

Monge was named, along with his friend Berthollet, one of the six members of the Commission des, Sciences et des Arts en Italie, set up by the Directory, to select the paintings, sculptures, manuscripts, and, valuable objects that the victorious army was to, bring back. He left Paris on 23 May 1796. His mission, took him to many cities in northern and central, Italy, including Rome, and allowed him to become, friendly with Bonaparte. At the end of October 1797 Monge returned to Paris, officially designated, with, General Fouis Berthier, to transmit to the Directory, the text of the Treaty of Campoformio.

Immediately after returning, Monge resumed his, former posts, as well as a new one, that of director, of the École Polytechnique. But his stay in Paris, was brief; at the beginning of February 1798 the, Directory sent him back to Rome to conduct a, political inquiry. While there, Monge took an active, interest in the organization of the short-lived Republic, of Rome. The following month, at the request of, Bonaparte, he took part in the preparations for the, Egyptian expedition. Although reluctant at first, he finally agreed to join the expedition. His boat, left Italy on 26 May 1798, joining Bonaparte's, squadron two weeks later. Monge arrived in Cairo, on 21 July and was assigned various administrative, and technical tasks. As president of the Institut, d'Égypte, created on 21 August, he played an important role in the many scientific and technical, projects undertaken by this body. He accompanied, Bonaparte on a brief trip in the Suez region, on the, disastrous Syrian expedition (February-June 1799), and, after another brief stay in Cairo, on his return, voyage to France (17 August-16 October). During this, period of three and a half years, in which he was for, almost the whole time away from France, Monge's, correspondence and communications to the Institut, d'Égypte show that he was working on new chapters, of his *Application de l'analyse à la ǵoḿtrie*. Moreover, the observation of certain natural phenomena, such as mirages, and the study of certain techniques, including metallurgy and the cultivation of the vine, provided him with fruitful sources for thought. Meanwhile, at ihe request of his wife and without, his knowledge, his *Ǵoḿtrie descriptive* was published, in 1799 by his friend and disciple J. N. Hachette, who limited himself to collecting Monge's École, Normale lectures previously published in the Śances.

On his return to Paris, Monge resumed his duties, as director of the École Polytechnique but relinquished, them two months later when, following the *coup d'etat* of 18 Brumaire, Bonaparte named him senator, for life. By accepting this position Monge publicly, attached himself to the Consulate. Although this, decision may seem to contradict his republican convictions and revolutionary faith, it can be explained, bv his esteem for and admiration of Bonaparte and, by his dissatisfaction with the defects and incompetence of the preceding regime. Dazzled by Napoleon, Monge later rallied to the Empire with the same, facility and accepted all the honors and gifts the, emperor bestowed upon him: grand officer of the Legion of Honor in 1804, president of the Senate, in 1806, count of Ṕluse in 1808, among others.

Monge had to divide his time among his family, his teaching of infinitesimal geometry at the École, Polytechnique, and his obligations as a member of, the Academy of Sciences and of the Cunseil de, Perfectionnement of the École Polytechnique, and, his duties as a senator. Further tasks were soon added. He was founder of the Socíĺ d'Encouragement pour, l'Industrie Nationale and vice-president of the commission responsible for supervising the preparation, and publication of the material gathered on the, Egyptian expedition, *Description de l'Égypte*. Even, though his duties as senator took him away on several, occasions from his courses at thécole Polytechnique, he maintained his intense concern for the school. He kept careful watch over the progress of the, students, followed their research, and paid close, attention to the curriculum and the teaching.

Most of Monge's publications in this period were, written for the students of the École Polytechnique. The wide success of the *Ǵoḿtrie descriptive* was, responsible for the rapid spread of this new branch, of geometry both in France and abroad. It was, reprinted several times; the edition of 1811 contained a, supplement by Hachette; and the fourth, posthumous, edition, published in 1820 by Barnab́ Brisson, included four previously unpublished lectures on, perspective and the theory of shadows.

In 1801 Monge published *Feullies d'analyse appliqúe à la ǵoḿtrie*, an expanded version of his, lectures on infinitesimal geometry of 1795. In 1802, working with Hachette, he prepared a brief exposition, of analytic geometry that was designed to replace, the few remarks on the subject contained in the *Feuilles*. Entitled *Application de l'alg̀bre ̀ l'analyse*, it was published separately in 1805; in 1807 it became, the first part of the final version of *Feuilles d'analyse*, now entitled *Application de l'analyse ̀ la ǵoḿtrie*. This larger work was republished in 1809 and again, in 1850 by J. Liouville, who appended important, supplements.

Aside from new editions of the *Trait́ Ĺmentaire, de statique*, revised by Hachette beginning with the, fifth edition (1810), and some physical and technical, observations made in Italy and Egypt and published, in 1799, Monge's other publications during this, period dealt almost exclusively with infinitesimal and, analytic geometry. For the most part they were, gradually incorporated into successive editions of, his books. His production of original scientific work, began to decline in 1805.

A decline likewise occurred in Monge's other, activities. Suffering from arthritis, he stopped teaching, at the École Polytechnique in 1809, arranging for, Arago to substitute for him and then to replace him. Although he wrote a few more notes on mathematics, and several official technical reports, his creative, period had virtually come to an end. In November 1812, overwhelmed by the defeat of the Grande, Arńe, he suffered a first attack of apoplexy, from, which he slowly recovered. At the end of 1813 he, was sent to his senatorial district of Liège to organize, its defenses but fled a few weeks later before the, advancing allied armies. Absent from Paris at the, moment of surrender, he did not participate in the, session of 3 April 1814, in which the Senate voted the, emperor's dethronement. He returned shortly afterward and resumed a more or less normal life. In 1815, during the [Hundred Days], he renewed his, contacts with Napoleon and even saw him several, times after Waterloo and the abdication. In October 1815, fearing for his freedom, Monge left France, for several months. A few days after his return to, Paris, in March 1816, he was expelled from the, Institut de France and harassed politically in other, ways. Increasingly exhausted physically, spiritually, and intellectually, he found his last two years especially painful. Upon his death, despite government, opposition, many current and former students at the École Polytechnique paid him tribute. Throughout the, nineteenth century mathematicians acknowledged, themselves as his disciples or heirs.

Scientific Work . Monge's scientific work encompasses mathematics (various branches of geometry and, mathematical analysis), physics, mechanics, and the, theory of machines. His principal contributions to, these different fields will be discussed in succession, even though his mathematical work constitutes a, coherent ensemble in which analytic developments, were closely joined with material drawn from pure, descriptive, analytic, and infinitesimal geometry, and even though his investigations in physics, mechanics, and the theory of machines were also, intimately linked.

Descriptive and Modern Geometry. Elaborated, during the period 1766–1775, Monge's important, contribution is known from his *Ǵomtrie descriptive*, the text of his courses at the École Normale de l'An III (1795), and from the manuscript of his lectures given, that year at the École Polytechnique. Before him, various practitioners, artists, and geometers, including, Albrecht Dürer, had applied certain aspects of this, technique. Yet Monge should be considered the, true creator of descriptive geometry, for it was he who, elegantly and methodically converted the group of, graphical procedures used by practitioners into a, general uniform technique based on simple and rigorous geometric reasoning and methods. Within a few years this new discipline was being taught, in French scientific and technical schools and had, spread to several other Continental countries.

Monge viewed descriptive geometry as a powerful, tool for discovery and demonstration in various, branches of pure and infinitesimal geometry. His, persuasive example rehabilitated the study and use of, pure geometry, which had been partially abandoned, because of the success of Cartesian geometry. Monge's, systematic use of cylindrical projection and, more, discreetly, that of central projection, opened the, way to the parallel creation of projective and modern geometry, which was to be the work of his disciples, particularly J.-V. Poncelet. The definition of the, orientation of plane areas and volumes, the use of the, transformation by reciprocal polars, and the discreet, introduction in certain of his writings of imaginary, elements and of elements at infinity confirms the importance of his role in the genesis of modern geometry.

Analytic and Infinitesimal Geometry. Analytic and, infinitesimal geometry overlap so closely in Monge's, work thai it is sometimes difficult to separate them. Whereas from 1771 to 1809 he wrote numerous, memoirs on the infinitesimal geometry of space, it, was not until 1795, in his lectures at the École Polytechnique, that he specifically developed analytic, geometry.

Nevertheless, even in his earliest works, Monge, sought to remedy the chief weaknesses of analytic, geometry, although this discipline was then for him, only an auxiliary of infinitesimal geometry. Rejecting, the restrictive Cartesian point of view that was still dominant, he considered analytic geometry as an, autonomous branch of mathematics, parallel to pure, geometry and independent of it. Consonant with this, approach, as earlv as 1772 and at the same time as, Lagrange, Monge systematically introduced into, the subject the elements defined by first-degree, equations (straight lines and planes) that had previously not been part of it. He also solved the basic, problems posed by this extension. Parallel with this, endeavor, he sought, following Clairaut and Euler, to make up for the long delay in the development of, three-dimensional anal)tic geomctrv. In addition, Monge introduced an absolute symmetry into the use, of the coordinate axes. He showed great analytic, virtuosity in his calculations, some of which display, except for the symbolism, a skillful handling of determinanls and of certain algorithms of vector calculus. His ability in this regard vcrv earl) allowed him to, establish the foundations of the geometry of the, straight line (in Plücker's sense), which he systematized, in 1795.

The first two editions (1795 and 1801) of Monge's, course in "analysis applied to geometry" at the École, Polytechnique contain as an introduction a brief, statement of the principles and fundamental problems, of this renewed analytic geometry, which was soon, taught in upper-level French schools. With his disciple, J. N. Haehette, Monge published in 1802 an important, memoir, "Application de l'algèbre` la ǵomtrie," which completed the preceding study, notably, regarding the theory of change of coordinates and, the theory of quadrics. In 1805 Monge collected these, various contributions to analytic geometry in a, booklet entitled *Application de l'algèbre` la ǵomtrie*, which in 1807 became the first part of his great, treatise *Application de l'analyse` la ǵometrie*. The, many articles that Monge and his students devoted, to individual problems of analytic geometry (change, of coordinates, theory of conies and quadratics, among others) in the *Journal de l'École polytechnique* and in the *Correspondance sur l' École polytechnique* attest to the interest stimulated by the discipline's, new orientation.

Throughout his career infinitesimal geometry remained Monger's favorite subject. Here his investigations were directed toward two main topics: families of surfaces defined by their mode of generation, which he examined in connection with the, corresponding partial differential equations, and the, direct study of the properties of surfaces and space curses. Since the first

topic is discussed below, only, the principal research relating to the second topic, will be presented here. In 1769 Monge defined the, evolutcs of a space curve and showed that these, curves are the geodesics of the developable envelope, of the family of planes normal to the given curve. In 1774, after having returned to this question in a, memoir presented in 1771, Monge completed the, study of developable surfaces outlined by Euler. Coneurrentiv utilizing geometric considerations and, analytic arguments, he established the distinction, between ruled surfaces and developable surfaces; gave simple criteria for judging, from its equation, whether a given surface is developable; applied these, results to the theory of shadows and penumbrae; and solved various problems concerning surfaces. In particular, he determined by means of descriptive, geometry the ruled surface passing through three, given space curves. Still more important is the memoir, on cuts and fills, of which Monge made two drafts (1776 and 1781). The point of departure was a, technical problem: to move a certain quantity of, earth, determining the trajectory of each molecule, in such a way that the total work done is a minimum. Through repeated schcmatizalions he derived the formulation a question concerning the theory of surfaces that he examined very generally, introducing such important notions as the congruence of straight lines, line of curvature, normal, and focal surface. This memoir served as a starting point for several of Monge's later works, as well as for important investigations by Malus in geometrical optics and by Dupin in infinitesimal geometry.

Several memoirs written between 1783 and 1787 contain numerous studies of families of surfaces, and some new results relating to the general theory, of surfaces and to the properties of certain space, curves.

In *Feuilles d'analyse appliquéè la gáoḿtrie* (1795 and 1801) Monge assembled, along with general, considerations regarding the theory of surfaces and, the geometric interpretation of partial differential, equations, monographs on about twenty families, of surfaces defined by their mode of generation. *Application de l'analyse` la ǵoḿtrie* (1807) includes, some supplementary material, notably attempts to, find families of surfaces when one of the nappes of, their focal surface is known. The manuscript of, Monge's course for 1805–1806 also contains important, additional findings (transformations by reciprocal, polars, conoids, etc.). The richness and originality in, Monge's lectures, qualities evident in this manuscript, and confirmed by the testimony of former students, explain whs so many French mathematicians can be, considered his direct followers. Among them we may, cite Tinseau and Meusnier at the École de Ḿzíres, Lacroix, Fourier, and Hachette at the École Normale, and Lancret, Dupin, Livet, Brianehon, Malus, Poncelet, Chasles, Laḿ, and still others at the École, Polytechnique. Certain aspects of their writings show the direct influence of Monge, who thus emerges as, a true *chef d'ècole*.

Mathematical Analysis. The theory of partial, differential equations and that of ordinary differential, equations occupies—often in close connection with infinitesimal geometry— an important place in Monge's, work. Yet, despite his great mastery of the techniques, of analysis and the importance and originality of, certain of the new methods he introduced, his writings, in this area are sometimes burdened by an excessive, number of examples and are blemished by insufficiently rigorous argumentation.

As early as 1771 the memoirs presented to the, Academy and the letters to Condorcet reflect two of, the guiding ideas of Monge's work: the geometric, determination of the arbitrary function involved in, the general solution of a partial defferential equation, and the equivalence established between the classification of families of surfaces according to their, mode of generation and according to their partial, differential equation. He returned to these questions, several times between 1771 and 1774, developing, many examples and extending his study to finite, difference equations. Also, in the memoir of 1775 on developable surfaces he discussed the partial, differential equation of developable surfaces and that, of ruled surfaces.

From 1773 to 1786 Monge carried out new research, in this area. In seven memoirs of varying importance, he presented flawlessly demonstrated results, and a, progressively elaborated outline of very fruitful new, methods. His essentially geometric inspiration drew, upon the ideas of his earliest papers and on the, division, introduced by Lagrange, of the integral, surfaces of a first-order partial differential equation, into a complete integral, a general integral, and a, singular integral. By means of his theory of characteristics Monge gave a geometric interpretation of, the method of the variation of parameters. In addition, he introduced such basic notions as characteristic, curve, integral curve, characteristic developable, trajectory of characteristics, and characteristic cone. Monge was also interested in second-order partial, differential equations.

In particular he created the theory of "Monge, equations"—equations of the type, $Ar + Bs + Ct + D = 0$. where A, B, C, D are functions of x, y, z, p, q, and, where p, q, r, s, and t have the classical meanings—and solved the equation of minimal surfaces. Investigating the theory of partial differential equations from, various points of view, Monge—despite some errors, and a somewhat disorganized and insufficiently, rigorous presentation—contributed exceptionally, fruitful methods of approaching this topic. For, example, he demonstrated the geometric significance, of the total differential equations that do not satisfy, the condition of integrability, thus anticipating J. F. Pfatrs treatment of the question in 1814-1815. Monge also introduced contact transformations, the, use of which was generalized by Lie a century later. In addition he determined the partial dilVereniial, equations of many families of surfaces and perfected, methods of solving and studying various types of, partial dilVereniial equations.

Monge resumed his research in this area in 1795–1796 and in 1803–1807, when he completed his courses, in infinitesimal geometry at the École Polyteehnique, with a view toward their publication. He perfected, the theories sketched in 1783–1786, corrected or made, certain arguments more precise, and studied the, area of their application.

Mechanics, Theory of Machines, and Technology. From the time he came to Ḿzières, Monge was, interested in the structure, functioning, and effects, of machines; in the technical and industrial problems, of fortification and construction; and in local

industry, particularly metallurgy. He held that technical, progress is a key factor governing ihe happiness of, humanity and depends csscntiail) on the rational, application of theoretical science. His interest in, physics, mechanics, and the theory of machines, derived in part from his view that they are the principal, factors of industrial progress and, therefore, of social, progress.

Monge discussed the theory of machines in his, course in descriptive geometry at the École Polytechnique (end of 1794). His ideas, employed by, Hachette in *Traiťĺmentaire des machines* (1809), were derived from the principle that the function of, every machine is to transform a motion of a given, type into a motion of another type. Although this, overly restrictive conception has been abandoned, it played an important role in the creation of the, theory of machines in the nineteenth century.

Monge's *Traiťĺmentaire de statique* (1788) was, a useful textbook, and its successive editions recorded, the latest developments in the subject, for example, the theory of couples introduced by Poinsot. The fifth, edition (1810) included important material on the, reduction of an arbitrary system of forces to two, rectangular forces.

The unusual experience that Monge had acquired, in metallurgy was frequently drawn upon by the, revolutionary government and then by Napoleon.

Physics and Chemistry. Although the details regarding Monge's contributions to physics are poorly, known, because he never published a major work in this feld, his reputation among his contemporaries, was solid. His main contributions concerned caloric, theory, acoustics (theory of tones), electrostatics, and, optics (theory of mirages).

In 1781 Monge was selected to be editor of the *Dictionnaire de physique of the Encycloṕdie ́mthodique*. He did not complete this task, but he did write, certain articles.

His most important research in chemistry dealt, with the composition of water. As early as 1781 he, effected the combination of oxygen and hydrogen in, the eudiometer, and in June-July and October 1783 he achieved the synthesis of water—at the same time, as Lavoisier and independently of him. Although, Monger's apparatus was much simpler, the results of, his measurements were more precise. On the other, hand, his initial conclusions remained tied to the, phlogiston theory, whereas Lavoisier's conclusions, signaled the triumph of his new ehemistry and the, overthrow of the traditional conception of the, elementary nature of water. Monge soon adhered to, the new doctrine. In February 1785 he took part in, the great experiment on the synthesis and analysis, of water; he was subsequently an ardent propagandist, for the new chemistry and actively participated in, its development.

In the experimental realm, in 1784 Monge achieved, in collaboration with Clouet, the first liquefaction of, a gas, sulfurous anhydride (sulfur dioxide). Finally, between 1786 and 1788 Monge investigated with, Berthollet and Vandermonde the principles of metallurgy and the composition of irons, cast metals, and, steels. This research enabled them to unite previous, findings in these areas, to obtain precise theoretical, knowledge by means of painstaking analyses, and, to apply this knowledge to the improvement of various, techniques.

BIBLIOGRAPHY

I. Orginal Works. A partial list of Monge's works is, given in Poggendorfff, II, 184–186, More precise and more, complete bibliographies are in L. de Launay, *Un grand, franeais: Monge…* (Paris, 1933), pp. 263–276, which, includes a list of MSS and portraits; and in R. Taton, *L'oeuvre scientifique de Gaspard Monge* (Paris, 1951), pp. 377-393, which contains lists of MSS, scientific correspondence, and memoirs presented to the Acad́mie des, techniques.

Works that were published separately are *Traiť áĺmentaire de statique* (Paris, 1788; 8th ed., 1846), trans, into, Russian, German, and English; *Avis aux ouvriers en fer, sur la fabrication de l'acier* (Paris, 1794), written with, Berthollet and Vandermonde; *Description de l'art de fabriquer les canons…* (Paris, 1794); *Ǵometric descriptive …* (Paris, 1799), a collection in 1 vol. of the lectures given in 1795 at the École Normale de l'An III and published in, the *Śances des ́coles normales …* (7th ed., Paris, 1847; repr. 1922), trans, into German, Italian, English, Spanish, and Russian; *Feuilles d' analyse appliqúe ` la ǵoḿtrie…* (Paris, 1801), a collection, with various additions, of lectures given at the École Polytechnique, which were published on separate sheets in 1795; *Application de l'algèbrè la ǵoḿtrie* (Paris, 1805), written with Hachette; and *Application de l'analyse a la ǵoḿtrie* (Paris, 1807), a new, ed. of the *Feuilles d' analyse appliqúe ` la ǵoḿtrie*, preceded, with special pagination, by *Application de l'algèbre, a la ǵoḿtrie* (new ed., 1809; 5th ed., 1850), J. Liouville, ed,. with several appendixes, including Gauss's *Disquisitiones circa superficies curvas*; trans, into Russian, with, commentary (Moscow, 1936).

II.Secondary Literaturte. The most recent biography, of Monge is P.-V. AuBry, *Monge, le savant ami de Napoĺcon: 1746–1818* (Paris, 1954). The most complete study of his scientific work is R. Taton, *L'oeuvre scientifique de, Gaspard Monge* (Paris, 1951).

A few older monographs are still worth consulting, in, particular the following, listed chronologically: C. Dupin, *Essai historique sur les services et les travaux scientifiques, de Gaspard Monge* (Paris, 1819; repr. 1964); F. Arago, "Biographie de Gaspard Monge …" in *Oeuvres de Francois Arago, Notices biographiques*, II (Paris, 1853; repr. 1964), 426–592, trans. into

German, English, and, Russian; E. F. Jomard, *Souvenirs sur Gaspard Monge et, ses rapports avec Napoĺon …* (Paris, 1853); L. de Launay, *Un grand franṣais: Monge, fondateur de l'École polytechnique* (Paris, 1933); and E. Cartan, *Gaspard Monge, sa vie, son oeuvre* (Paris, 1948).

A very complete bibliography of other works dealing, with Monge is given in R. Taton, L'oeuvre scientifique, pp. 396–425. This list should be completed by some more, recent studies, listed chronologically: C. Bronne, "La, śnatorerie de Monge," in *Bulletin de la Socí́beige, d'etudes napoĺoniennes*, no. 9 (1953), 14–19; Y. Laissus, "Gaspard Monge et l'exṕdition d'Égypte (1798–1799)," in *Revue de synthèse*. **81** (1960), 30–336; R. Taton, "Quelques lettres scientifiques de Monge," in *84 Congŕs, des socítes savantes, Dijon, Section des sciences* (Paris, 1960), pp. 81–86; J. Duray, "Le śnateur Monge au ch̀teau de Seraing (près de Liège)," in *Bulletin de la Socí́belge d'etudes napoĺoniennes*, no. 36 (1961), 5–17; A. Birembaut, "Deux lettres de Watt, ṕre et fils, à Monge," in *Annales historiques de la Ṙvolution fraṇaise*, **35** (1963), 356–358; J. Booker, "Gaspard Monge and His Effect on, Engineering Drawing and Technical Education," in *Transactions of the Newcomen Society*, **34** (1961–1962), 15–36; and R. Taton, "La première note math́matique de, Gaspard Monge," in *Revue d'histoire des sciences*, **19** (1966), 143–149.

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