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(*b*. Nice, France, 29 April 1876; *d*. Paris, France, 22 January 1975)

mathematics.

Paul Montel was the son of Aristide and Anais (Magiolo) Montel. His father was a photographer. He was educated in the Lycée of Nice, and in 1894 he was admitted to the École Normale Supérieure in Paris. After graduation in 1897 he taught classes in several provincial lycées preparing students for the competitive entrance examinations to the École Polytechnique and other engineering schools. He enjoyed teaching and liked a quiet life with plenty of leisure to devote to literature and travel, and he might have remained all his life a lycée professor if his friends had not urged him not to waste his talents and to start writing a thesis. He therefore returned to Paris to work on a doctorate, which he obtained in 1907. He did not become a university professor in Paris until 1918, occupying in the interim several teaching jobs in lycées and technical schools. During the German occupation he was dean of the Faculty of Science, and he was able to uphold the dignity of the French university in spite of the arrogance of the occupiers and the servility of their collaborators. Montel retired in 1946. He married late in life and had no children.

Most of Montel's mathematical papers are concerned with the theory of analytic functions of one complex variable, a very active field among French mathematicians between 1880 and 1940. The idea of compactness had emerged as a fundamental concept in analysis during the nineteenth century: provided a set is bounded in R^n, it is possible to define for any sequence (x_n) of points of the set a subsequence (x_m) which converges to a point of R^n (the Bolzano-Weierstrass theorem). Riemann had sought to extend this extremely useful property to sets *E* of functions of real variables, but it soon appeared that boundedness of *E* was not sufficient.

Around 1880 G. Ascoli introduced the additional condition of equicontinuity of *E*, which implies that *E* has again the Bolzano-Weierstrass property. But at the beginning of the twentieth century Ascoli's theorem had very few applications, and it was Montel who made it popular by showing how useful it could be for analytic functions of a complex variable. His fundamental concept is what he called a normal family, which is a set *H* of functions defined in a domain $D \subset C$, taking their values in the Riemann sphere *S* and meromorphic in *D*, and satisfying the following condition: from any sequence of functions of *H* it is possible to extract a subsequence that, in every compact subset of *D*, converges uniformly either to a holomorphic function or to the point ∞ of *S*.

Montel's central observation is that if *H* consists of uniformly bounded holomorphic functions in *D*, it is a normal family; this is a consequence of the Cauchy integral and of Ascoli's theorem. From this criterion follow many others; for instance, if the values of the functions of a set *H* belong to a domain Δ that can be mapped conformally on a bounded domain, then *H* is a normal family. This is the case in particular when Δ is the complement of a set of two points in the complex plane **C** .

Montel showed how the introduction of normal families may bring substantial simplifications in the proofs of many classical results of function theory, such as the mapping theorem of Riemann and Hadamard's characterization of entire functions of finite order. An ingenious application is to the proof of Picard's theorem on essential singularities: suppose *O* is an essential singularity of a function *f* holomorphic in Δ: $O|z| \leq 1$. Then Picard's theorem asserts that $f(z)$ takes on all finite complex values, with one possible exception, as *z* ranges through Δ. It can be proved by observing that if there are two values that *f* does not take in Δ, then the family of functions $f_n(z) = f(z/2^n)$ in the ring Γ : $\frac{1}{2} \leq |z|1$ would be a normal family, and there would be either a subsequence (f_m) with $|f_m(z)| \leq M$ in Γ, or a subsequence with $|f_m(z)| \geq 1/M$ in Γ, contradicting the assumption that *O* is an essential singularity of *f*.

Other applications made by Montel and his students concern univalent and multivalent functions and algebroid functions. He also investigated what he called quasi-normal families *H*, which are such that in the domain of definition *D* each point has a neighborhood in which *H* is a normal family, with the exception of a finite number of "irregular" points; the consideration of quasi-normal families leads to other applications to complex function theory.

Montel was also interested in the relations between the coefficients of a polynomial and the location of its zeros in the complex plane. For instance, if in a polynomial

$$1 + a_1x + \cdots + a_{n-1}x^{p-1} + x^p + a_{n1}x^{n1} + \cdots + a_{n1}x^{n1}$$

the number p and the number k are given (but the a_j the n_i are arbitrary), then the polynomial always has a root of absolute value at most

Montel often was invited to lecture in countries where most mathematicians understood French, such as Belgium and Egypt, and in South America. He was especially honored in Rumania, which he visited often and where he had several students. He was the recipient of many honors and was elected a member of the French Academy of Sciences in 1937.

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