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(*b*. Darmstadt, Germany, 10 January 1905; *d*. Frankfurt, Germany, 26 November 1977)

mathematics.

Ruth Moufang was the younger daughter of Dr. Eduard Moufang, an industrial chemist, and his wife, Else Fecht Moufang. Her interest in mathematics was first stimulated at the Realgymnasium in Bad Kreuznach, which she attended from 1921 to 1924. From 1925 to 1930 she studied mathematics at the University of Frankfurt. In 1929 she took her techer's examination, and in October 1931 she received her Ph. D. with a dissertation on projective geometry supervised by Max Dehn. She held a fellowship in Rome during 1931 and 1932, and the following academic year a teaching assignment at the University of Königsberg.

Moufang then returned to Frankfurt, where she held teaching assignments from 1934 to 1936. During this period she continued working on the geometry of the projective plane, and in 1936 she completed her habilitation thesis to qualify as a university lecturer. By this time, however, the Nazi government had made a distinction between the habilitation and the *venia legendi* (which conferred the right to teach). On 9 February 1937 Moufang became the third woman in Germany to receive the habilitation in mathematics. A month later, however, the Ministry of Education informed her that she could not receive a *Dozentur*. A *Dozent* had to qualify as a leader of an almost exclusively male student body, argued the ministry, and "in the future the preconditions for the fruitful activity of a female *Dozent* are lacking."

With the help of a friend. Moufang found employment as an industrial mathematician at the Krupp research laboratories in Essen; she worked there from November 1937 to August 1946, during which time she authored and coauthored several papers, most of them on applied elasticity theory. In 1946 Moufang was asked to return to the University of Frankfurt, where she was given the *veina legendi*. There she held a lecturer's position until her appointment as associate professor in December 1947; she became full professor in February 1957. In the years after the war she published almost nothing, although she was a successful teacher.

Moufang's work from 1931 to 1937 was the main starting point for a new specialty in mathematics, the study of projective planes, in which geometrical and algebraical structures are closely interrelated. At the turn of the century David Hilbert had shown the existence of non-Desarguian planes. With her dissertation of 1931 Moufang started the systemic study of such planes. Her adviser, Max Dehn, had posed the problem of surveying the interdependence of closure theorems like Desargues's in planes. Such theorems state the closure of a geometric configuration if certain incidences of points and lines are given. In the case of a plane generated by four points, Moufang showed that all closure theorems can be derived from one special case of Desargues's theorem in which the vertices of one of the perspective triangles lie on the sides of the other.

In 1932 Moufang studied the plane generated by five points and showed the equivalence of the above theorem, which she called D_9, with the theorem of the complete quadrilateral. The latter can be used in Coordinatization to deduce all algebraic laws for a field except associativity. This is a special case of what is now called a "Moufang plane." In two further papers (1933, 1934) she took up the concept of an alternative division ring (introduced by Max Zorn in 1930), an algebraic structure in which all laws for a field hold except the associative law, for which a weaker "alternation law" is valid. Moufang proved the fundamental theorem for Moufang planes: that if D_9 or, equivalently, the theorem of the complete quadrilateral holds in a projective plane, then and only then can it be coordinatized by an alternative division ring (of characteristic not 2). The main theorem in this field was then proved by Skornjakov (1950) and by Bruck and Kleinfeld (1951); that any alternative division ring of characteristic other than 2 is either associative or a Cayley-Dickson algebra over its center. In 1934 Moufang also studied the multiplicative structure of alternative division rings, which led to what today is called a "Moufang loop."

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II. Secondary Literature. A brief biography and description of Moufang's work, including a list of publications (incomplete in applied mathematics), is in Bhama Srinivasan, "Ruth Moufang 1905–1977," in *Mathematical Intelligencer*, **6** no. 2 (1984), 51–55.

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