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(*b*. Joensuu, Finland, 22 October 1895; *d*. Helsinki, Finland, 28 May 1980)

mathematics.

Nevanlinna came from a Swedish-speaking Finnish family. His father, Otto Wilhelm Nevanlinna, was a noted mathematician and teacher; his mother was Margarete Romberg Nevanlinna. Rolf was the second of their four children.

Nevanlinna could already read and write when he entered primary school. Apparently he was so advanced over his classmates that he grew bored and left school for a year and a half. In 1904 the family moved to Helsinki, where school was more challenging and he learned German and French, starting the development of his superb gift for languages. Perhaps his best teacher was his father, who taught him mathematics and physics in secondary school.

When Nevanlinna graduated from secondary school in 1913, his chief interests were classics and mathematics. Between graduation and enrolling at university, he read the *Introduction to Higher Analysis* of Ernst Lindelöf, a cousin of his father and the outstanding scientist at Helsinki University. It kindled in him the enthusiasm for analysis that led to his life's work.

On 4 June 1919, after receiving the doctorate, Nevanlinna married his cousin Mary Elise Selin; they had four children. In 1918 he had been exempted from conscription for Mannerheim's war of liberation because of his low weight of 110 pounds, and instead served as a clerk.

In 1945, while helping to organize a chamber music society, Nevanlinna met Sinikka Kallio-Visapää, an author and translator (particularly of the works of Thomas Mann). His marriage was dissolved, and he and Sinikka were married in 1958. They had one daughter.

In 1919 Nevanlinna became a schoolteacher, since there were no jobs in the Finnish universities, and for a time joined the Salama Insurance Company, for which his brother Frithiof worked, while continuing to teach eighteen classes a week. He became a docent at the University of Helsinki in 1922 and professor in 1926.

During the 1920's Nevanlinna developed the theory of value distribution that bears his name. It is concerned with the distribution of roots of equations $f(z) = a$, or a-values. Here $f(z)$ is a function of the complex variable z, which is everywhere either differentiable or takes the value infinity so that $1/f(z)$ is differentiable. Such functions are called meromorphic. In 1880 Charles émile Picard had proved that a-values exist except for at most two values of a. For instance, $f(z) = e^z$ is never 0 or infinite, and so has no 0 values or ∞ values.

Nevanlinna turned this quantitative statement into a theory of unprecedented precision. He introduced a characteristic function $T(r, f)$, which refers to the behavior of f in a circle $|z| \le r$. For any a, Nevanlinna's first fundamental theorem states that as $r \to \infty$,

$N(r, a) + m(r, a) = T(r)$ + a bounded term, where $N(r, a)$ measures the number of a-values in $|z| < r$ and $m(r, a)$ measures the closeness of $f(z)$ to a on $|z| = r$. The second fundamental theorem says that for any q values a_1 to a_9.

$$m(r, a_1) + m(r, a_2) + \ldots + m(r, a_v) < 2T(r) + S(r),$$

Where $S(r)$ is in general small compared with $T(r)$. In particular $N(r, a)$ is never much larger than $T(r)$ and is much smaller than $\frac{1}{3}$ $T(r)$ for at most two values of a. Nevanlinna's original result was for $q = 3$. The extension to general q was suggested by J.E. Littlewood and E.F. Collingwood in 1924.

The characteristic function $T(r)$ has allowed a much closer study of analytic functions, and it has also been useful in the study of many other situations. An analysis of multiple roots of equations $f(z) = a$ and of linear combinations of meromorphic functions with few zeros and poles has proved very fruitful.

Another concept that has been very valuable is Nevanlinna's invention of harmonic measure in 1935. The theory of harmonic measure, originally published in a series of papers, was also presented in two books, *Le théor` de Picard-Borel et la théorie des fonctions méromorphes*(1929) and *Eindeutige analytische Funktionen*(1936). Although Nevanlinna continued to write

books and papers throughout his life, his reputation rests on these two works. The German book in particular contains many important concepts derived from such authors as Lars Ahlfors, Élie Cartan, and O. Frostman, as well as the Nevanlinna theory proper.

Nevanlinna traveled extensively after the age of thirty and was warmly received by mathematicians in many countries. He was visiting professor at Göttingen in 1936 and 1937 and guest professor at the Eidegenössische Technische Hochschule in Zurich from 1946 to 1973. He was rector of Helsinki University from 1941 to 1945 and president of the International Mathematical Union from 1959 to 1962. In 1948 he became one of the twelve members of the newly established Finnish Academy. He held honorary doctorates from eight universities and was an honorary or foreign member of more than a dozen scientific societies, including the Institut de France.

BIBLIOGRAPHY

I. Original Works. Important works by Nevanlinna include *Le théorème de Picard-Borel et la th´orie des fonctions méromorphes*(Paris, 1929); *Eindeutige analytische Funktionen*(Berlin, 1936; 2nd ed., 1953), translated by Phillip Emig as *Analytic Functions*(New York, 1970); and *Absolute Analysis*, Phillip Emig, trans. (Berlin and New York, 1973), written with Frithiof Nevanlinna.

II. Secondary Literature. Walter K. Hayman, “Rolf Nevanlinna,” in *Bulletin of the London Mathematical Society*, **14** (1982), 419–436, has a list of Nevanlinna’s published books and papers.

Walter K. Hayman