encyclopedia.com

Noether, Amalie Emmy | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
10-13 minutes

(*b*. Erlangen, Germany, 23 March 1882; *d*. Bryn Mawr, Pennsylvania, 14 April 1935) *Mathematics*.

Emmy Noether, generally considered the greatest of all female mathematicians up to her time, was the eldest child of Max Noether, research mathematician and professor at the University of Erlangen, and Ida Amalia Kaufmann. Two of Emmy's three brothers were also scientists. Alfred, her junior by a year, earned a doctorate in chemistry at Erlangen. Fritz, two and a half years younger, became a distinguished physicist; and his son, Gottfried, became a mathematician.

At first Emmy Noether had planned to be a teacher of English and French. From 1900 to 1902 she studied mathematics and foreign languages at Erlangen, then in 1903 she started her specialization in mathematics at the University of Göttingen. At both universities she was a nonmatriculated auditor at lectures, since at the turn of the century girls could not be admitted as regular students. In 1904 she was permitted to matriculate at the University of Erlangen, which granted her the Ph.D., *summa cum laude*, in 1907. Her sponsor, the algebraist Gordan, strongly influenced her doctoral dissertation on algebraic invariants. Her divergence from Gordan's viewpoint and her progress in the direction of the "new" algebra first began when she was exposed to the ideas of Ernst Fischer, who came to Erlangen in 1911.

In 1915 Hilbert invited Emmy Noether to Göttingen. There she lectured at courses that were given under his name and applied her profound invariant-theoretic knowledge to the resolution of problems which he and Felix Klein were considering. In this connection she was able to provide an elegant pure mathematical formulation for several concepts of Einstein's general theory of relativity. Hilbert repeatedly tried to obtain her appointment as *Privatdozent*, but the strong prejudice against women prevented her "habitation" until 1919. In 1922 she was named a *nichtbeamteter ausserordentlicher Professor* ("unofficial associate professor"), a purely honorary position. Subsequently, a modest salary was provided through a *Lehrauftrag* ("teaching appointment") in algebra. Thus she taught at Göttingen (1922–1933), interrupted only by visiting professorships at Moscow (1928–1929) and at Frankfurt (summer of 1930).

In April 1933 she and other Jewish professors at Göttingen were summarily dismissed. In 1934 Nazi political pressures caused her brother Fritz to resign from his position at Breslau and to take up duties at the research institute in Tomsk, Siberia. Through the efforts of Hermann Weyl, Emmy Noether was offered a visiting professorship at Bryn Mawr College; she departed for the United States in October 1933. Thereafter she lectured and did research at Bryn Mawr and at the Institute for Advanced Study, Princeton, but those activities were cut short by her sudden death from complications following surgery.

Emmy Noether's most important contributions to mathematics were in the area of abstract algebra, which is completely different from the early algebra of equation solving in that it studies not so much the results of algebraic operations (addition, multiplication, etc.) but rather their formal properties, such as associativity, commutativity, distributivity; and it investigates the generalized systems that arise if one or more of these properties is not assumed. Thus, in classical algebra it is postulated that the rational, the real, or the complex numbers should constitute a "field" with respect to addition and multiplication, operations assumed to be associative and commutative, the latter being distributive with respect to the former. One of the traditional postulates, namely the commutative law of multiplication, was relinquished in the earliest example of a generalized algebraic structure (William Rowan Hamilton's "quaternion algebra" of 1843) and also in many of the 1844 Grassmann algebras. The entities in such systems and in some of the research of Emmy Noether after 1927 are still termed numbers, albeit hypercomplex numbers. In further generalization the elements of an algebraic system are abstractions that are not necessarily capable of interpretation as numbers, and the binary operations are not literally addition and multiplication, but merely laws of composition that have properties akin to the traditional operations.

If Hamilton and Grassmann inspired Emmy Noether's later work, it was Dedekind who influenced the abstract axiomatic "theory of ideals" which Noether developed from 1920 to 1926. The Dedekind ideals—which are not numbers but sets of numbers—were devised in order to reinstate the Euclidean theorem on unique decomposition into prime factors, a law which breaks down in algebraic number fields. Two of the generalized structures which Noether related to the ideals are the "group" and the "ring".

A group is more general than a field because it involves only a single operation (either an "addition" or a "multiplication") which need not be commutative. It is, then, a system $\{S, \circ\}$ where S is a set of elements, $\circ$ is a closed associative binary operation, and S contains a unit element or identity as well as a unique inverse for every element. A ring is a system $\{S \oplus, \otimes\}$ which is a commutative group with respect to $\oplus$, an "addition", and which is closed under a "multiplication", that is, a second binary associative operation $\otimes$, which is distributive with respect to the first operation. Finally, a subset of a ring with a commutative multiplication $\otimes$ is called an "ideal" if it is a subgroup of the additive group of the ring—for this it is sufficient that the difference of any two elements of the subset belong to that set—and if it contains all products of subset elements by arbitrary elements of the ring. In a ring with a noncommutative multiplication, there are left ideals and right ideals.

Emmy Noether showed that the ascending chain condition is important for ideal theory. A ring satisfies that condition if every sequence of ideals C_1, C_2, C_3,… in the ring—such that each ideal is a proper part of its successor—has only a finite number of terms. Noether demonstrated that for a commutative ring with a unit element the requirement is equivalent to each of two other requirements: namely, that every ideal in the ring have a finite basis, that is, that the ideal consist of the set of all elements

$x_1a_1 + x_2a_2 + \dots + x_na_n$,

where the a_i are fixed elements of the ring and the X_i are any elements whatsoever in the ring, and that, given any nonempty set of ideals in the ring, there be at least one ideal which is "maximal" in that set.

Having formulated the concept of primary ideals—a generalization of Dedekind's prime ideals—Noether used the ascending chain condition in order to prove that an ideal in a commutative ring can be represented as the intersection of primary ideals. Then she studied the necessary and sufficient conditions for such an ideal to be the product of "prime power ideals". A somewhat different aspect of ideal theory was her use of polynomial ideals to rigorize, generalize, and give modern pure mathematical form to the concepts and methods of algebraic geometry as they had first been developed by her father and subsequently by the Italian school of geometers.

In another area Emmy Noether investigated the noncommulative rings in linear algebras like the Hamilton and Grassmann systems. An "algebra" is a ring in which the two binary operations are supplemented by a unary operation, an external or scalar multiplication, that is, a multiplication by the elements (scalars) of a specified field. From 1927 to 1929 Emmy Noether contributed notably to the theory of representations, the object of which is to provide realizations of noncommutative rings (or algebras) by means of matrices or linear transformations in such a way that all relations which involve the rings addition and/or multiplication are preserved; in other words, to study the homomorphisms of a given ring into a ring of matrices. From 1932 to 1934 she was able to probe profoundly into the structure of noncommutative algebras by means of her concept of the *verschränktes* ("cross") product. In a 1932 paper that was written jointly with Richard Brauer and Helmut Hasse, she proved that every "simple" algebra over an ordinary algebraic number field is cyclic; Weyl called this theorem "a high water mark in the history of algebra."

Emmy Noether wrote some forty-five research papers and was an inspiration to Max Deuring, Hans Fitting, W. Krull, Chiungtze Tsen, and Olga Taussky Todd, among others. The so-called Noether school included such algebraists as Hasse and W. Schmeidler, with whom she exchanged ideas and whom she converted to her own special point of view. She was particularly influential in the work of B. L. van der Waerden, who continued to promote her ideas after her death and to indicate the many concepts for which he was indebted to her.

BIBLIOGRAPHY

I. Original Works. Among Emmy Noether's many papers are "Invarianten beliebiger Differentialausdrücke," in *Nachrichten Von der Gesellschaft der Wissenschaften zu Göttingen* (1918), 37–44; "Moduln in nichtkommutativen Bereichen, insbesondere aus Differential-and Differenzenausdrü," in *Matlrematische Zeitschrift*, 8 (1920), 1–35, written with W. Schmeidler; "Idealtheorie in Ring-bereichen," in *Mathematische Annalen*, **83** (1921), 24–66 "Abstrakter Aufbau der Idealtheorie in algebraischen Zahlund Funktionenkörpern," *ibid*., **96** (1927), 26–61; "Über minimale Zerfallungskörper irreduzibler Darstellungen" in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin* (1927), 221–228, written with R. Brauer; "Hyperkomplexe Grossen and Darstellungs-theorie," in *Mathematische Zeitschrift*, 30 (1929), 641–692; "Beweis eines Hauptsatzes in der Theorie der Algebren," in *Journal für die reine und angewandte Mathematik*, 167 (1932), 399–404, written with R. Brauer and H. Hasse; and "Nichtkommutative Algebren," in *Mathematische Zeitschrift*, **37** (1933), 514–541.

II. Secondary Literature. For further information about Noether and her work, see A. Dick, "Emmy Noether," in *Revue de mathmatiques elementaries*, supp. 13 (1970); C. H. Kimberling, "Emmy Noether," in *American Mathematical Monthly*, **79** (1972), 136–149; E. E. Kramer, *The Nature and Growth of Modern Mathematics* (New York, 1970), 656–672; B.L. van der Waerden, "Nachruf auf Emmy Noether," in *Mathematische Annalen*, 111 (1935), 469–476; and H. Weyl, "Emmy Noether," in *Scripta mathematica*, **3** (1935), 201–220.

Edna E. Kramer