Pappus of Alexandria | Encyclopedia.com

(*b*. Alexandria, *fl*. a.d. 300–350)

mathematics, astronomy, geography.

In the silver age of Greek mathematics Pappus stands out as an accomplished and versatile geometer. His treatise known as the *Synagoge* or *Collection* is a chief, and sometimes the only, source for our knowledge of his predecessors' achievements.

The *Collection* is in eight books, perhaps originally in twelve, of which the first and part of the second are missing. That Pappus was an Alexandrian is affirmed by the titles of his surviving books and also by an entry in the *Suda Lexicon*[1]. The dedication of the seventh and eighth books to his son Hermodorus[2] provides the sole detail known of his family life. Only one of Pappus' other works has survived in Greek, and that in fragmentary form—his commentary on Ptolemy's *Syntaxis* (the *Almagest*). A commentary on book X of Euclid's *Elements*, which exists in Arabic, is generally thought to be a translation of the commentary that Pappus is known to have written, but some doubts may be allowed. A geographical work, *Description of the World*, has survived in an early Armenian translation.

The dates of Pappus are approximately fixed by his reference in the commentary on Ptolemy to an eclipse of the sun that took place on the seventeenth day of the Egyptian month Tybi in the year 1068 of the era of Nabonasar. This is 18 October 320 in the Christian era, and Pappus writes as though it were an eclipse that he had recently seen.[3] The *Suda Lexicon*, which is followed by Eudocia, would make Pappus a contemporary of Theon of Alexandria and place both in the reign of Theodosius I (A.D. 379–395), but the compiler was clearly not well informed. The entry runs: "Pappus, of Alexandria, philosopher, lived about the time of the Emperor Theodosius the Elder, when Theon the Philosopher, who wrote on the *Canon of* Ptolemy, also flourished. His books are: *Description of the world, Commentary on the Four Books of Ptolemy's Great Syntaxis, Rivers of Libya, Interpretation of Dreams*." The omission of Pappus' chief work and the apparent confusion of the *Syntaxis* with the *Tetrabiblos* of Ptolemy[4] does not inspire confidence. The argument that two scholars could not have written in the same city, on the same subject, at the same time, without referring to each other may not be convincing, for that is precisely what scholars are liable, deliberately or inadvertently, to do. But detailed examination shows that when Theon wrote his *commentary on the Syntaxis* he must have had pappus' commentary before him.[5] A scholium to a Leiden manuscript of chronological tables, written by Theon, would place Pappus at the turn of the third century, for opposite the name Diocletian (A.D. 284–305) it notes: "In his time Pappus wrote."[6] This statement cannot be reconciled with the eclipse of A.D. 320, but it is more than likely that Pappus' early life was spent under Diocletian, for he would certainly have been older than fifteen when he wrote his commentary on the *Syntaxis*.

The several books of the *Collection* many well have been written as separate treatises at different dates and later brought together, as the name suggests. It is certain that the *Collection*, as it has come down to us, is posterior to the *Commentary on the Syntaxis*, for in book VIII Pappus notes that the rectangle contained by the perimeter of a circle and its radius is double the area of the circle, "as Archimedes showed, and as is proved by us in the commentary on the first book of the *Mathematics [sc.,* the *Syntaxis mathematica* of Ptolemy] by a theorem of our own."[7] A. Rome concludes that the *Collection* was put together about a.d. 340, but K. Ziegler states that a long interval is not necessary, and that the *Collection* may have been compiled soon after a.d. 320.[8] It has come down to us from a single twelfth-century manuscript, Codex Vaticanus Graecus 218, from which all the other manuscripts are derived.[9]

T.L. Heath judiciously observes that the *Collection*, while covering practically the whole field of Greek geometry, is a handbook rather than an encyclopedia; and that it was intended to be read with the original works, where extant, rather than take their place. But where the history of a particular topic is given, Pappus reproduces the various solutions, probably because of the difficulty of studying them in many different sources. Even when a text is readily available, he often gives alternative proofs and makes improvements or extensions.[10] The portion of book II that survives, beginning with proposition 14, expounds Apollonius' system of large numbers expressed as powers of 10,000. It is probable that book I was also arithmetical.

Book III is in four parts. The first part deals with the problem of finding two mean proportionals between two given straight lines, the second develops the theory of means, the third sets out some "paradoxes" of an otherwise unknown Erycinus, and the fourth treats of the inscription of the five regular solids in a sphere, but in a manner quite different from that of Euclid in his *Elements*, XIII. 13–17.

Book IV is in five sections. The first section is a series of unrelated propositions, of which the opening one is a generalization of Pythagoras' theorem even wider than that found in Euclid VI.31. In the triangle

ABC let any parallelograms *ABED, BCFG* be drawn on *AB, AC* and let *DE, FG* meet in *H*. Join *HB* and produce it to meet *AC* in *K*. The sum of the parallelograms *ABED, BCFG* can then be shown to the equal to the parallelogram contained by *AC, HB* in an angle equal to the sum of the angles *BAC, DHB*. (It is, in fact, equal to the sum of *ALNK, CMNK*; that is, to the figure *ALMC*, which is easily shown to be a parallelogram having the angle *LAC* equal to the sum of the angles *BAC, DHB*.)

The second section deals with circles inscribed in the figure known as the ἄρβηλος or "shoemaker's knife." It is formed when the diameter *AC* of a semicircle

ABC is divided in any way at *E* and semicircles *ADE*, *EFC* are erected. The space between these two semicircles and the semicircle *ABC* is the ἄρβηλος. In a series of elegant theorems Pappus shows that if a circle with center *G* is drawn so as to touch all three semicircles, and then a circle with center *H* to touch this circle and the semicircles *ABC*, *ADE*, and so on *ad infinitum*, then the perpendicular from *G* to *AC* is equal to the diameter of the circle with center *G*, the perpendicular from *H* to *AC* is double the diameter of the circle with center *H*, the perpendicular from *K* to *AC* is triple the diameter of the circle with center *K*, and so on indefinitely. Pappus records this as "an ancient proposition" and proceeds to give variants. This section covers as particular cases propositions in the *Book of Lemmas* that Arabian tradition attributes to Archimedes.

In the third section Pappus turns to the squaring of the circle. He professes to give the solutions of Archimedes (by means of a spiral) and of Nicomedes (by means of the conchoid), and the solution by means of the quadratrix, but his proof is different from that of Archimedes. To the traditional method of generating the quadratrix (see the articles on Dinostratus and Hippias of Elis), Pappus adds two further methods "by means of surface loci," that is, curves drawn on surfaces. As a digression he examines the properties of a spiral described on a sphere.

The fourth section is devoted to another famous problem in Greek mathematics, the trisection of an angle. Pappus' first solution is by means of a νευ^σις or verging—the construction of a line that has to pass through a certain point—which involves the use of a hyperbola. He next proceeds to solve the problem directly, by means of a hyperbola, in two ways; on one occasion he uses the diameter-and-ordinate property (as in Apollonius), and on another he uses the focus-directrix property. This property is proved in book VII. Pappus then reproduces the solutions by means of the quadratrix and the spiral of Archimedes; he also gives the solution of νευ^σις which he believes Archimedes to have unnecessarily assumed in *On Spirals*, proposition 8.

In the preface to book V, which deals with isoperimetry, Pappus praises the sagacity of bees who make the cells of the honeycomb hexagonal because of all the figures which can be fitted together the hexagon contains the greatest area. The literary quality of this preface has been warmly praised. Within the limits of his subject, pappus looks back to the great Attic writers from a world in which Greek had degenerated into Hellenistic. In the first part of the book Pappus appears to be reproducing Zenodorus fairly closely; in the second part he compares the volumes of solids that have equal surfaces. He gives an account of thirteen semiregular solids, discovered and discussed by Archimedes (but not in any surviving works of that mathematician) that are contained by polygons all equilateral and equiangular but not all similar. He then shows, following Zenodorus, that the sphere is greater in volume than any of the regular solids that have surfaces equal to that of the sphere. He also proves, independently, that, of the regular solids with equal surfaces, that solid is greater which has the more faces.

Book VI is astronomical and deals with the books in the so-called *Little Astronomy*—the smaller treatises regarded as an introduction to Ptolemy's *Syntaxis*. In magistral manner he reviews the works of Theodosius, Autolycus, Aristarchus, and Euclid, and he corrects common misrepresentations. In the section on Euclid's *Optics*, Pappus examines the apparent form of a circle when seen from a point outside the plane in which it lies.

Book VII is the most fascinating in the whole *Collection*, not merely by its intrinsic interest and by what it preserves of earlier writers, but by its influence on modern mathematics. It gives an account of the following books in the so-called *Treasury of Analysis* (those marked by an asterisk are lost works): Euclid's *Data and Porisms*,* Apollonius' *Cutting Off of a Ratio, Cutting Off of an Area*,**Determinate Section*,**Tangencies*,*, *Inclinations*,*, *Plane Loci*,* and *Conics*. In his account of Apollonius' *Conics*, Pappus makes a reference to the "locus with respect to three or four lines" (a conic section); this statement is quoted in the article on Euclid (IV, 427 *ad fin*.). He also adds a remarkable comment of his own. If, he says, there are more than four straight lines given in position, and from a point straight lines are drawn to meet them at given angles, the point will lie on a curve that cannot yet be identified. If there are five lines, and the parallelepiped formed by the product of three of the lines drawn from the point at fixed angles bears a constant ratio to the parallelepiped formed by the product of the other two lines drawn from the point and a given length, the point will be on a certain curve given in position. If there are six lines, and the solid figure contained by three of the lines bears a constant ratio to the solid figure formed by the other three, then the point will again lie on a curve given in position. If there are more than six lines it is not possible to conceive of solids formed by the product of more than three lines, but Pappus surmounts the difficulty by means of compounded ratios. If from any point straight lines are drawn so as to meet at a given angle any number of straight lines given in position, and the ratio of one of those lines to another is compounded with the ratio of a third to a fourth, and so on (or the ratio of the last to a given length if the number of lines is odd) and the compounded ratio is a constant, then the locus of the point will be one of the higher curves. Pappus had, of course, no symbolism at his disposition, nor did he even use a figure, but his meaning can be made clearer by saying that if $p_1, p_2, \ldots, p_n$ are the lengths of the lines drawn at fixed angles to the lines given in position, and if (*a* having a given length and *k* being a constant)

or

then the locus of the point is a certain curve.

In 1631 Jacob Golius drew the attention of Descartes to this passage in Pappus, and in 1637 “Pappus’ problem,” as Descartes called it, formed a major part of his *Géométrie*.[11] Descartes begins his work by showing how the problems of conceiving the product of more than three straight lines as geometrical entities, which so troubled Pappus, can be avoided by the use of his new algebraic symbols. He shows how the locus with respect to three or four lines may be represented as an equation of degree not higher than the second, that is, a conic section which may degenerate into a circle or straight line. Where there are five, six, seven, or eight lines, the required points lie on the next highest curve of degree after the conic sections, that is, a cubic; if there are nine, ten, eleven, or twelve lines on a curve, one degree still higher, that is, a quartic, and so on to infinity. Pappus’ problem thus inspired the new method of analytical geometry that has proved such a powerful tool in subsequent centuries. (See the article on Descartes, IV, 57.)

In his *Principia* (1687) Newton also found inspiration in Pappus; he proved in a purely geometrical manner that the locus with respect to four lines is a conic section, which may degenerate into a circle. It is impossible to avoid seeing in Newton’s conclusion to lemma XIX, cor. ii, a criticism of Descartes: “Atque ita Problematis veterum de quatuor lineis ab *Euclide* incaepti et ab *Apollonio* continuati non calculus, sed compositio Geometrica, qualem Veteres quaerebant, in hoc Corollario exhibetur.”[12] But in this instance it was Descartes, and not Newton, who had the forward vision. Pappus observes that the study of these curves had not attracted men comparable to the geometers of previous ages. But there were still great discoveries to be made, and in order that he might not appear to have left the subject untouched, Pappus would himself make a contribution. It turns out to be nothing less than an anticipation of what is commonly called “Guldin’s theorem.”[13] Only the enunciations, however, were given, which state

Figures generated by complete revolutions of a plane figure about an axis are in a ratio compounded (*a*) of the ratio [of the areas] of the figures, and (*b*) of the ratio of the straight lines similarly drawn to [*sc*. drawn to meet at the same angles] the axes of rotation from the respective centers of gravity. Figures generated by incomplete revolutions are in a ratio compounded (*a*) of the ratio [of the areas] of the figures and (*b*) of the ratio of the arcs described by the respective centers of gravity; it is clear that the ratio of the arcs is itself compounded (1) of the ratio of the straight lines similarly drawn [from the respective centers of gravity to the axis of rotation] and (2) of the ratio of the angles contained about the axes of rotation by the extremities of these straight lines.

Pappus concludes this section by noting that these propositions, which are virtually one, cover many theorems of all kinds about curves, surfaces, and solids, “in particular, those proved in the twelfth book of these elements.” This implies that the *Collection* originally ran to at least twelve books.

Pappus proceeds to give a series of lemmas to each of the books he has described, except Euclid’s *Data,* presumably with a view to helping students to understand them. (He was half a millennium from Apollonius and elucidation was probably necessary.) It is mainly from these lemmas that we can form any knowledge of the contents of the missing works, and they have enabled mathematicians to attempt reconstructions of Euclid’s *Porisms* and Apollonius’ *Cutting Off of an Area, Plane Loci, Determinate Section, Tangencies,* and *Inclinations*. It is from Pappus’ lemmas that we can form some idea of the eighth book of Apollonius’ *Conics*.

The lemmas to the *Cutting Off of a Ratio* and the *Cutting Off of an Area* are elementary, but those to the *Determinate Section* show that this work amounted to a theory of involution. The most interesting lemmas concern the values of the ratio $AP \cdot PD : BP \cdot PC$, where (A, D), (B, C) are point-pairs on a straight line and P is another point on the straight line. Pappus investigates the “singular and least” values of the ratio and shows what it is for three different positions of P.

The lemmas to the *Inclinations* do not call for comment. The lemmas to the second book of the *Tangencies* are all concerned with the problem of drawing a circle so as to touch three given circles, a problem that Viéte and Newton did not consider it beneath their dignity to solve.[14] The most interesting of Pappus’ lemmas states: Given a circle and three points in a straight line external to it, inscribe in the circle a triangle, the sides of which shall pass through the three points.

The lemmas to the *Plane Loci* are chiefly propositions in algebraic geometry, one of which is equivalent to the theorem discovered by R. Simson, but generally known as Stewart’s theorem:[15] If A, B, C, D are any four points on a straight line, then

$$AD^2.BC+BD^2.CA+CD^2.AB+BC.CA.AB=0.$$

The remarkable proposition that Pappus gives in his description of Euclid’s *Porisms* about any system of straight lines cutting each other two by two has already been set out in modern notation in the article on Euclid (IV, 426–427). The thirty-eight lemmas that he himself provides to facilitate an understanding of the *Porisms* strike an equally modern note. Lemma 3, proposition 129 shows that Pappus had a clear understanding of what Chasles called the anharmonic ratio and is now generally called the cross-ratio of four points. It proves the equality of the cross-ratios that

are made by any two transversals on a pencil of four lines issuing from the same point. The transversals are, in fact, drawn from the same points on one of the straight lines—in Figure 3 they are $HBCD$ and $HEFG$, cutting the lines $AH, AL, AF,$ and AG—but it is a simple matter to extend the proof, and Pappus proves that

that is to say, the cross-ratio is thus invariant under projection.

Lemma 4, proposition 130 shows, even more convincingly than the lemmas to the *Determinate Section,* that Pappus had an equally clear grasp of involution. In Figure 4, *GHKL* is a quadrilateral and *ABCDEF* is

any transversal cutting pairs of opposite sides and the diagonals in (*A*,*F*), (*C*,*D*), (*B*,*E*). Pappus shows that

(Strictly, what Pappus does is to show that if, in the figure, which he does not set out in detail, this relationship holds, then *F, G, H* lie on a straight line, but this is equivalent to what has been said above.) This equation is one of the ways of expressing the relationship between three pairs of conjugate points in involution. That Pappus gives these propositions as lemmas to Euclid's *Porisms* implies that they must have been assumed by Euclid. The geometers living just before Euclid must therefore have had an understanding of cross-ratios and involution, although these properties were not named for 2,250 years.

Lemma 13, proposition 139 has won its way into text books of modern geometry as "The Theorem of Pappus"[16] It establishes that if, from a point *C* two transversals *CE, CD* cut the straight lines *AN, AF, AD* (see Figure 5) so that *A, E, B* and *C, F, D* are two sets of collinear points, then the points *G, M, K* are collinear. *GMK* is called the "Pappus line" of the two sets of collinear points.

In the second of the two lemmas that Pappus gives to the *Surface Loci,* he enunciates and proves the focus-directrix property of a conic, which, as we have seen, he had already once employed. There is only one other place in any surviving Greek text in which this property is used—the fragment of Anthemius' *On Remarkable Mechanical Devices*. G. L. Toomer, however, has recently discovered this property in an Arabic translation of Diocles' treatise *On Burning Mirrors* in Mashhad (Shrine Library, MS 392/5593) and Dublin (Chester Beatty Library, Arabic MS 5255). But Pappus' passage remains the only place in ancient writing in which the property is proved.

Book VIII is devoted mainly to mechanics, but it incidentally gives some propositions of geometrical interest. In a historical preface Pappus justifies the claim that mechanics is a truly mathematical subject as opposed to one of merely utilitarian value. He begins by defining "center of gravity" —the only place in Greek mathematics where it is so defined—gives the theory of the inclined plane; shows how to construct a conic through five given points; solves the problem of constructing six equal hexagons around the circumference of a circle so as to touch each other and a seventh equal hexagon at the center; discourses on toothed wheels; and in a final section (which may be wholly interpolated) gives extracts from Heron's description of the five mechanical powers: the wheel and axle, the lever, the pulley, the wedge, and the screw.

Commentary on the Almagest. A commentary by Pappus on book V (with lacunae) and book VI of Ptolemy's *Syntaxis exists* in the Florentine manuscript designated L (ninth century) and in various other manuscripts. But this commentary is only part of a larger original. In the *Collection* Pappus refers to his commentary (*scholion*) on the first book of the *Almagest,* and in the surviving sixth book he makes the same reference, repeating a proof of his own for Archimedes' theorem about the area of a circle which, he says, he had given in the first book. In the compilation of uncertain authorship known as the *Introduction to the Almagest* there is a reference to a method of division "according to the geometer Pappus," which would seem to hark back to the third book.[17] In the fifth book of the commentary Pappus refers to a theorem in connection with parallax proved in his fourth book.[18] Although there is no direct reference to the second book, there is sufficient evidence that he commented on the first six books, and he may have written on all thirteen. The date of the commentary, as we have seen, must be soon after 320.

At the outset of his fifth book Pappus gives a summary of Ptolemy's fourth book, and at the beginning of his sixth book he summarizes Ptolemy's fifth book, which suggests that his commentary was a course of lectures. This theory is borne out by the painstaking and methodical way in which he explains, apparently for an audience of beginners, the details of Ptolemy's theory.

Ptolemy's fourth book introduces his lunar theory, and he explains the "first of simple anomaly" (irregularity of the movements of the moon) by postulating that the moon moves uniformly round the circumference of a circle (the epicycle), the center of which is carried uniformly round a circle concentric with the ecliptic. Pappus, following Ptolemy closely, explains in his fifth book that this needs correction for a second anomaly, which disappears at the new and full moons but is again noticeable when the moon is at the quadratures—provided that it is not then near its apogee or perigee, and iregularity later called evection. He also explains in detail Ptolemy's hypothesis that the circle on which the epicycle moves (the deferent) is eccentric with the ecliptic, and that the center of the eccentric circle itself moves uniformly round the center of the earth. To account for certain irregularities not explained by these anomalies, Ptolemy postulates a further correction which he calls prosneusis (that is, inclination or verging). In this context prosneusis means that the diameter of the epicycle which determines apogee and perigee is not directed to the center of the ecliptic but to a point on the line joining the center of the eccentric and the center of the ecliptic produced, and as far distant from the latter as the latter is from the former. After a gap in the manuscript, Pappus begins his comment again in the middle of this subject and proceeds to deal with a further complication. He states that the true position of the moon may not be where it is seen in the heavens on account of parallax, which may be neglected for the sun but not for the moon. He gives details for the construction of a "parallactic instrument" (an alidade) used for finding the zenithal distances of heavenly bodies when crossing the meridian. He had previously given details of "an astrolabe" (really an armillary sphere) described by Ptolemy.[19] He also follows Ptolemy closely in his deduction of the sizes and distances of the sun and moon, the diameter of the shadow of the earth in eclipses, and the size of the earth.

In the sixth book, again following Ptolemy closely, Pappus explains the conditions under which conjunctions and oppositions of the sun and moon occur. This explanation leads to a study of the conditions for eclipses of the sun and moon and to rules for predicting when eclipses will occur. The book closes with a study of the points of first and last contact during eclipses.

Pappus, like Theon after him, not only follows Ptolemy's division into chapters but enumerates theorems as Ptolemy does not. It is clear that Theon had Pappus' commentary before him when he wrote over a century later, and in some cases Theon lifted passages directly from Pappus.

Commentary on Euclid's Elements . Eutocius[20] refers to a commentary by Pappus on the *Elements* of Euclid and it probably extended to all thirteen books. In Proclus' commentary on book **I** there are three references to Pappus,[21] and it is reasonable to believe that they relate to Pappus' own commentary on the *Elements* as they do not relate to anything in the *Collection*. Pappus is said to have pointed out that while all right angles are equal to one another, it is not true that an angle equal to a right angle is always

a right angle—it may be an angle formed by arcs of circles and thus cannot be called a right angle. He is also alleged to have added a superfluous axiom: If unequals are added to equals, the excess of one sum over the other is equal to the excess of one of the added quantities over the other. He also added a complementary axiom about equals added to unequals, as well as certain axioms that can be deduced from the definitions. He gave a neat alternative proof of Euclid 1.5 (the angles at the base of an isosceles triangle are equal) by comparing the triangle *ABC* with the triangle *ACB*, that is, the same triangle with the sides taken in reverse order (Figure 6).

Eutocius states that Pappus, in his commentary on the *Elements*, explains how to inscribe in a circle a polygon similar to a polygon inscribed in another circle. This would doubtless be in his commentary on book XII, and Pappus probably solved the problem in the same manner as a scholiast to XII. 1, that is, by making the angles at the center of the second circle equal to the angles at the center of the first.[22]

If Pappus wrote on books I and XII it is likely that he also commented on the intermediate books, and the fact that he commented on book X is attested by a scholiast to Euclid's *Data*[23] and by the *Fihrist*, in which it is stated that the commentary was in two parts.[24]

A two-part commentary on the tenth book of Euclid's *Elements* does actually exist in Arabic,[25] and it is usually identified with that of Pappus. It was discovered in a Paris manuscript by F. Woepcke in 1850, but the manuscript lacks diacritical marks and Woepcke himself read the consonantal skeleton of the author's name as Bls, which he interpreted as meaning Valens, probably Vettius Valens, an astronomer of the age of Ptolemy. [26] Heiberg showed this interpretation to be impossible, and was the first scholar to identify the commentary with that which Pappus was known to have written.[27] H. Suter pointed out that the Arabic for Bls could easily be confused with Bbs, and as there is no P in Arabic, Pappus would be the author indicated.[28] This was accepted by T. L. Heath,[29] and indeed generally, but when Suter's translation of Woepcke's text was published in 1922[30] he raised the question whether the prolixity and Neoplatonic character of the treatise did not indicate Proclus as the author. In the latest study of the subject (1930) William Thomson denied the charges of prolixity and mysticism and accepted the authorship of Pappus.[31] It must be admitted that the commentary is in a wholly different style from the severely mathematical nature of the *Collection*, or even of the more elementary commentary on the *Almagest*, and the question of authorship cannot be regarded as entirely free from doubt.

The superscription to the first part of the commentary and the subscription to the second part state that the Arabic translation is the work of Abū 'Uthman al-Dimishqī (*fl. ca*. 908–932), who also translated the tenth book of Euclid's *Elements*. The postscript to the second part adds that the copy of the cpmmentary was written in 969 by Aḥmad ibn Muḥammad ibn 'Abd al-Jalīl, that is, the Persian geometer generally known as al-Sizjī (*ca*. 951–1029).

Some two dozen passages in the commentary have parallels in the scholia to Euclid's book X, sometimes remarkably close parallels. The simplest explanation is that the scholiast made his marginal notes with Pappus' commentary in front of him.

Euclid's book X is a work of immense subtlety, but there is little in the commentary that calls for comment. The opening section has an interest for the historian of mathematics as it distinguishes the parts played by the Pythagoreans, Theaetetus, Euclid, and Apollonius in the study of irrationals. It also credits Theaetetus with a classification of irrationals according to the different means.[32] He is said to have assigned the medial line to geometry (is the geometric mean between x, y), the binomial to arithmetic ($\frac{1}{2}[x + y]$ is the arithmetic mean between x, y), and the apotome to harmony (the harmonic mean $[2xy]/[x + y]$ between x, y is $[(2xy)/(x^2 - y^2)] \cdot [x - y]$, which is the product of a binomial and an apotome.)

Other Mathematical Works . Marinus, in the final sentence of his commentary on Euclid's *Data*,[33] reveals that Pappus also commented on the *Data*. Pappus apparently showed that Euclid's teaching followed the method of analysis rather than synthesis. Pappus also mentions a commentary that he wrote on the *Analemma* of Diodorus, in which he used the conchoid of Nicomedes to trisect an angle.[34]

The *Fihrist* includes among Pappus' works "A commentary on the book of Ptolemy on the *Planisphaerium*, translated by Thābit into Arabic." The entry leaves it uncertain whether Thābit ibn Qurra (*d*. 901) translated Ptolemy's work or Pappus' commentary, but Hājjī Khalīfa states that Ptolemy was the author of a treatise on the *Planisphaerium* translated by Thabit. He also adds that Ptolemy's work was commented on by "Battus al Roûmi [that is, late Greek], an Alexandrian geometer." "Battus" is clearly "Babbus." that is Pappus.[35]

Geography . The *Description of the World* mentioned in the *Suda Lexicon* has not survived in Greek, but the *Geography* bearing the name of the Armenian Moses of Khoren (although some scholars see in it the work of Anania Shirakatsi) appears to be a translation, or so closely based on Pappus' work as to be virtually a translation. The *Geography*, if correctly ascribed to Moses, was written about the beginning of the fifth century. The archetype has not survived, and the manuscripts contain both a long and a short recension. The character of Pappus' work may be deduced from two passages of Moses, or the pseudo-Moses, which may be thus rendered:[36] "We shall begin therefore after the *Geography* of Pappus of Alexandria, who has followed the circle or the special map of Claudius Ptolemy" and "Having spoken of geography in general, we shall now begin to explain each of the countries according to Pappus of Alexandria." From these and other passages J. Fischer[37] deduced that Pappus' work was based on the world map and on the special maps of Ptolemy rather than on the text itself, and as Pappus flourished only a century and a half after Ptolemy it is a fair inference that the world map and the special maps date back to Ptolemy himself. Pappus appears to have written with Ptolemy's maps as his basis, but about the world as he knew it in the fourth century.

Nothing is known of the second geographical work, *Rivers of Libya*, mentioned in the *Suda Lexicon*, or of the *Interpretation of Dreams*. The interpretation of dreams is akin to astrology, and there would be nothing surprising in a work on the subject by an ancient mathematician.

Music . It is possible that the commentary on Ptolemy's *Harmonica*, which was first edited by Wallis as the work of Porphyry, is, from the fifth chapter of the first book on, the work of Pappus. Several manuscripts contain the first four chapters only, and Lucas Holstein found in the Vatican a manuscript containing a definite statement that porphyry's commentary was confined to the first four chapters of the first book and that Pappus was responsible for the remainder. Montfaucon also noted the same manuscript under the title "Pappi De Musica." Wallis did not accept the attribution because the title of the whole work and the titles of the chapters imply taht it is wholly the work of Porphyry and because he could detect no stylistic difference between the parts. But the titles prove nothing as Porphyry no doubt did comment, or intended to comment, on the whole work, and only missing parts would have been taken from another commentary, and arguments based on differences of style, especially in a technical work, are notoriously difficult. Hultsch and Jan were satisfied that Pappus was the author, but During was emphatically of the opinion that the whole is the work of Porphyry, and Ver Eecke agreed. [38] It must be left an open question.

Hydrostatics . An Arabic manuscript discovered in Iran by N. Khanikoff and published in 1860 under the title *Book of the Balance of Wisdom, an Arabic Work on the Water Balance, Written by al-Khazini in the Twelfth Century*[39] attributes to Pappus an instrument for measuring liquids and describes it in detail. The instrument is said by Khanikoff to be nearly identical with the volumeter of Gay-Lussac. If the attribution is correct—and there seems no reason to doubt it—the instrument may have been described in the missing part of the eighth book of the *Collection* or it may have had a place in a separate work on hydrostatics, of which no other trace has survived.

An Alchemical Oath . An oath attributed to "Pappus, philosopher" in a collection of alchemical writings may be genuine—if not *vero*, it is at least *ben trovato*—and if so it may tell us something of Pappus' syncretistic religious views in an age when paganism was retreating before Christianity. It is an oath that could have been taken equally by a pagan or a Christian, and it would fit in with the dates of Pappus. It could be gnostic, it has a Pythagorean element in it, there may be a veiled reference to the Trinity, and there is a Byzantine ring to its closing words. It reads: "I therefore swear to these, whoever thou art, the great oath, I declare God to be one in form but not in number, the maker of heaven and earth, as well as the tetrad of the elements and things formed from them, who has furthermore harmonized our rational and intellectual souls with our bodies, who is borne upon the chariots of the cherubim and hymned by angelic throngs."[40]

A Vatican manuscript containing Ptolemy's *Handy Tables* has on one folio a short text about the entry of the sun into the signs of the zodiac, which F. Boll has shown must refer to the second half of the third century and which E. Honigmann attributes to Pappus. But this is no more than an unsubstantiated guess, which Boll himself refrained from making.[41]

A Florentine manuscript catalogued by Bandini notices Η*μ*ερoδρ*ό*μιον Πάππου τω^ν διεπόντων καί πολευόντων, that is, daily tables of governing and presiding stars compiled by Pappus.[42]

NOTES

1.*Suda Lexicon*, Adler, ed., vol. Pars IV (Leipzig, 1935), P 265, p. 26.

2. Pappus, *Collectio*, III. 1, F. Hultsch, ed., 1, 30.4; VII.1, Hultsch, ed., II, 634.1. Nothing more is known of Hermodorus or of Pandrosion and Megethion, to whom the third and fifth books are dedicated; or of his philosopher-friend Hierius, who pressed him to give a solution to the problem of finding two mean proportionals (Hultsch, ed., III, 3–8). A phrase in Proclus, *In primum Euclidis*, Friedlein, ed. (Leipzig, 1873; repr. Hildesheim, 1967), p. 429.13, ol … repl Harrová, implies that he had a school.

3. A. Rome, *Commentaires de Pappus et de Théon d' Alexandrie sur l' Almageste*, I (Rome, 1931), 180.8–181.23, *Studi e Testi*, no. 54 (1931). The eclipse is no. 402 in F. K. Ginzel, *Spezialler Kanon der Sonnen und Mond Finsternisse* (Berlin, 1899), p. 87, and no. 3642 in T. von Oppolzer, *Canon der Finsternissen* (Vienna, 1887), repr. translated by Owen Gingerich (New York, 1962), p. 146. Rome, who first perceived the bearing of this eclipse on the date of Pappus, argues that if the total, or nearly total, eclipse of A. D. 346 had taken place, Pappus would certainly have chosen it for his example, and that the better eclipse of A. D. 291 was already too distant to be used (A. Rome, *op. cit.,* pp. x–xiii).

4. So A. Rome, *op. cit*., I, xvii, note 1, suggests. This is more convincing than the conjecture of F. Hultsch, *op. cit*., III, viii, note 3, that *Δ* is a copyist's error for *II*.

5. A. Rome, *op. cit*., II. lxxxiii, *Studi e Testi*, no, 72 (1936).

6. Leiden MS, no, 78. of Theon's ed. of the *Handy Tables*, fol. 55. This was first noted by J. van der Hagen, *Observationes in Theonis Fastos Graecos priores* (Amsterdam, 1735). p. 320, and his view was followed by H. Usener, "Vergessenes III," in *Rheinisches Museum*, n.s. **28** (1873), 403–404, and F. Hultsch, *op. cit*., III, vi–vii, but none of these scholars realized the significance of Pappus' reference to the eclipse of a.d. 320.

7. Pappus, *Collectio* VIII. 46, *op. cit*., III, 1106.13–15. Rome *op. cit*., I, 254, note 1, gives reasons for thinking that the third theorem of book V of the *Collectio* is a fragment, all that now survives, of book I of the *Commentary on the Syntaxis*, and that it is an interpolation by an ed.

8. A. Rome, see previous note; K. Ziegler, in Pauly-Wissowa, XVIII (Waldsee, 1949), col. 1094.

9. F. Hultsch *op. cit*., I, p. vii–xiv.

10. Thomas Heath, *A Hisory of Greek Mathematics*, II (Oxford, 1921), 357–358. A full and excellent conspectus of the *Collection* is given by Heath, *loc. cit*., pp. 361–439; Gino Loria, *Le scienze esatte nell'antica Grecia*, 2nd ed. (Milan, 1914), pp. 658–700; and Paul Ver Eecke, *Pappus d'Alexandrie: La Collection mathématique*, I (Paris–Bruges, 1933), xiii–cxiv.

11. René Descartes, *Des matières de la géométrie* (Leiden, 1637), book I, 304–314, book II, 323–350; David Engene Smith and Marcia C. Latham, *The Geometry of René Descartes With a Facsimile of the First Edition* (New York, 1925; repr. 1954), book I, 17–37, book II, 59–111.

12. Isaac Newton, *Philosophiae naturalis principia mathematica* (London, 1687; repr. London, 1953), "De motu corporum," lib. 1, sect. 5, XIX, pp. 74–75.

13. Pappus, VII,42, *op. cit*., II, 682.7–15. The whole passage in which this occurs is attributed by Hultsch to an interpolator, but without reasons given, by Ver Eecke (*op. cit*., I, xcvi) for unvonvinving stylistic reasons and lack of connection with the context. But Heath pertinently observes (*A History of Greek Mathematics*, II, 403) that no Greek after Pappus would have been capable of framing such an advanced proposition. Ver Eecke (*op. cit*., I. xcv, cxxiii) observes that Paul Guldin (1577–1643) could not have been inspired by the passage in Pappus as Commandino did not include it in his first ed. (Pesaro, 1588) and he could not have seen the second ed. (Bologna, 1660), augmented with this passage by Mamolessius. But this conclusion is an error; the passage is in the first no less than the second ed. See also the article on Guldin.

14. F. Vieta, *Apollonius Gallus* (Paris, 1600), problem x, pp. 7–8; Isaac Newton, *Arithmetica universalis* (Cambridge, 1707), problem xli *ad finem*, pp. 181–182, 2nd ed. (London, 1722), problem xlvii *ad finem*, p. 195; *Principia* (London, 1687; repr. London, 1953), lemma XVI, pp. 67–68.

15. Robert Simson, *Apollonii Pergaei locorum planorum libri II restituti* (Glasgow, 1749), pp. 156–221; Matthew Stewart, *Some General Theorems of Considerable Use in the Higher Parts of Mathematics* (Edinburgh, 1746), pp. 1–2. See also Moritz Cantor, *Vorlesungen über Geschichte der Mathematik,***III** (Leipzig, 1898), 523–528.

16. For example, E. A. Maxwell, *Geometry For Advanced Pupils* (Oxford, 1949), p. 97. The term "Pappus' Theorem" is thus used by Renaissance and modern geometers in two different ways.

17. C. Henry, *Opusculum de multiplicatione et divisione sexagesimalibus, Diophanto vel Pappo tribuendum* (Halle, 1879), p. viii; A. Rome, *op. cit.,***1,** xvi.

18. A. Rome, *op. cit.,***I,** 76.19–77.1.

19. For a reconstruction of the astrolabe and parallactic instrument as described by Pappus, with illustrations, see A. Rome, *Annales de la Société scientifique de Bruxelles,***47** (1927), 77–102, 129–140, and *op. cit.,***1,** 3–5, 69–77.

20. Eutocius, *Commentarü in libros Archimedis De Sphaera et cylindro,* p. 1.13, *ad init., Archimedis opera omnia,* J. L. Heiberg, ed., 2nd ed., III (Leipzig, 1915), corr. repr. Evangelos S. Stamatis (Stuttgart, 1972), P. 28.19–22: οπως μέν οὔύ ἑστιν εἱς τόν δοθεντα ͵γκλον πολύλωνον ελλρά΅Ψαι ὁμοιον τῷ εν ετερῳ ελλελραμμενῳ, δῆλον, εἰρηται δε καί ΠάΠΠῳ εἱς τόύΠόμνημα τῶν ἑτοιϰείων.

21. Proclus, *In primum Euclidis,* Friedlein, ed., pp. 189.12–191.4, 197.6–198.15, 249.20–250.19.

22.*Euclidis opera omnia,* J. L. Heiberg and H. Menge, eds., V (Leipzig, 1888), scholium 2, 616.6–617.21.

23.*Ibid.,* VI (Leipzig, 1896), scholium 4 *ad definitiones,* 262.4–6:

24. H. Suter, "Das Mathematiker Verzeichniss im Fihrist des Ibn abi Ja'kub an–Nadim," in *Zeitschrift für Mathematik und Physik,***37** (1892), suppl. (or *Abhandlungen zur Geschichte der Mathematik,***6**), p. 22. The whole entry runs, in English: "Pappus the Greek. His writings are: A Commentary on the book of Ptolemy concerning the representation of the sphere in a plane, translated by Thābit into Arabic. A commentary on the tenth book of Euclid, in two parts."

25. Bibliothèque Nationale (Paris), MS no. 2457 (Supplément arabe de la Bibliothèque impériale no. 952.2). The manuscript contains about fifty treatises, of which nos. 5 and 6 constitute the two books of the commentary.

26. Woepcke described the manuscript and translated four passages into French in his "Essai d'une restitution de travaux perdus d'Apollonius sur les quantités irrationelles," in *Mémoires présentés par divers savants à l' Académie des sciences*, **14** (1856), 658–720. He developed his theory about the authorship in *The Commentary on the Tenth Book of Euclid's Elements by Bls,* which he published anonymously and without date or place of publication. Woepcke read the name of the author in the title of the first book of the commentary as B.los (the dot representing a vowel) and in other manuscripts as B.lis, B.n.s, or B.l.s.

27. J. L. Heiberg, *Litterärgeschichtliche Studien über Euklid* (Leipzig, 1882), pp. 169–170. Heiberg points out that one of the manuscripts cited by Woepcke states that "B.n.s le Roumi" (that is, late Greek) was later than Claudius Ptolemy, while the *Fihrist* says that "B.l.s le Roumi" wrote a commentary on Ptolemy's *Planisphaerium*. As Vettius Valens lived under Hadrian, he was therefore older than Ptolemy—an elder contemporary. Moreover the *Fihrist* gives separate entries to B.l.s and Valens. See also Suter, *op. cit.,* p. 54, note 92.

28. H. Suter, "Das Mathematiker Verzeichniss in Fihrist," pp. 22, 54, note 92.

29. T. L. Heath, *The Thirteen Books of Euclid's Elements,* 2nd ed., III (Cambridge, 1905, 1925; repr. New York, 1956), 3; Heath, *A History of Greek Mathematics,* I (Oxford, 1921), 154–155, 209, II, 356.

30. H. Suter, "Der Kommentar des Pappus zum X Buche des Eukleides," in *Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin,***4** (1922), 9–78; see p. 78 for the question of authorship.

31. William Thomson and Gustav Junge, *The Commentary of Pappus on Book X of Euclid's Elements* (Cambridge, Mass., 1930; repr. New York, 1968), pp. 38–42.

32. There is nothing in the opening section about the rational and irrational being "given," as Pappus is stated by a scholiast (see note 23) to have maintained at the beginning of his commentary. This may be evidence against the ascription of the Arabic treatise to Pappus.

33.*Euclidis opera omnia,* J. L. Heiberg and H. Menge, eds., VI, 256.22–25.

34. F. Hultsch, *op. cit.,* I, 246.1–3.᾿Αν́□ μμα, as in Ptolemy's work with that title, means the projection of the circles of a celestial sphere on the plane. Neither the work of Diodorus nor the commentary of Pappus has survived. In Ptolemy's work certain segments of a semicircle are required to be divided into six equal parts, and it is easy to see how Pappus would need to trisect an arc or angle.

35. H. Suter, "Das Mathematiker Verzeichniss im Fihrist," p. 22 (see note 24, supra); Hājjī Khalīfa, *Lexikon bibliographicum et encyclopaedicum,* G. Fluegel, ed., V (London, 1850), 61–62, no. 9970, *s.v.* Kitab testih el koret. The *Planisphaerium* is a system of stereographic projection by which points on the heavenly sphere are represented on the plane of the equator by projection from a pole.

36. Translated from the French of P. Arsène Soukry, *Géographie de Moīse de Corène d'après Ptolémée* (Venice, 1881), p. 7.

37. J. Fischer, "Pappus und die Ptolemaeus Karten," in *Zeitschrift der Gesellschaft fur Erdkunde zu Berlin,***54** (1919), 336–358.

38. John Wallis, *Claudii Ptolemaei Harmonicorum libri* III (Oxford, 1682), reprinted in *Opera mathematica,* III (Oxford, 1699); the commentary is on pp. 183–355 of the latter work, and the authorship is discussed on p. 187. It has been edited in modern times by Ingemar Düring as *Porphyrios' Kommentar zur Harmonienlehre des Ptolemaios* (Göteborg, 1932). His discussion of the authorship is on pp. xxxvii–xxxix. Lucas Holstenius, *Dissertatio De vita et scriptis Porphyrii* (Rome, 1630), c. vi, p. 55: Neque tamen in universum ϱ́ □ *νικν̄* opus scripsit Porphyrius, sed in quatuor duntaxat prima capita: Cetera dein Pappus pertexuit. Ita enim in alio manuscripto Vaticano titulus indicat: . Bernard de Montfaucon, *Bibliotheca bibliothecarum manuscriptorum nova,* I (Paris, 1739), 11B. Paul Ver Eecke, *Pappus d'Alexandrie: La Collection mathématique,* I (Paris–Bruges, 1933), cxv–cxvi. F. Hultsch, *op. cit.,* III, xii. C. Jan, *Musici scriptores graeci* (Leipzig, 1895; repr. Hildesheim, 1962), p. 116 and note 1.

39. See *Journal of the American Oriental Society,* **6** (1860), 40–53; and the article on al-Khāzinī, IV, 338–341; and bibliography, 349–351.

40. C. G. Grumer, *Isidis, Christiani et Pappi philosophi Iusjurandum chemicum nunc primum graece et latine editum* (Jena, 1807); M. Berthelot and C. E. Ruelle, *Collection* des anciens alchimistes grecs (Paris, 1888), pp. 27–28, traduction, pp. 29–30; Paul Tannery, "Sur la réligion des derniers mathématiciens de l'antiquité," in *Annales de philosophie chrétienne,* **34** (1896), 26–36, repr. in *Mémoires scientifiques,* **2** (1912), 527–539, esp. pp. 533–535. Tannery seems inclined to think that the oath may be correctly attributed to Pappus the mathematician, and he speculates that he may have been a gnostic.

41. Vaticanus Graecus 1291, fol. 9r. F. Boll, "Eine illustrierte Prakthandschrift der astronomischen Tafeln des Ptolemaios," in *Sitzungsberichte der Königliche Bayerische Akademie der Wissenschaften, philosophisch-philologischen und historischen Classe,* **29** (1899), 110–138. E. Honigmann, *Die sieben Klimata und die* (Heidelberg, 1929), p. 73.

42. Codex Laurentianus XXXIV plut. XXVIII; A. M. Bandini, *Catalogus Bibliothecae Laurentianae,* II (Florence, 1767), 61.

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The only translation of the whole extant text into a modern language is that of Paul Ver Eecke, *Pappus d'Alexandrie, La Collection Mathématique: oeuvre traduite pour la premiere fois du grec en français avec une introduction et des notes,* 2 vols. (Paris–Bruges, 1993). A German translation of books III and VIII is given by C. J. Gerhardt, *Die Sammlung des Pappus von Alexandrien, griechisch und deutsch herausgegeben,* 2 vols. (Halle, 1871).

The *Collection* first became known to the learned world when Commandino included Latin translations of various extracts in his editions of Apollonius (Bologna, 1566) and Aristarchus (Pesaro, 1572). After Commandino's death, his complete Latin trans. of the extant Greek text, except the unknown fragment of book II, appeared as *Pappi Alexandrini Mathematicae Collectiones a Federico Commandino Urbinate in Latinum conversae et commentariis illustratae* (Pesaro, 1588). Reprints appeared in 1589 (Venice) and 1602 (Pesaro) and a second ed. was published by C. Manolessius in 1660 (Bologna); despite the editor's claims it was inferior to the first ed.

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The extant Greek text of book V was printed, with numerous errors and together with Theon's commentary, at the end of the *editio princeps* of the *Almagest*. This ed. was published by Grynaeus and Camerarius (Basel, 1538), but contained no mention of Pappus on the title page. F. Hultsch began, but was not able to complete, an ed. of the commentary by Pappus and Theon; see his "Hipparchos über die Grosse und Entfernung der Sonne," in *Berichte über die Verhandlungen der königlich sachsischen Gesellschaft der Wissenschaften,* Philologisch-Historische Klasse, **52** (1900), 169–200. This work is vitiated by a fundamental error—what he thought was a working over of Pappus' text by Theon was really the same text—an error he would undoubtedly have recognized had he been able to continue his research.

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Collection. The works cited in the previous paragraph are helpful. See also Robert Simson, *Apollonii Pergaeii locorum planorum libri II restituti* (Glasgow, 1749); "De porismatibus tractatus," in *Opera quaedam reliqua*(Glasgow, 1776), pp. 315–594; Michel Chasles, *Les trois livres de porismes d'Euclide, retablis pour la premiere fois, d'apres la notice et les lemmes de Pappus* (Paris, 1860); Paul Tannery, "L'arithmetique des Grecs dans Pappus" in *Memoires de la Societe des sciences physiques et naturelles de Bordeaux,***3** (1880), 3510–371, repr. in *Memoires Scientifiques,***1** (1912), pp. 80–105; "Note sur le probleme de Pappus" ("Pappus' problem" in the sense used by Descartes), in *Oeuvres de Descartes,* C. Adam and P. Tannery, eds., VI (Paris, 1902), 721–725, repr. in *Mémoires scientifiques,***3** (1915), 42–50; J. S. Mackay, "Pappus on the Progressions" (a translation of Pappus on means), in *Proceedings of the Edinburgh Mathematical Society,***6** (1888), 48–58; J. H. Weaver, "Pappus," in *Bulletin of the American Mathematical Society,* 23 (1916–1917), 127–135; "On Foci of Conics," *ibid.,* 361–365; N. Khanikoff, "Analysis and Extracts of *Book of the Balance of Wisdom,* an Arabic Work on the Water Balance, Written" by al–Khazini in the Twelfth Century, in *Journal of the American Oriental Society,***6** (1860), lecture 1, ch. 7, 40–53; and al–Khazini, *Kitab mizan al-hikma* (Hyderabad, Deccan, A.H. 1359 [A.D. 1940–1941]). For further references see Bibliography to article on al–Khazini, in *Dictionary of Scientific Biography,* IV, 349–351. An article by Malcolm Brown, "Pappus, Plato and the Harmonic Mean," is promised for *Phronesis*.

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