Parseval Des Chênes, Marc-Antoine | Encyclopedia.com

(*b*. Rosières-aux-Salines, France, 27 April 1755; *d*. Paris, France, 16 August 1836)

mathematics.

Little is known of Parseval's life or work. He was a member of a distinguished French family and described himself as a squire; his marriage in 1795 to Ursule Guerillot soon ended in divorce. An ardent royalist, he was imprisoned in 1792 and later fled the country when Napoleon ordered his arrest for publishing poetry against the regime. He was nominated for election to the Paris Academy of Sciences in 1796, 1799, 1802, 1813, and 1828; but the closest he came to being elected was to place third to Lacroix in 1799.

Parseval's only publications seem to have been five memoirs presented to the Academy of Sciences. The Second of these (dated 5 April 1799) contains the famous Parseval theorem, given here in his own notation:

If there are two series

$A + Bf + Cf^2 + Ff^3 + \ldots = T$

as well as the respective sums T, T', then we obtain the sum of the series

$Aa + Bb + Cc + Ff + \ldots = V$

by multiplying T by T' and, in the new function $T \times T'$, substituting

for the variable f, which will yield the function V'. Then for f substitute

which will yield the new function V''. We then obtain

u being made equal to 180° after integrating.

According to Parseval, the theorem was suggested by a method of summing special cases of series of products, presented by Euler in his *Institutiones calculi differentialis* of 1755. He believed the theorem to be self-evident, suggesting that the reader multiply the two series and recall that $(\cos u + i\sin u)^m = \cos mu + i \sin mu$, and gave a simple example that would "confirm its validity." He noted that it could be used only if the imaginaries in V' and V'' cancel one another, and he hoped to overcome this inconvenience. This hope was realized in a note appended to his next memoir (dated 5 July 1801), in which he gave a simplified version of the theorem. In modern notation the theorem states:

If, in the series $M = A + Bs + Cs^2 + \ldots$ and $m = a + bs + cs^2 + \ldots$, s is replaced by $\cos u + i \sin u$, and the real and imaginary parts are separated so that

$M = P + Qi$

and

$m = p + qi$

then

(There is an error in Parseval's statement: the 2 in the right-hand side of the last equation is missing.)

In his memoirs, which were not published until 1806, Parseval applied his theorem to the solution of certain differential equations suggested by Lagrange and l'Alembert. The theorem first appeared in print in 1800, in Lacroix's *Traité des différences et des séries* (p. 377). By 1810 Delambre, in his *Rapport historique sur les progrès des sciences mathématiques depuis 1789, et sur leur état actuel*, could report that Prony had given, and published, lectures at the École Polytechnique taking Parseval's procedure into account and that Poisson had used a method dependent on an equation of this type. Since then dozens of equations have been called Parseval equations, although some only remotely resemble the original. Although Parseval's method involves trigonometric series, he never tried to find a general expression for the series coefficients; and hence he did not contribute directly to the theory of Fourier series. It should be noted that although Parseval viewed his theorem as a formula for summing infinite series, it was taken up at the end of the century as defining properties in more abstract treatments of analysis.

BIBLIOGRAPHY

I. Original Works. Parseval's five memoirs appeared in *Mémoirs présentés à l'Institut des Sciences, Lettres et Arts, par divers savans, et lus dans ses assemblées. Sciences mathématiques et physiques. (Savans étrangers.)*, **1** (1806): "Mémoire sur la résolution des équations aux différences partielles linéaires du second ordre" (5 May 1798), 478492; "Mémoire sur les séries et sur l'intégration compléte d'une équation aux différences partielles linéaires du second ordre, à coefficiens constants" (5 Apr. 1799), 638–648; "Intégration gén´rale et compléte des équations de la propagation du son, l'air étant considéré avec ses trois dimensions" (5 July 1801), 379–398); "Intégration générale et compléte de deux équations importantes dans la mécanique des fluides" (16 Aug. 1803), 524–545; and "Méthode générale pour sommer, par le moyen des intégrales définies, la suite donnée par le théoréme de M. Lagrange, au moyen de laquelle il trouve une valeur qui satisfait à une équation algébrique ou transcendente" (7 May 1804), 567–586.

II. Secondary Literature. A brief biography is in *Généalogies et souvenirs de famille; les Parseval et leurs alliances pendant trois siècles, 1594–1900,* b (Bergerac, 1901), 281–282. The memoirs are described in Niels Nielsen, *Géomètres français sous la Révolution* (Copenhagen, 1929), 192–194. The relation of Parseval's theorem to the work of Fourier is discussed in Ivor Grattan-Guinness, *Joseph Fourier, 1768–1830* (Cambridge, Mass., 1972), 238–241, written with J. R. Ravetz.

Hubert C. Kennedy