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(*b*. Clermont-Ferrand, Puy-de-Dôme, Frnace, 19 Jume 1623; *d*. Paris, France 19 August 1662) *mathematics, mechanical computation, physics, epistemology*.

Varied, original, and important, although often the subject of controversy, Pascal's scientific work was intimately linked with other aspects of his writings with his personal life, and with the development of several areas of science. Consequently a proper understanding of his contribution requires a biographical framework offering as precise a chronology as possible.

Pascal's mother, Antoninette Begon, died when he was three; and the boy was brought up by his fater Etienne, who took complete charge of his education, In 1631 the elder pascal left Clermont and moved to Paris with his son two daughters, Gilberte (1620–1987), who married Florin Peiert in 1641, and Jacqueline (1625–1661), who entered the convent of port-Royal in 1652.

The young pascal began his scientific studies about 1635 with the reading of Euclid's *Elements*. His exceptional abilities, immediately and strikingly apparent, aroused general admiration. His sister Gilberts Perier left an account, more doting than objective, of her brothers's life and in particular, of his first contacts with mathematics. According to her Pascal accompanied his father to the meetings of the "Académie Parisienne" soon after its founding by Mersenne in 1635 and played an important role in it from the first This assertion, however, is nor documented; and it appears more likely that it was at the beginning of 1639 that pascal, not yeat sixteen, began to participate in the activities of Mersenne's academy. In that year Girard Desargues had just publihsed hi *Brouillon project d'une atteinte aux événemens des recontres du cone avec un plan* but his orginality, his higly personal style and vocabulary, and his refusal to use Cartesian algebraic symbols baffled most contemporary mathematicians. As the only one to appreciate the richness of this work, which laid the foundations of projective geometry and of a unified theory of conic secions, Pascal became desargue's principle disciple in geometry.

Projective Geometry . Grasping the significance of Desargues's new conception of conics , Pascal adopted the basic ideas of the *Brouillon project* the introduction of elements at infinity the definition of a conic as any plane section of a cone with a circular base; the study of conics as perspectives of circles; and the involution determined on any straight line by a conic and the opposite sides of an inscribed quadrilater. As early as june 1639 pascal made his first great discovery, that of property equilvalnt to the theorem now known as Pascal's mystic hexagram according to it, the three points of intersection of the pairs of opposite sides of a hexagon inscribed in a conic are collinear.[1] He also soon saw the possibility of basing a comprehensive projective study of conics on his property. (The property amounts to an elegant formulation, in geometric language of the to a single conic). Next he wrote *Essay* pour les *coniques* (February, 1640) a pamphlet of which only a few copies were published [1].[2] A plan for further research, illustrated with statements of several typical propositions that he had already discovered, the *Essay* constituted the outline of a great treatise on conics that he just conceived and begun to prepare.

Pascal seems to have made considerable progress by December 1640, having deduced from his theorem most of the propositions contained in the *Conics of* A pollonius [3] however, he worked only intermittently on completing the treatise. Although Desargues and Mersenne alluded to the work in November 1642 and 1644, respectively, it was apparently not until March 1648 that Pascal obtained a purely geometric definitive general solution to the celebrated problem of Pappus, which had furnished Descartes with the principal example for illustrating the power of his new analytic geometry (1637).[4] Pascal's success marked an important step in the elaboration of his treatise on conics, for it demonstrated that in this domain projective geometry might prove as effective as the Cartesian analytic methods. Pascal therefore reserved the sixth, and final, section of his treatise, "Des lieux solides" (geometric loci composed of conics), for this problems.

In 1654 Pascal indicated that he had nearly completed the treatise[12], conceived "on the basis of a single proposition"—a work for which he had "had the idea before reaching the age of sixteen" and which he then "constructed and put in order He also mentioned some special geometric problems to which his projective method could usefully be applied: circles or spheres defined by three or four conditions; conics determined by five elements (points or tangents); geometic loci five elements (points or tangents); goe conics and a general method of perspective.

Pascal made no further mention of this treatise, which was never published, It seems that only Leibniz saw it in manuscript, and the most precise details known about the work were provided by him. In a letter [23] of 30 August 1676 to E. Périer, one of Pascal's heirs, Leibniz stated that the work merited publications and mentioned a number of points con cerning tis contents, which he divided into six parts (1)the projective generation (2) the definition and properties of the "mystic hexagram"—Pascal's theorem and its applications; (3) the projective theory of poles and polars and of centers and diameters (4) various properties related to the classic definitions of conics on the basis of their axes and foci; (5) *contacts conquies*, the construction of conics defined by five elements (points or tangents); and (6) solid loci (the problem of Pappus). Besides reading notes on a number of passages of Pacscal's treatise [15], Leibniz's papers preserve the text of the first part, "Generation conisectionum" [14].

The content and inspiration of this introductory chapter are readily apparent from the full title: "The Generation of Conics, Tangents, and Secants; or the Projection of the Circumference, Tangents, and Secants of the Circle for Every Position of the Eye and of the Plane of the Figure." The text presents in an exceptionally elegant form the basic ideas of projective geometry already set forth, in a much less explicit fashion, in Desargues's *Brouillon project*[5]. Although these few elements of Pascal's treatise preserved by Leibniz do not provide a complete picture of its contents, they are sufficient to show the richness and clarity of Pascal's conceptions once he had become fully aware of the power of projective methods. It is reasonable to assume that publication of this work would have hastened the development of projective geometry, impeded until then by the obscurity of Desargues's writings and by their limited availability. Despite the efforts of Philippe de la Hire,[6] the ultimate disappearance of the treatise on conics and the temporary eclipse of both *Essay pour les coniques* (which was not republished until 1779) and Desargues's *Brouillon project* (rediscovered in 1864) hindered the progress of projective geometry. It was not truly developed until the nineteenth century, in the work of Poncelet and his successors. Poncelet, in fact, was one of the first to draw attention to the importance of Pascal's contribution in this area.

Pascal was soon obliged to suspend the contact with the "Académie Parisienne" that had encouraged the precocious flowering of his mathematical abilities. In 1640 he and his sisters joined their father, who since the beginning of that year had been living in Rouen as a royal tax official. From the end of the 1640 until 1647 Pascal made only brief and occasional visits to Paris, and no information has survived concerning his scientific activity at the beginning of this long provincial interlude. Moreover, in 1641 he began to suffer from problems of health that several times forced him to give up all activity. From 1642 he pursued his geometric research in a more or less regular fashion; but he began to take an interest in a new problem, to the solution of which he made a major contribution.

Mechanical Computation . Anxious to assist his father, whose duties entailed a great deal of accounting, Pascal sought to mechanize the two elementary operations of arithmetic, addition and subtraction. Toward the end of 1642 he began a project of designing a machine that would reduce these operations to the simple movements of gears. Having solved the theoretical problem of mechanizing computation, it remained for him to produce such a machine that would be convenient, rapid, dependable, and easy to operate. The actual construction, however, required relatively complicated wheel arrangements and proved to be extremely difficult with the rudimentary and inaccurate techniques available. In this venture Pascal displayed remarkable practical sense, great concern for efficiency, and undeniable stubbornness. Supervising a team of workers, he constructed the first model within a few months but, judging it unsatisfactory, he decided to modify and improve it. The considerable problems he encountered soon discouraged him and caused him to interrupt his project. At the beginning of 1644 encouragement from several people, including the chancellor of France, Pierre Seguier, induced Pascal to resume the development of his "arithmetic machine." After having constructed, in his words, "more than fifty models, all different," he finally produced the definitive model in 1645. He himself organized the manufacture and sale of the machine.

This activity is the context of Pascal's second publication, an eighteen-page pamphlet [2] consisting of a "Lettre dédicatoire" to Séguier and a report on the calculating machine—its purpose, operating principles, capabilities, and the circumstances of its construction ("Avis nécessaire à ceux qui auront curiosité de voir ladite machine et de s'en servir "). The text concludes with the announcement that the machine can be seen in operation and purchased at the residence of Roberval. Pascal's first work of this scope, the pamphlet is both a valuable source of information on the guiding ideas of his project and an important document on his personality and style.

It is difficult to estimate the success achieved by Pascal's computing machine, the first of its kind to be offered for sale—an earlier one designed by W. Schickard (1623) seems to have reached only the prototype stage. Although its mechanism was quite complicated, Pascal's machine functioned in a relatively simple fashion—at least for the two operations to which it was actually applied.[7] Its high price, however, limited its sale and rendered it more a curiosity than a useful device. It is not known how many machines were built and sold; seven still exist in public and private collections.[8] For a few years Pascal was actively involved in their manufacture and distribution, for which he had obtained a monopoly by royal decree (22 May 1649) [22]. In 1652 he demonstrated his machine during a lecture before fashionable audience and presented one to Queen Christina of Sweden. For some time, however, he had been directing his attention to problems of a very different kind.

Raised in a Christian milieu, Pascal had been a practicing Catholic throughout his youth but had never given any special consideration to problems of faith. Early in 1646, however, he became converted to the austere and demanding doctrine of Saint-Cyran (Jean Duvergier de Hauranne), whose views were close to those of the Jansenists. This event profoundly marked the rest of Pascal's life. The intransigence of his new convictions was underscored at Rouen between February and April 1647, when Pascal and two friends denounced certain bold theological positions defended by Jacques Forton de Saint-Ange. This change in attitude did not, however, prevent Pascal from embarking on a new phase of scientific activity.

Fluid Statics and the Problem of the Vacuum. To understand and evaluate Pascal's work in the statics of gases and liquids, it is necessary to trace the origins of the subject and the establish a precise chronology. In his *Discorsi* (1638) Galileo had noted that a suction pump cannot raise water to more than a certain height, approximately ten meters. This observation, which seemed to contradict the Aristotelian theory that nature abhors a vacuum, was experimentally verified about 1641 by R. Maggiotti and G. Berti. V. Viviani and E. Torricelli modified the experiment by substituting mercury for water, thereby reducing the height of the column to about seventy-six centimeters. Torricelli announced the successful execution of this experiment in two letters to M. Ricci of 11 and 28 June 1644. Describing the experiment in detail, he gave a correct interpretation of it based on the weight of the external column of air and the reality of the existence of the vacuum.[9] Mersenne, informed of the work of the Italian scientists, attempted unsuccessfully to repeat the experiment, which for some time fell into neglect.

In October 1646 Mersenne's friend P. Petit, who was passing through Rouen, repeated the experiment with the assistance of Etienne and Blaise Pascal. At the end of November 1646 Petit described the event in a letter to Pierre Chanut. Meanwhile, Pascal, seeking to arrive at firm conclusions, had repeated the experiment in various forms, asserting that the results contradicted the doctrine of the horror vacui. Profiting from the existence at Rouen of an excellent glassworks, Pascal conducted a series of further experiments in January and February 1647. He repeated Torricelli's experiment with water and wine, using tubes of different shapes, some as long as twelve meters, affixed to the masts of ships. These experiments became known in Paris in the spring of 1647. Gassendi wrote the first commentary on them, and Mersenne and Roberval undertook their own experiments. The first printed account of the entire group of Pascal's experiments was *Discours sur le vide* by P. Guiffart, of Rouen, written in April 1647 and published in August of that year. Just as it was published, word reached Paris that a barometric experiment had been conducted at Warsaw in July 1647 by V. Magni, who implicitly claimed priority. Roberval responded on 22 September with a Latin *Narratio* (published at Warsaw in December), in which he established the priority of Torricelli's and Pascal's experiments and revealed new details concerning the latter.

Pascal soon intervened directly in the debate. During the summer of 1647 his health had deteriorated; and he left Rouen with his sister Jacqueline to move to Paris, where their father joined them a year later. Henceforth, Pascal maintained contacts both with the Jansenists of Port-Royal and with the secular intellectuals of Pairs, who were greatly interested in the interpretain of the experiments with the vacuum. He had two discussions on this topic with Descartes (23 and 24 September), who may have suggested that he compare barometric observations made at different altitudes.[10] This idea was also proposed by Mersenne in his *Reflexiones physico-mathematicae* (beginning of October 1647).[11] At this time Pascal wrote a report of his experiments at Rouen, a thirty-two-page pamphlet published in October 1647 *as Experiences nouvelles touchant le vide [3]*. In this "abridgment" of a larger work that the planned to write, Pascal admitted that his initial inspiration derived from the Italian barometric experiment and stated that his primary goal was to combat the idea of the impossibility of the vacuum. From his experiments he had deduced the existence of an apparent vacuum, but he asserted that the existence of an absolute vacuum was still an unconfirmed hypothesis. Consequently his pamphlet makes no reference to the explanation of the barometric experiment by means of the weight of the air, proposed by Torricelli in 1644.[12] According to his sister Jacqueline, however, Pascal had been a firm proponent of this view from 1647.[13] In any case his concern was to convince his readers; he therefore proceeded cautiously, affirming only what had been irrefutably demonstrated by experiment.

Despite his moderate position, Pascal's rejection of the theory of the impossibility of the vacuum involved him in vigorous debate. With the publication of the *Expériences nouvelles*, a friend of Descrates's, the Jesuit Estienne Noël, declared in a letter to Pascal that the upper portion of Torricelli's tube was filled with a purified air that had entered through the pores of the glass.[14] In a dazzling reply (29 October 1647) [4] Pascal clearly set forth the rules of his scientific method and vigorously upheld his position. Several days later Noël reaffirmed the essence of his views but expressed a desire to end the dispute.[15] It was indirectly resumed, however, after Noël published a new and violent critique of the Experiences nouvelles.[16] In a letter to his friend F. Le Pailleur [5], Pascal refuted Noël's second letter and criticized his recent publication. In April 1648 Étienne Pascal entered the debate against Noël.[17] The dispute soon ended, however, when Noël published a much more moderate Latin version of his short treatise.[18]

During this controversy scientists in Paris had become interested in the problem of the vacuum, devoting many experiments to it and proposing a number of hypotheses to explain it. Having participated in discussions on the topic, Pascal conceived one of the variants of the famous experiment of the vacuum within the vacuum, designed to verify the hypothesis of the column of air.[19] He seems, however, to have expected a still better confirmation of the hypothesis from a program of simultaneous barometric observations at different altitudes (at Clermont-Ferrand and at the summit of Puy de Dôme), the execution of which he entrusted to his brother-in-law, perier. One of these observations, now known as the "Puy de Dôme experiment," was carried out on 19 September 1648. Pascal immediately published a detailed, twentypage account of it, *Recit de la grande experience de l'equilibre des liqueurs. … [6]*, consisting principally of Perier's letter and report. In a short introduction he presented the experiment as the direct consequence of his *Experiences nouvelles,*and the text of a letter of 15 November 1647 to Perier, in which he explained the goal of the experiment and the principle on which it was based. He concluded by pointing out his analogous experiment at the Tour St. Jacques in Paris and by announcing his conversion to the principles of the existence of the absolute vacuum and of the weight of air.

The *Récit*, which marks an important phase of Pascal's research on the vacuum, gave rise to two heated controversies.[20] The first arose at the end of the seventeenth century, when several authors denied Pascal's priority with regard to the basic principle of the Puy de Dôme experiment. This question, however, is of only secondary importance. While it appears that the principle was formulated simultaneously—on the basis of different presuppositions—by Pascal, Descartes, and Mersenne, only Pascal tested it and integrated it into an exceptionally cogent chain of reasoning.

The second controversy was launched in 1906–1907 by F.Mathieu, who challenged both Pascal's scientific originality and his honesty. He accused Pascal of having fabricated the letter to Périer of 15 November 1647 after completion of the event, in order to take credit for the experiments of the vacuum within the vacuum and of Puy de Dôme. Although the heated debate that ensued did not produce any unanimously accepted conclusions, it did stimulate research that brought to light many unpublished documents. In an assessment of the question J. Mesnard, after examining the arguments and clarifying many points, suggests that Pascal probably did send the contested letter to Perier on 15 November 1647 but may have altered the text slightly for publication. This compromise judgment is probably close to the truth.

At the beginning of 1649 Périer, following Pascal's instructions, began an uninterrupted series of barometric observations designed to ascertain the possible relationship between the height of a column of mercury at a given location and the state of the atmosphere. The *expérience continuelle,* which was a forerunner of the use of the barometer as an instrument in weather forecasting, lasted until March 1651 and was supplemented by parallel observations made at Paris and Stockholm.[21] Pascal continued working on a major treatise on the vacuum; but only a few fragments, dating from 1651, have survived: a draft of a preface [7] on the relationships between reason and authority and between science and religion, and two short passages published by Périer in 1663.[22] In June 1651 a Jesuit accused Pascal of claiming credit for Torricelli's experiment. In two letters [9, 10], of which only the first was printed, Pascal recounted—with several serious errors—the history of that experiment and laid claim to the idea of the Puy de Dôme experiment.

Pascal soon put aside his great treatise on the vacuum in order to write a shorter but more synthetic work. Divided into two closely related parts, this work is devoted to the laws of hydrostatics and to the demonstration and description of the various effects of the weight of air. It was completed about the beginning of 1654 and marked the end of Pascal's active research in physics. It was published posthumously by Perier, along with several appendices, in 1663 as *Traités de l' équilibre des liqueurs et de la pesanteur de la masse de l' air* … [13]. The fruit of several years of observations, experiments, and reflection, it is a remarkable synthesis of new knowledge and theories elaborated since the work of Stevin and Galileo. The highly persuasive *Traités*, assembling and corrdiating earlier results and recent discoveries, are characterized above all by their rigorous experimental method and by the categorical rejection of Scholasticism. In hydrostatics Pascal continued the investigations of Stevin, Galileo, Torricelli, and Mersenne. He clearly set forth the basic principles of the science, although he did not fully succeed in demonstrating them satisfactorily. In particular he provided a lucid account of the fundamental concept of pressure.

The untoward delay in the publication of the *Traités* obviously reduced its timeliness; for in the meanwhile the study of the weight of air and the existence of the vacuum had been profoundly affected by the work of Otto von Guericke and Robert Boyle.[23] In this area, in fact, the *Traités* essentially systematized, refined, and developed experiments, concepts, and theories that, for the most part, had already been discussed in the *Expériences nouvelles* and the *Récit*, Pascal's influence, therefore, must be measured as much by the effect of these preliminary publications and the contemporary writings of Mersenne and Pecquet, which reflect his thinking, as by posthumous *Truités*.[24] This influence was certainly considerable, for it partially conditioned all subsequent research on the subject; but it cannot easily be separated from that of, for instance, Roberval and Auzout, who participated in the rapid progress of research on the vacuum at Paris in 1647 and 1648. Nevertheless, for their synthetic treatment of the subject, clarity, and rigor, the *Traités* are indisputably a classic of seventeenth-century science.

Although from October 1646 Pascal had been deeply interested in problems of the vacuum, he was often impeded in his research by poor health and by religious concerns. The death of his father in September 1651 and the entry of his sister Jacqueline into the convent of Port-Royal in January 1652 marked a turning point in his life. In better health and less preoccupied with religious problems, he pursued his scientific work while leading a more worldly existence. Beginning in the summer of 1653 he frequently visited the duke of Roannez. Through the duke he met the Chevalier de Méré, who introduced him to the problems of games of chance. At the beginning of 1654, in an address [12] to the Académie Parisienne de Mathématique, which was directed by F. Le Pailleur, Pascal listed the works on geometry, arithmetic, and physics that he had already completed or begun writing and mentioned, in particular, his recent research on the division of stakes.[25]

Calculus of Probabilities. The Arithmetical Triangle. The year 1654 was exceptionally fruitful for Pascal. He not only did the last refining of his treatises on geometry and physics but also conducted his principal studies on arithmetic, combinatorial analysis, and the calculus of probability. This work can be seen in his correspondence with Fermat [16] and his *Traité du triangle arithmétique* [17].

Pascal's correspondence with Fermat between July and October 1654 marks the beginning of the calculus of probability. Their discussion focused on two main problems. The first concerned the probability that a player will obtain a certain face of the die in a given number of throws. The second, more complex, consisted in determining, for any game involving several players, the portion of the stakes to be returned to each player if the game is interrupted. Fermat succeeded in solving these problems by using only combinatorial analysis. Pascal, on the other hand, seems gradually to have discovered the advantages of the systematic application of reasoning by recursion. This recourse to mathematical induction, however, is not clearly evident until the final section of the *Traité du triangle arithmétique,* of which Fermat received a copy before 29 August 1654.

The *Traité* was printed in 1654 but was not distributed until 1665. Composed partly in French and partly in Latin, it has a complex structure; but the discovery of a preliminary Latin version of the first part makes it easier to trace its genesis.[26] Although the principle of the arithmetical triangle was already known,[27] Pascal was the first to make a comprehensive study of it. He derived from it the greatest number of applications, the most important and original of which are related to combinatorial analysis and especially to the study of the problems of stakes. Yet it is impossible to appreciate Pascal's contribution if it is considered solely from the perspective of combinatorial analysis and the calculus of probability. Several modern authors have shown that Pascal's letters to Fermat and the *Traité du triangle arithmétique* can be fully understood only when they are seen as preliminary steps toward a theory of decision.[28]

As E. Coumet has pointed out, Pascal's concern, beyond the purely mathematical aspect of the problems, was to link decisions and uncertain events. His aim was not to define the mathematical status of the concept of probability—a term that he did not employ—but to solve the problem of dividing stakes. This innovative effort must therefore be viewed in the context of the discussions conducted by jurists, theologians, and moralists in the sixteenth and seventeenth centuries on the implications of chance in the most varied circumstances of individual and community life. Unrecognized until recently, this aspect of Pascal's creative work is revealed in its full significance in the light of recent ideas on game theory and decision theory.

On the other hand, Pascal's research on combinatorial analysis now appears much less original. Considered in the context of the vigorous current of ideas on the subject in the sixteenth and seventeenth centuries, it is noteworthy less for the originality of its results than for the clarity, generality, and rigor with which they are presented.[29] Pascal's contribution to the calculus of probability is much more direct and indisputable: indeed, with Fermat he laid the earliest foundations of this discipline.[30] The *Traité du triangle arithmétique* contains only scattered remarks on the subject; in addition, only a part of the correspondence with Fermat [16] has been preserved, and its late publication (1679 and 1779) certainly reduced its direct influence. Fortunately, through Huygens the original contribution of Pascal and Fermat in this area became quickly known. During a stay in Paris in 1655 Huygens was informed in detail of their work, and he recast their ideas in the light of his own conceptions in his *Tractatus de ratiociniis in aleae ludo*. With its publication in 1657 the essential elements of the new science were revealed.[31] Nevertheless, the calculus of probability did not experience further development until the beginning of the eighteenth century, with Jakob I Bernoulli, P. R. de Montmort, and A. de Moivre.

Unsatisfied by his worldly life and intense scientific activity, Pascal was again drawn to religious concerns. Following a second conversion, during the famous "nuit de feu" of 23 November 1654, he abandoned his scientific work in order to devote himself to meditation and religious activity and to assist the Jansenists in their battle against many enemies, particularly the Jesuits. Working anonymously, between 13 January 1656 and 24 March 1657 Pascal composed the eighteen *Lettres provinciales* with the assistance of his friends from Port-Royal, Antoine Arnauld and Pierre Nicole. A masterpiece of polemic, this eminent contribution to the debate then agitating Christian doctrine was first published as a collection in 1657 under the pseudonym Louis de Montalte. Although Pascal produced other ploemical writings, he worked primarily on preparing a defense of Christianity directed to nonbelievers. This unfinished project was the source of several posthumously published writings, the most important being the *Pensées,* published in 1670. The object of numerous commentaries and penetrating critical studies, this basic work fully displays Pascal's outstanding philosophical and literary talents.

Although concerned above all with meditation and religious activities during this period, Pascal was not totally estranged from scientific life thanks to his friends, particularly Carcavi. Around 1657, at the request of Arnauld, Pascal prepared a work entitled *Éléments de géométrie,* of which there remain only a few passages concerning methodology: the brief "Introduction à la géométrie," preserved among Leibniz's papers [18]; and two fragments, "De l'esprit géométrique" and "De l'art de persuader" [19]. Finally, in 1658 Pascal undertook a brilliant, if short-lived, series of scientific studies.

The Calculus of Indivisibles and the Study of Infinitesimal Problems. During 1658 and the first months of 1659 Pascal devoted most of his time to perfecting the "theory of indivisibles," a forerunner of the methods of integral calculus. This new theory enabled him to study problems involving infinitesimals: calculations of areas and volumes, determinations of centers of gravity, and rectifications of curves.

From the end of the sixteenth century many authors, including Stevin (1586), L. Valerio (1604), and Kepler (1609 and 1615), had tried to solve these fundamental problems by using simpler and more intuitive methods than that of Archimedes, which was considered a model of virtually unattainable rigor.[32] The publication in 1635 of Cavalieri's *Geometria* marked the debut of the method of indivisibles;[33] its principles, presentation, and applications were discussed and elaborated in the later writings of Cavalieri (1647 and 1653) and in those of Galileo (1638), Torricelli (1644), Guldin (1635–1641), Gregory of Saint-Vincent (1647), and A. Tacquet (1651). (The research of Fermat and Roberval on this topic remained unpublished.)[34] The method, which assumed various forms, constituted the initial phase of development of the basic procedures of integral calculus, with exception of the algorithm.

Pascal first referred to the method of indivisibles in a work on arithmetic of 1654, "Potestatum numericarum summa."[35] He observed that the results concerning the summattion of numerical powers made possible the solution of certain quadrature problems. As an example he stated a known result concerning the integral of x_n for whole n, in modern notation.[36] This arithmetical interpretation of the theory of indivisibles permitted Pascal to give a sufficiently precise idea of the order of infinitude[37] and to establish the natural relationship between "la mesure d'une grandeur continue" and "la sommation des puissances numériques." In the fragment "De l'esprit géométrique" [19], composed in 1657, he returned to the notion of the indivisible in order to specify its relationship to the notions of the infinitely small and of the infinitely large and to refute the most widespread errors concerning it.

At the beginning of 1658 Pascal believed that he had perfected the calculus of indivisibles by refining his method and broadening its field of application. Persuaded that in this manner he had discovered the solution to several infinitesimal problems relating to the cycloid or *roulette,* he decided to challenge other mathematicians to solve these problems[38] Although rather complicated, the history of this contest is worth a brief recounting because of its important repercussions during a crucial phase in the birth of infinitesimal calculus. In an unsigned circular distributed in June 1658, Pascal stated the conditions of the contest and set its closing date at 1 October [20a]. In further unsigned circulars and pamphlets [20], issued between July 1658 and January 1659, he modified or specified certain of the conditions and announced the results. He also responded to the criticism of some participants and sought to demonstrate the importance and the originality of his own solutions.

Most of the leading mathematicians of the time followed the contest with interest, either as participants (A. de Lalouvère and J. Wallis) or as spectators working on one or several of the questions proposed by Pascal or on related problems—as did R. F. de Sluse, M. Ricci, Huygens, and Wren.[39] Their solutions having been judged incomplete and marred by errors, Lalouvére and wallis were eliminated. Their heated reactions to this decision were partially justified by the bias it displayed and the commentaries that accompabied it.[40] This bias, which also appears in certain passages of Pascal's *Histoire de la roulette* [20b, 20d], was the source of intense polemics concerning, in particular, the importance of Torricelli's original contribution.[41] At the end of the contest Pascal published his own solutions to some of the original problems and to certain problems that had been added in the meantime. In December 1658 and January 1659 he brought out, under the pseudonym A. Dettonville, four letters setting forth the principles of his method and its applications to various problems concerning the cycloid, as well as to such questions as the quadrature of surfaces, cubature of volumes, determination of centers of gravity, and rectification of curved lines. In February 1659 these four pamphlets were collected in *Lettres de A. Dettonville contenant quelques-unes de ses inventions de géométrie* … [21].

This publication of some 120 pages has a very complex structure. The first of the *Lettres* consists of five sections with independent paginations, and the three others appear in inverse order of their composition.[42] Thus only by returning to the original order is it possible to understand the logical sequence of the whole, follow the development of Pascal's method, and appreciate the influence on it of the new information he received and of his progress in mastering infinitesimal problems.[43]

When he began the contest, Pascal knew of the methods and the chief results of Stevin, Cavalieri, Torricelli, Gregory of Saint-Vincent, and Tacquet; but he was not familiar with the bulk of the unpublished research of Roberval and Fermat. Apart from this information, and in addition to the arithmetical procedures that he applied, starting in 1654, to the solution of problems of the calculus of indivisibles, Pascal possessed a new method inspired by Archimedes. It was elaborated on a geometric foundation, its point of departure being the principle of the balance and the concepts of static moment and center of gravity. Pascal learned of the importance of the results obtained by Fermat and Roberval—notably in the study of the cyucloid—at the time he issued his first circular. This information led him to modify the subject of the contest and to develop his own method further. Similarly, in August 1658, when he was informed of the result of the rectification of the cycloid, Pascal extended rectification to other arcs of curves and then undertook to determine the center of gravity of these arcs, as well as the area and center of gravity of the surfaces of revolution generated by their revolution about an axis. Consequently the *Lettres* present a method that is in continual development, appearing increasingly complex as it becomes more precise and more firmly based. The most notable characteristics of this work, which remained unfinished, are the importance accorded to the determination of centers of gravity, the crucial role of triangular sums and statical considerations, its stylistic rigor and elegance, and the use of a clear and precise geometric language that partially compensates for the absence of algebraic symbolism.[44] Among oustanding contributions of the work are the discovery of the equality of curvature of the generalized cycloid and the ellipse; the deepening of the concept of the indivisible; a first step toward the concept of the definite integral and the determination of its fundamental properties; and the indirect recourse to certain methods of calculation, such as integration by parts.

Assimilated and exploited by Pascal's successors, these innovations contributed to the elaboration of infinitesimal methods. His most productive contribution, however, appears to have been his implicit use of the characteristic triangle.[45] Indeed, Leibniz stated that Pascal's writings on the characteristic triangle were Pascal's writings on the characteristic triangle were and especially fruitful stimulus for him.[46] This testimony from one of the creators of infinitesimal calculus indicates that Pascal's work marked an important stage in the transition from the calculus of indivisibles to integral calculus. Pascal was unable, howeverm to transcend the overly specific nature of his conceptions. Neither could he utilize to full effect the power and generality of the underlying methods nor develop the results he obtained. This partial failure can be attributed to two causes. First, his systematic refusal to adopt Cartesian algebraic symbolism prevented him from realizing the necessity of the formalization that permitted Leibniz to create the integral calculus. Second, his preoccupation with mystic concerns led him to interrupt his research only a short time after he had begun it.

Early in 1659 Pascal again fell gravely ill and abandoned almost all his intellectual undertakings in order to devote himself to prayer and to charitable works.[47] In 1661 his desire for solitude increased after the death of his sister Jacqueline and a dispute with his friends from Port-Royal. Paradoxically, it was at this time that Pascal participated in a project to establish a public transportation system in Paris, in the form of carriages charging five *sols* per ride—a scheme that went into effect in 1662.[48] Some writers have asserted that Pascal's doctrinal intransigence had diminished in this period to such a point that at the moment of his death he renounced his Jansenist convictions, but most of the evidence does not support this interpretation.

Pascal was a complex person whose pride constantly contended with a profound desire to submit to a rigorous, Augustinian insistence on self-denial. An exceptionally gifted polemicist, moralist, and writer, he was also a scientist anxious to help solve the major problems of his day. He did not, it is true, produce a body of work distinguished by profound creativity, on the model of such contemporaries as Descartes, Fermat, and Torricelli. Still, he was able to elucidate and systematize several rapidly developing fields of science (projective geometry, the calculus of probability, infinitesimal calculus, fluid statics, and scientific methodology) and to make major original contributions to them. In light of this manifold achievement Pascal, a leading opponent of Descartes, was undoubtedly one of the outstanding scientists of the midseventeenth century.

NOTES

1. The first known formulation of this theorem was as lemma 1 of *Essay pour les coniques*. It clearly differs from the modern statement by not referring explicitly to the inscribed hexagon and by apparently being limited to the case of the circle (even though the corresponding figure illustrates the case of the ellipse). According to remarks made by Leibniz, it seems that this theorem, in its hexagonal formulation and under the name "hexagramme mystique," held a central place in Pascal's treatise on conics, now lost. The fact that the *Essay pour les coniques* contains only statements without demonstrations makes it impossible to ascertain the precise role Pascal assigned to this theorem in 1640.

2. The numbers in square brackets refer to the corresponding works listed in sec. 1 of the bibliography. For a more detailed study of the *Essay,* see R. Taton, in *Revue d'histoire des sciences,* **8** (1955), 1–18, and in *L'oeuvre scientifique de Pascal* (Paris, 1964), 21–29; and J. Mesnard, ed., *Blaise Pascal*. *Oeuvres complétes,* II (1971), 220–225 (cited below as Mesnard).

3. See Mersenne's letter to Theodore Haak of 18 Nov. 1640, in Mesnard, II, 239.

4. On the references by Desargues and Mersenne, see *ibid.,* 279–280, 299. On the problem of Pappus, see Mersenne's letter to Constantijn Huygens of 17 Mar. 1648 in C. Huygens,*Oeuvres complétes de Christiaan Huygens, publiées par la Société Hollandaise des Sciences,* II (1888), 33, and in Mesnard, II, 577–578. On Descartes, see Taton, in *L'oeuvre scientifique de Pascal,* 45–50; and M.S. Mahoney, "Descartes: Mathematics and Physics," in *DSB,* IV, 56.

5. See Taton, in *L'oeuvre scientifique*…,55–59 (for "Generatio conisectionum") and 53–72 (for the treatise as a whole). See also his "Desargues," in *DSB,* IV, 46–51.

6. See Taton, "La Hire, Philippe de," in *DSB,* VII, 576–578.

7. See D. Diderot, "Arithmétique (Machine)," in *Encyclopédie,* I (1751), 680–684.

8. See J. Payen, in L'oeurre scientifique tie Pascal, 229–247.

9. See, in particular, C. De Waard, L'e_vpcrience barenu4rique, ses amcccc/ents et ses explications (Thouars, 1936), 110–123; M. Gliozzi, –Origine e sviluppi dell'espericnza torricelliana,— in Opere de Erangelista Torricelli, G. Loria and G. Vassura, eds., IV (Faenza, 1944), 231–294; and W. E. K. Middleton, The History of the Barometer (London, 1964).

10. Jacqueline Pascal gave some details of these meetings in a letter to her sister Gilberte of 25 Sept. 1647. (See Mesnard, II, 478–482.) In a letter to Mersenne of 13 Dec. 1647 and in two letters to Carcavi of I I June and 17 Aug. 1649 (see ibid., 548–550, 655–658, 716–719) Descartes stated that he had suggested this idea, which was the origin of the celebrated Puy de Dome experiment of 19 Sept. 1648, to Pascal.

11. See ibid., 483–489.

12. Torricelli held that the space above the column of mercury was empty. Considering the horizontal plane determined by the exterior level of the mercury, he asserted that the weight of the column of mercury equaled the weight of a column of air of the same base, which implied simultaneously the existence of the vacuum, the weight of the air, and the finiteness of the terrestrial atmosphere. In 1651 Pascal admitted that he was aware of Torricelli's explanation as early as 1647 (see ibid., 812), but he insisted that at that time the explanation was only a conjecture; it had yet to be verified by experiment, and for this reason he undertook the experiment of Puy de Dome.

13. Letter to Gilberte Pascal of 25 Sept. 1647 (see ibid., 482).

14. See ibid., 513–518.

15. See ibid., 528–540.

16. It was a brief work with the picturesque title Le plein du ride (Paris, 1648); see Mesnard, 11, 556–558. This work was reprinted by Bossut in Oeuvres de Blaise Pascal, C. Bossut, ed., IV (The Hague, 1779), 108–146.

17. See Mesnard, II, 584–602.

18. E. Noel, Plenont experimenlis novis confirmation (Paris, 1648); see Mesnard, II, 585.

19. This experiment is mentioned without details in Pascal's Récit… (see Mesnard, II, 678). The reality of the experiment is confirmed by the quite precise description of it that Noel gave in his Gravitas comparata (Paris, 1648); on this point see Mesnard, II, 635–636, which presents the Latin text, a French translation, and an explanatory diagram derived from an earlier

study by P. Thirion. The principle of this experiment consists of conducting Toricclli's experiment in an environment where the pressure can be varied from atmospheric pressure to zero. Other variants were devised at almost the same time by Robcrval (Mesnard, II, 637–639) and by Auzout (ibid., 767–771). A fourth variant, easier to carry out in practice, is described in Pascal's Traitcs de l'cquilibre ties liqueurs et de la pesantcur de la masse de fair… (ibid., 1086–1088).

20. See ibid., 653–676.

21. F. Perier published an account of them in 1663 as an appendix to Pascal's Traitcs tie l'cqui/ibre… (pp. 195–209); see Mesnard, 11, 738–745. The fact that the first observations made at Stockholm were carried out by Descartes appears to indicate that he had become reconciled with Pascal.

22. The preface was not published until 1779, when it appeared under the title –De I'autorite en matiere de philosophic— (Bossut, II, 1–12). The passages published by Pericr appear at the end of Pascal's Traitcs de l'cquilibrc… (pp. 141–163).

23. See F. Krafft, –Guericke,— in DSB, V, 574–576; and C. Webster, –The Discovery of Boyle's Law and the Concept of the Elasticity of Air in the Seventeenth Century,— in Archive for History of the Exact Sciences, 2 (1965), 441–502, esp. 447–458.

24. M. Mersenne, Reflecvione.s plgvsico-ma thematieae (Paris, 1647); and J. Pecquet, Erperintenta nova anatomica (Paris, 1651). To these storks should be added publications by Noel, already cited, as well as those of Roberval and of V. Magni (see Webster, op. cit.), and, above all, the correspondence of scientists from Italy, France, England, Poland, and other European countries.

25. The word used in French to designate this problem, parti, is the past participle (considered as the noun form) of the verb partir, understood in the sense of –to share.— The problem consists in finding, for a game interrupted before the end, the way of dividing the stakes among the players in proportion to their chances of winning at the time of interruption.

26. See Mesnard, II, which provides an introduction to the texts (pp. 1166–1175) and the texts themselves, both of the first printing, in Latin with French translation (pp. 1176–1286), and of the second, with translation of the Latin passages (pp. 1288–1332).

27. This figure, in more or less elaborated forms that were equivalent to lists of coefficients of the binomial theorem, appeared as early as the Middle Ages in the works of Nasir al-Din al Tusi (1265) and Chu Shih-chieh (1303). The arithmetical triangle reappeared in the sixteenth and seventeenth centuries in the writings of Apian (1527), Stifel, Scheubel, Tartaglia, Bombelli, Peletier, Trenchant, and Oughtred. But Pascal was the first to devote to it a systematic study linked to many questions of arithmetic and combinatorial analysis.

28. See, for example, E. Coumet, –La theorie du hasard est-elle nee par hasard?" in Annales. Economies, soeietes, civilisations (1970), 574–598, as well as the studies of G.-T. Guilbaud (1952) and the other works on operational research, cybernetics, game theory, and other fields cited in Coumet's article (p. 575, notes I and 2).

29. See E. Coumet, –Mersenne, Frenicle et ('elaboration de ('analyse combinatoire clans la premiere moitie du XVIIe siecle— (a typescript thesis, Paris, 1968), and –Mersenne : Denombrements, repertoires, numerotations de permutations,— in Mathcmatiques et sciences humaines, 10 (1972), 5–37.

30. See 1. Todhunter, A History of the Mathematical Theory of Probability Front the Time of Pascal to That of Laplace (Cambridge-London, 1865; repr. New York, 1949), 7–21.

31. See F. Van Schooten, Exereitationam mathetnaticartan libri quinque (Leiden, 1657), 519–534, and H. J. M. Bos's article on Huygens in DSB, VI, 600.

32. On Archimedes see the article by M. Clagett in DSB, 1, 213–231, esp. 215–222, for his infinitesimal methods and 229 for the diffusion of his writings in the sixteenth and seventeenth centuries. It should be noted that at this period mathematicians were aware only of his rigorous method of presentation, which Gregory of Saint-Vincent termed the —method of exhaustion.— Archimedes' much more intuitive method of discovery did not become known until the rediscovery of his Method in 1906. On the infinitesimal work of Stevin, Valerio, and Kepler, see C. B. Boyer, The Concept of Calc ulus (New York, 1949), 98–111.

33. B. Cavalieri, Geometria indimisihilibas conlinaoraat tiara quadam ratione promola (Bologna, 1635). On this subject see Boyer, op. c it., pp. 111–123; A. Koyre, in Etudes d'histoire dc' la pensce scietuifigae (Paris, 1966), 297–324; and the article on Cavalieri by E. Carruccio in DSB, III, 149–153.

34. See Boyer, op. tit., pp. 123–147, 154–165.

35. Reprinted in Mesnard, 11, 1259–1272; see esp. 1270–1272. This work is the next to last-but also one of the earliest written-of the brief treatises making up the Traitc du triangle arithmctique [17].

36.“The sum of all the lines of any degree whatever is to the larger line and to the higher degree as unity is to the exponent of the higher degree— (Mesnard, 11, 1271). On Pascal’s infinitesimal work see H. Bosmans, in Archivio di storia dclla scien_a, 4 (1923), 369–379; Boyer, op. cit., pp. 147–153; F. Russo, in L’oeurre scientifique de Pascal (Paris, 1964), 136–153; and P. Costabel, ibid., 169–206.

37.“In the case of a continuous magnitude (*grandeur continue*), magnitudes of any type (*genre*), when added in any number desired to a magnitude of higher type, do not increase it at all. Thus, points add nothing to lines, [nor] lines to surfaces, [nor] surfaces to solids, or, to use the language of numbers in a treatise devoted to numbers, roots do not count with respect to squares, [nor] squares with respect to cubes.... Therefore, lower degrees should be neglected as possessing no value” (Mesnard, II, 1271–1272).

38. The cycloid is the curve generated by a point *M* of the circumference of a circle (*C*) that rolls without sliding on a straight line *D*. *AB*, the base of the cycloid, is equal to $2\pi rr$ (where *r* is the radius of the circle *C*). Derived curves are obtained by the displacement of a point M’ situated on the interior (curtate cycloid) or M” on the exterior (prolate cycloid) of the moving circle. Defined by Roberval in 1637,

these curves had served since that year—under the name of *roulettes*, trochoids, or cycloids—as key examples for the solution of various problems pertaining to the infinitesimal calculus. These problems included the construction of tangents to plane curves by the use of the method of indivisibles, the determination of plane areas, the calculation of volumes, and the determination of centers of gravity. The cycloid thus played an important role in the patient efforts that resulted in the transition from the method of indivisibles to the infinitesimal calculus. Between 1637 and 1647 Roberval, then Fermat and Descartes, and finally Torricelli were particularly interested in the solution of infinitesimal problems associated with the cycloid; and bitter priority disputes broke out between Roberval and Descartes and then between Roberval and Torricelli. But in June 1658, when Pascal distributed his first circular, it appears that he had only a very imperfect knowledge of prior work on this subject.

The practice of setting up a contest was very common at the time. A similar contest, initiated by Fermat in January 1657 on questions of number theory, continued to set Fermat against some of the participants, notably Wallis. See O. Becker and J. E. Hofmann, *Geschiclate tier Mathenatik* (Bonn, 1951), 192–194.

The contest problem was the following: Given an arch of the cycloid of base *AB* and of axis *CF*, one considers the semicurvilinear surface *CZY* defined by the curve, the axis, and a semichord *ZY* parallel to the base. The problem is to find (1) the area of *CZY* and its center of gravity; (2) the volumes of the solids V_1, and V_2 generated by the revolution of *CZY* about *CY* and about *ZY*, as well as their centers of gravity; and (3) the centers of gravity of the semisolids obtained by cutting V_1 and V_2 by midplanes.

39. In his *Histoire cle la roulette* [20b], Pascal mentions the results sent to him by these four authors and notes, in particular, the rectification of the arch of the cycloid communicated to him by Wren. He points out that he has extended this operation to an arbitrary arc *AZ* originating at the summit of the cycloid and that he has determined the center of gravity of this arc *AZ*, as well as the areas and centers of gravity of the surfaces of revolution generated by the rotation of *AZ* about the base or about the axis of the cycloid. Carcavi, the president of the jury, also mentioned the results sent by Fermat, particularly those on the areas of the surfaces of revolution.

40. See A. Lalouvere, *Veterun geometria promota in septena de cycloide libris* (Toulouse, 1660); and J. Wallis, *Tractatus duo, prior de cycloide, posterior de cissoide* (Oxford, 1659). On the latter publication see K. Hara, “Pascal et Wallis au sujet de la cycloide,” in two parts: the first in *Annals of the Japanese Association for the Philosophy of’Science*, **3** , no. 4.1969), 36–57, and the second in *Gallia* (Osaka), nos. 10–11.1971), 231–249.

41. See, in particular, C. Dati, *Lettera delta vera storia delta cicloide* (Florence, 1663).

42. This question is raised by K. Hara, in “Quelques additions a 1’examen des textes mathematiques de Pascal,” in *Gallia* (Osaka), no. 7.1962); by P. Costabel, in *L’oeuvre scientifique c/c Pascal,* 169–198. and by J. Mesnard, in Mesnard, 1, 31–33.

43. The original order is reproduced in vol. III of Mesnard’s ed. of Pascal’s works (in preparation).

44. See Bosmans, op. cit.; Boyer, op. cit., pp. 147–153. Russo, op. cit., pp. 136–153. and Costabel, *ibid*., pp. 169–206.

45. See Russo, op. cit., pp. 149–151. It should be noted that the expression “characteristic triangle” was introduced not by Pascal but by Leibniz. See also Boyer, op. cit., pp. 152–153. Boyer points out that this figure had previously been used by Torricelli and Roberval and even by Snell (1624). In modern notation, the characteristic triangle at a point $M(x[_0], y[_0])$ of a plane curve (*C*) of equation y = f(x) is a right triangle, the first two sides of which, parallel to the axes Ox and Oy, are of length alx and cly; its diagonal, of length as, is parallel to the tangent to the curve (C) at M.

46. See a letter from Leibniz to Jakob I Bernoulli of Apr. 1703, in Leibniz, Matheanatische Schriften, C. L Gerhardt, ed., III (Halle, 1856), 72–73. This letter is reproduced by J. Itard in Histoire gen%rale des sciences, 2nd ed., 11.Paris, 1969), 245–246. For other statements by Leibniz concerning his knowledge of Pascal's writtings, see P. Costabel, in *L'oeuvre scientifique de Pascal*, 201–205.

47. Pascal wrote again to Fermat (10 Aug. 1660), met Huygens (5 and 13 Dec. 1660), and conversed with the duke of Roannez on the force of rarefied air and on flying. These are the few indications that we have regarding Pascal's scientific activity during the last three years of his life.

48. See M. Duclou, *Les carrosses` cinq sols* (Paris, 1950).

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c. *Oeuvres completes de Blaise Pascal,* J. Chevalier, ed. (Paris, 1954), in Bibliothèque de la Pleiade," abbrev. a s PL.

d. *Blaise Pascal. Oeuvres completes*, J. Mesnard, ed., 2 vols. to date (Paris, 1964–1971); a bbrev. as Mesnard. This last ed., which surpasses all previous ones, so far comprises only vol. 1 (*Introduction generale and Documents generaux*) and vol. II (*Oeuvres diverses, 1623–1654*). It has been used in preparing this article.

Each reference to a passage in one of these eds. will consist of the abbreviation, the volume number, year of publication of the volume, and page number. The list below includes most of Pascal's surviving scientific writings, cited in the order in which they were written. For each writing there is the title, its presumed date of composition, and its various eds.: the first (indicated as "orig." if published during Pascal's lifetime and as "1st ed." if posthumous) and the chief subsequent eds. (in separate vols. and in the sets of complete works cited above, as well as any other ed. containing important original material).

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2. *Lettre d́dicatoire` Monseigneur le Chancelier sur le sujet de la machine nouve/lenient invent́ par le sieur B. P. pour faire toutes sortes d'općrations d'arilthmetique par un mouvement ŕgĺ sans plume ni jetons avec rut avis ńcessaire` ceux qui auront curiosit́ de voir ladite machine et de s'en servir* (1645). Orig. (Paris, 1645); Bossut, IV (1779), 7–24; G.E., 1 (1908), 291–314; PL (1954), 347–358; Mesnard, 11 (1971), 329–341.

3. *Exṕriences nouvelles touchant le vide*…. Avec un discours sur le ḿme sujet… d́d́` Monsieur Pascal, conseiller du roi… par le sieur B. P. son fils. Le tout ŕduit en abŕg et donń par nuance d'un plus grand trait́ sur le ḿme sujet *(Sept.-early Oct. 1647). Orig.* (Paris, Oct. 1647); Bossut, IV (1779), 51–68; G.E., 11 (1908), 53–76; PL (1954), 359–370; Mesnard, 11 (1971), 493–508.

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12. “Celeberrimae matheseos academiae Parisiensis” (Paris, 1654). 1st. ed., Bossut, IV (1779), 408–411; G.E., 111 (1908), 293–308; PL (1954), 71–74, 1400–1404 (French trans.); Mesnard, 11 (1971), 1121–1135 (with French trans.).

13. *Traiťs de lʹquilibre des liqueurs et de la pesanteur de la masse de l’air. Contenant l’explication des causes de diners gets de la nature qui n’avaient point etc bien connus jttsgttes ici, et particulièrement de ceux que l’on avait attribués d I’horrcur du vide* (completed at the latest in 1654). 1st ed. (Paris, Nov. 1663). The text of the Traites corresponds to pp. 1–140; pp. 141–163 reproduce the only two fragments known of the great treatise on the vacuum prepared by Pascal in 1651; the rest of the volume contains (pp. 164–194) a repr. of the Ŕcit de la grande experínce… and (pp. 195–232) texts by F. Perier and others. Important subsequent eds. are those of 1664 and 1698; Bossut, IV (1779), 222–325; G.E., ILL (1908), 143–292, and IX (1914), 352; PL (1954), 412–471; Mesnard, 1 (1964), 679–689 (preface by F. Perier), and 11 (1971), 739–745 (account of Perier’s observations), 787–798 (two “Fragments d’un traite du vide”), and 1036–1101 (the actual Traites).

14. “Generatio conisectiomun” (completed about 1654). 1st ed. in Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin, 1 (1892), 197–202 (edited by C. 1. Gerhardt); G.E., **II** (1908), 234–243; PL (1954), 66–70, 1382–1387 (French trans.); Mesnard, II (1971), 1108–1119.

15. Leibniz’s notes on Pascal’s treatise on conics (the notes date from 1676, but the treatise was finished about 1654). 1st ed. (partial) in *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin*, **1** (1892), 195–197, edited by C. I. Gerhardt; G.E., II (1908), 227–233; P. Costabel, in *L’oeuvre scientifique de Pascal* (1964), 85–101 (with French trans.); Mesnard, II (1971), 1120–1131 (with French trans.).

16. Correspondence with Fermat (July-Oct. 1654). 1st ed., p. Fermat, *Varia opera mathematica*…(Toulouse, 1679), 179–188(for the three letters by Pascal; for the other four see Bossut); Bossut, IV (1779), 412–445; Fermat, *Oeuvres*. P. Tannery and C. Henry, eds., II (1894), 288–314, and III (1896), 310–311; PL (1954), 74–90; Mesnard, II (1971), 1132–1158.

17.*Traite du triangle arithmetique, ovev quelques petits traites sur la meme matlere*(1654). 1st ed. (Paris, 1665). Without the first four pages (title page, foreword, and table of contents) and the plate, this work was printed during Pascal’s lifetime (1654) but was not distributed. It consists of four parts: the “Traité du triangle arithmétique” itself; two papers devoted to various applications of the triangle; and a fourth paper on numerical orders, powers, combinations, and multiple numbers that is formed of seven sections, the first in French and the rest in Latin. J. Mesnard has identified a preliminary Latin Version of the part of this treatise that was published in French.

Subsequent eds.: Bossut, V (1779), 1–134; *Oeuvres completes de Pascal*, C. Lahure, ed., II (1858), 415–494 (with French trans. of the Latin passages); G.E., III (1908), 311–339, 341–367, 433–598, and XI (1914), 353, 364–390; PL (1954), 91–171, 1404–1432 (translations); Mesnard, II (1971), 1166–1332—repr. with French trans. of the entire preliminary Latin ed., *Triangulus arithmeticus*, followed by the new sections of the *Traité*.

18. “Introduction à la géométrie” (written about the end of 1657). 1 st ed. in *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin*, **1** (1892), 202–204 (C.I.Gerhardt, ed.); G.E. IX(1914), 291–294; PL(1954), 602–604, 1476; Itard, in *L’oeuvre scientifique de Pascal* (1964), 102–119.

19. “De l’esprit geometrique” and “De l’art de persuader” (written about 1657–1658). 1st ed. (partial), P. N. Desmolets in *Continuation des mémoires de littérature et d’histoire*, V, pt. 2 (Paris, 1728), 271–296; Bossut, II (1779), 12–38, 39–57; G.E., IX (1914), 240–290; PL (1954), 574–602.

20. Various items pertaining to the cycloid competition (June 1658-Jan. 1659).

a. Three circulars addressed to the contestants: the first in Latin (June 1658); the second in Latin (July 1658); the third in French and Latin (dated 7 Oct. in the French text and 9 Oct. in Latin version).

b. *Histoire de la roulette …* (10 Oct. 1658), also in Latin, *Historia trochoidis* (same date).

C. *Recit de l'examen et du jugement des ecrits envoyes pour les prix proposes publiquement sur le sujet de la roulette…*(25 Nov.1658).

d.*Suite de l'hisoire de la roulette…*(12 Dec. 1658. with an addition on 20 Jan. 1659); the Latin version exists only in MS.

A more detailed description of this group of writings is provided by L. Scheler, in *L'oeuvre scietifique de Pascal* (1964), 30–31, and in Mesnard, I (1964), 163–167. subsequent eds. are Bossut, V(1779), 135–213; G.E., VII (1914), 337–347, and VIII (1914), 15–19, 155–223, 231246, 289–319; PL (1954), 180–223, 1433–1435 (French trans of the circulars of June and July 1658)

21. *Lettres de A. Dettonville contenant queues-unes de ses inventions de géométrie …* (Paris, Feb. 1659). This vol. contains a title page (written after the rest of contents), four sheets of plates, and four letters published between Dec. 1658 and Jan. 1659 (in the order 1, 4, 3, 2).

Letter no. 1: *Lettre de A. Dettonville à Monsieur de Carcavy, en lui envoyant: Une méhtode générale pour trouver les centres de gravité de toutes sortes de grandeurs. Un traité des trilignes et de leurs onglets. Un traité des sinus du quart de cercle. Un traité des solides circulaires. Et enfin un traité général de la roulette, contenant la solution de tous les problèmes touchant la roulette qu'il avait proposés publiquement au mois de juin 1658.* Orig. (Paris, 1658).

Leter non. 2; *Lettre de A. Dettoneille a monsteur A.D. D.S. en luis envoyant; La demunstration a la manlere des anciens de l'egalite des lignes spirale et parabolique. Orig.* (Paris, 1658).

Letter no. 3; *Lettre de A. Destionville a monsicur de sluze, chanooine de la cathedrale de Liege, en lui envoyant: La dimension et lecentre de gravite de l'escalier. La dimension et le centre de gravite des traiangles cylindriques. La dimension d' un solide forme par le moyen d'une spirale autour d' un cone, Orig.* (Paris, 1658)

Letter no.4: *Lettre de A. Dettonville a Monsieur Huggyens [sic] de Zulichem, en lui encoyant; La dimenston des lignes de toutes sortes de roulettes, lesquelles il montre eire egales a des lignes elliptiques. Orig.*(Paris, 1659).

Later eds.: Bossut, V(1779), 229–452; G.E. VIII(1914), 247–288, 325–384, and IX (1914), 1–149, 187–204; PL (1954), 224–340, 1436–1437; a faces. of the original ed. has recently appeared (London, 1966).

Two other important documents relating to Pascal's scientific work are the following:

22. The License for his calculating machine (22 May 1649). 1st ed. in *Recueil de diverses pieces pour servir a l' histoire de Port-Royal* (Utrecht, 1740), 244–248; Bossut, IV(1779), 30–33; G.E., II (1908), 399–404; Mesnard, II(1971), 711–715.

23. Letter from Leibniz to Etienne Perier of 30 Aug. 1676 concerniong pascal's treatise on conies. 21st ed., Bossut, V (1779), 459–462; G.E. II(1908), 193–194; PL (1954), 63–65; J.Mesnbard and R. Taton, in L'oeuvre scientifique de Pascal (1964), 73–84.

II. Secondary Litieratuer. A very complete bibliography of stugies on Pascal's scientific work published before 1925 can be found in A. Maire, *Bibliographic generale des oeuvres de Pascal*, 2nd ed., **1** , *Pascal sovant* (Paris, 1925), Most of the more recent works on the subject (except for those dealing with the cycloid) are cited in the bibliographies in Mesnard, II (1971) — geometry,227228, 1108; combinatorial theory and the calculus of probatbility, 1135, 1175; the calculating machine, 327–328; physics, 349, 4959, 513, 675–676, 777, 804, 1040; miscellaneous, 1031.

Two general studies in particular should be mentioned: P. Humbert, *L'oeuvre scientifique de Pascal* (Paris, 1947), a survey written for a broad audience; and *L'oeuvre scientifique de Pascal* (Paris, 1964), a joint effort that restates the main aspects of Pascal's career and scientific work (with the exception of the theory of combinations and the calculus of probability). Other recent studies worth consulting are A. Koyre, "Pascal savant," in *Blaise Pascal, l'homme et l'oeucre* (Paris, 1956). pp. 259–285; K. Hara, "Examen des textes mathématiques dans les oeuvers complètes de Pascal d'apres les Grands Ecrivains de la France." in Gallia (Osaka). no. 6 (1961); "Quelques additions a l'examen des textes mathematiques de Pascal," no. 7 (1962); and "pascal et Wallis au sujet de la cycloide, I," in *Annals of the Jopan Assocation for Philosaphy of Sciences*, **3** , no. 4 (1969), 166–187; "Pascal et Wallis …, II,' in *Gallia*, nos. 10–11 (1971), 231–249; and :Pascal et wallis…, III," In *Japanese Studies in the History of Science*, no.10(1971)95–112; N. Bourbaki, Elements d'histoire des mathematiques, 2nd ed. (Paris, 1969), see index; M. E. Baron, *The Origins of the Infinitesimal Calculus* (London, 1969), esp. 196–205; and E. Coumet, "La theorie du hasard est-elic nee par hadard?" in *Annales. Economies, societes, Civilisations*, **5** (May-June 1970) 574–598.

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