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(*b*. Elberfeld, Germany, 16 June 1801; *d*. Bonn, Germany, 22 May 1868)

mathematics, physics.

Plücker was descended from a Rhenish merchant family of Aix-la-Chapelle (Aachen). After graduating from the Gymnasium in Düsseldorf, he studied at the universities of Bonn, Heidelberg, Berlin, and Paris until 1824, when he earned his doctorate *in absentia* from the University of Marburg. In 1825 he became *Privatdozent* at the University of Bonn, where in 1828 he was promoted to extraordinary professor. In 1833 he served in Berlin simultaneously as extraordinary professor at the university and as teacher at the Friedrich Wilhelm Gymnasium. In 1834 he became ordinary professor at the University of Halle. He then served as full professor of mathematics (1836–1847) and physics (1847–1868) at Bonn, where he succeeded Karl von Münchow. In 1837 he married a Miss Altstätten; his wife and one son survived him.

Although Plücker was educated primarily in Germany, throughout his life he drew much on French and English science. He was essentially a geometer but dedicated many years of his life to physical science. When Plücker began his work in mathematics, the only German mathematician of international repute was Gauss. In 1826, however, Crelle founded, in Berlin, his *Journal für die reine und angewandte Mathematik;* and the work of Plücker, Steiner, and others soon became well known. Their field of research was not the differential geometry of Monge and Gauss, but rather the analytic and projective geometry of Poncelet and Gergonne. But differences between the synthetic school in geometry, of which Steiner was the head in Berlin, and Plücker's analytical school—together with a conflict of personality between the two men—resulted in Plücker"s being resident at Berlin for only a year.

In 1828 Plücker published his first book, volume I of *Analytisch-geometrische Entwicklungen*, which was followed in 1831 by volume II. In each volume he discussed the plane analytic geometry of the line, circle, and conic sections; and many facts and theorems—either discovered or known by Plücker—were demonstrated in a more elegant manner. The point coordinates used in both volumes are nonhomogeneous affine; in volume II the homogeneous line coordinates in a plane, formerly known as Plücker's coordinates, are used and conic sections are treated as envelopes of lines. The characteristic features of Plücker's analytic geometry were already present in this work, namely, the elegant operations with algebraic symbols occurring in the equations of conic sections and their pencils. His understanding of the so-called reading in the formulas enabled him to achieve geometric results while avoiding processes of elimination, and his algebraic elegance was surpassed in some matters only by Hesse. Plücker's careful treatment, in the first book, of conic sections that osculate with one another in different degrees is still noteworthy.

In 1829 Plücker introduced the so-called triangular coordinates as three values that are proportional to the distances of a point from three given lines. Simultaneously, Mobius introduced his barycentric coordinates, another type of homogeneous point coordinates. In his *Analytisch-geometrische Entwickhmgen*, however, Plücker used only nonhomogeneous point coordinates. At the end of volume II he presented a detailed explanation of the principle of reciprocity, now called the principle of duality. Plücker, who stood in the middle of the Poncelet-Gergonne controversy, was inclined to support Poncelet's position: Plücker introduced duality by means of a correlation polarity and not in the more modern sense (as in Gergonne) of a general principle. Thus Plücker's work may be regarded as a transitional stage preceding the pure projective geometry founded by Staudt.

After 1832 Plücker took an interest in a general treatment of plane curves of a higher degree than the second. Although his next book, *System der analytischen Geometrie, insbesondere eine ausführliche Theorie clear Kurven 3. Ordnung enthaltend* (1835), discussed general (or projective) point and line coordinates for treating conic sections, the greater part of the book covered plane cubic curves. Plücker's consideration of these curves began with the following theorem by Poncelet. The three finite points where the three asymptotes of a cubic intersect the curve lie on a straight line. Analytically this theorem is equivalent to the possibility of writing the curve equation in the form $pqr + \lambda s^3 = 0$ (p, q, r, s linear forms). A cubic curve is determined by the 4 lines with equations $p = 0$, $q = 0$, $r = 0$, $s = 0$, and one point on the curve. Plücker gave constructions for the cubics thus determined. A real affine classification based upon these constructions leads to 219 different types.

Plücker devoted the greater part of *Theorie der algebraischen Kurven* (1839) to the properties of algebraic curves in the neighborhood of their infinite points. He considered not only the asymptotic lines, but also asymptotic conic sections and other curves osculating the given cubic in a certain degree. For the asymptotic lines he corrected some false results given by Euler in *Introduction in anatysin infinitorum* (1748).

Although the increasing predominance of projective and birational geometry abated interest in these particulars about the behavior of curves at infinity, the second part of *Theorie der algebraischen Kurven* was of more permanent value. It contained a new treatment of singular points in the plane, a subject previously discussed in Cramer's work (1750) on the theory of curves. Plücker's work also resolved several doubts concerning the relation between the order and class of curves in the work of Poncelet and Gergonne.

In his 1839 publication Plücker proved the following celebrated formulas, known as "Plücker's equations":

$k = n(n - 1) - 2d - 3s, n = k(k - 1) - 2\delta - 3\sigma,$

connecting the order n and class k of a curve, which contains as singular points d double points and s cuspidal points, and as dual singularities δ double tangents and σ inflexions. The same assumptions validate Plücker's formulas of the second group:

$\sigma = 3n(n - 2) - 6d - 8s, s = 3k(k - 2) - 6\delta - 8\sigma.$

A cubic without singular points therefore contains nine inflexional points, and Plücker discovered that no more than three inflexional points can be real. He thus prepared the foundation for the results later obtained by Hesse. In the last chapter of *Geometrie der algebraischen Kurven* Plücker dealt with plane quartic curves and developed a full classification of their possible singular points. A nonsingular quartic curve that possesses twenty-eight double tangents is the central fact in his theory of these curves.

Although Plücker's treatment of quartic curves and his theorems on their configuration were all wrong (Hesse later corrected his errors), Plücker had a clear insight, which eluded his predecessors, into the meaning of the so-called Cramer paradox and its generalizations. The crux of the paradox is that $1/2\ n(n + 3) - 1$ common points of two curves of degree $n \geq 3$ determine another set of $1/2(n - 1)(n - 2)$ common points. Severi, in his conferences and books on enumerative geometry, underlined the so-called Plücker-Clebsch principle in the following form; If a system of algebraic equations, depending on certain constants, generally has no common solution—except when the constants fill certain conditions—then the system in this latter case has not only one, but also infinitely many solutions.

In 1829, independent of Bobillier, Plücker extended the notion of polars (previously known only for conic sections) to all plane algebraic curves. He also studied the problem of focal points of algebraic curves, the osculation of two surfaces, and wave surface, and thus became concerned with algebraic and analytic space geometry. This field was also discussed in *System der Geometrie des Raumes in neuer analytischer Behandltmgsweise* (1846), in which he treated in an elegant manner the known facts of analytic geometry. His own contributions in this work, however, were not as significant as those in his earlier books.

After 1846 Plücker abandoned his mathematical researches and conducted physical experiments until 1864, when he returned to his work in geometry. His mathematical accomplishments during this second period were published in *Neue Geometrie des Raumes gegriindet auf die Betrachtung der Geraden als Raum-element*, which appeared in 1868. Plücker's death prevented him from completing the second part of this work, but Felix Klein, who had served as Plücker's physical assistant from 1866 to 1868, undertook the task. Plücker had indicated his plans to Klein in numerous conversations. These conversations served also as a source for Plücker's ideas in *Neue Geometrie*, in which he attempted to base space geometry upon the self-dual straight line as element, rather than upon the point or in dual manner upon the plane as element. He thus created the field of line geometry, which until the twentieth century was the subject of numerous researches (see Zindler, "Algcbraische Liniengeometrie," in *Encyktopädie der mathematisehen Wissensehaftet***III** , 2 [1921], 973–1228).

Plücker's work in line geometry can be related to several earlier developments: the notion of six line coordinates in space as well as a complex of lines intersecting a rational norm curve had already been discussed by Cayley; the researches of Poinsot and Möbius on systems of forces were closely related to line geometry; and researches on systems of normal to a surface were made by Monge and later generalized by W. R. Hamilton to a differential geometry of ∞^2rays.

Notwithstanding these developments, Plücker's systematic treatment of line geometry created a new field in geometry. He introduced in a dual manner the six homogeneous line coordinates $p_{i,j}$ now called "Plücker's coordinates," among which a quadratic relation $Q_4(P_{i,j}) = 0$ exists. Subsequent work by Klein and Segre interpreted line geometry of R_3 as a geometry of points on a quadric Q_4 of P_5. But this development, as well as further generalizations of an S_k geometry in S_n to be interpreted as a point geometry on a Grassmannian variety $G_{n,k,}$ was not anticipated by Plücker. who restricted his work to the domain of ordinary space and conceived therein a four-dimensional geometry with the line as element.

Plücker's algebraic line geometry was distinct, however, from the differential line geometry created by Hamilton. Plücker introduced the notions (still used today) of complexes; congruences; and ruled surfaces for subsets of lines of three, two, or one dimension. He also classified linear complexes and congruences and initiated the study of quadratic complexes, which were defined by quadratic relations among Plücker's coordinates. (Complex surfaces are surfaces of fourth order and class and are generated by the totality of lines belonging to a quadratic complex that intersects a given line.) These complexes were the subject of numerous researches in later years, beginning with Klein's doctoral thesis in 1868. In Neue Geometric Plücker again adopted a metrical point of view, which effected extended calculations and studies of special cases. His interest in geometric shapes and details during this period is evident in the many models he had manufactured.

In assessing Plücker's later geometric work it must be remembered that during the years in which he was conducting physical research he did not keep up with the mathematical literature. He was not aware, for example, of Grassmann's *Die Wissenschaft der exten-siren Grösslehre oder die Ausdehnungslehre* (1844), which was unintelligible to almost all contemporary mathematicians.

Plücker was professor of mathematics and physics at the University of Bonn; he is said to have always been willing to remind other physicists that he was competent in both fields. It is particularly noteworthy that Plücker chose to investigate experimental rather than theoretical physics; Clebsch, in his celebrated obituary on Plücker, identified several relations between Plücker's mathematical and his physical preoccupations. In geometry he wished to describe the different shapes of cubic curves and other figures, and in physics he endeavored to describe the various physical phenomena more qualitatively. But in both cases he was far from pursuing science in a modern axiomatic, deductive style.

Plücker's guide in physics was Faraday, with whom he corresponded. His papers of 1839 on wave surface and of 1847 on the reflection of light at quadric surfaces concerned both theoretical physics and mathematics, though often counted among his forty-one mathematical papers. Plücker also wrote fifty-nine papers on pure physics, published primarily in *Annalen der Physic and Chemie and Philosophical Transactions of the Royal Society*. He investigated the magnetic properties of gases and crystals and later studied the phenomena of electrical discharge in evacuated gases. He and his collaborators described these phenomena as precisely as the technical means of his time permitted. He also made use of an electromagnetic motor constructed by Fessel and later collaborated with Geissler at Bonn in constructing a standard thermometer. Plücker further drew upon the chemical experience of his pupil J. W. Hittorf in his study of the spectra of gaseous substances, and his examination of the different spectra of these substances indicates that he realized their future significance for chemical analysis.

In 1847 Plücker discovered the magnetic phenomena of tourmaline crystal: and in his studies of electrical discharges in rarefied gases he anticipated Hittorf's discovery of cathodic rays. His discovery of the first three hydrogen lines preceded the celebrated experiments of R. Bunsen and G. Kirchhoff in Heidelberg. Although Plücker's accomplishments were unacknowledged in Germany, English scientists did appreciate his work more than his compatriots did. and in 1868 he was awarded the Copley Medal.

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