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(b. Pithiviers, Loiret, France 21 June 1781; d. Paris, France, 25 April 1840)

mathematical physics.

Poisson was an example of those scientists whose intellectual activity was intimately linked to a great number of educational or administrative duties and to the authority derived from them. This responsibility and authority earned him more misunderstanding than esteem, and his reputation in the French scientific community was challenged during his lifetime as well as after his death. It was only outside France that certain results of his prodigious activity were best understood and considered worthy of perpetuating his memory. His life and work are thus of special interest for the history and philosophy of science. The same institution that gave him his training the newly founded École Polytechnique, also assured his success. An exemplary product of a certain type of training and of a particular attitude toward scientific research, he devoted his life to both, exhausting in their service his remarkable capacity for hard work. His activities, which continued unabated through a succession of political regimes, exercised a major influence on French science. Although his ambition to continue Laplace's work by giving a true summa of mathematical physics was not to be realized, his numerous efforts toward this goal offer a lesson concerning the application of mathematics to natural phenomena that is still worth examining.

Poisson came from a modest family. His health, like that of several older siblings who died in childhood, was weak; and his mother had to entrust him to a nurse. His father, formerly a soldier, had been discriminated against by the noble officers, and after retiring from military service he purchased a low ranking administrative post. Apparently it was he who first taught Poisson to read and write. The Revolution, which the elder Poisson welcomed with enthusiasm, enabled him to become president of the district of Pithiviers, which post afforded him access to information useful in choosing a career for his son. The latter had been entrusted to an uncle named Lenfant in Fontainebleau in order to learn surgery, but he lacked the prerequisite manual dexterity and showed little interest in the profession. Having failed as an apprentice, he was guided by his father toward those professions to which access had been eased by measures recently adopted by the republican regime. In 1796 he was enrolled at the École Centrale of Fontainebleau, where he soon displayed a great capacity for learning and was fortunate in having a dedicated teacher. He made rapid progress in mathematics and was encouraged to prepare for the competitive entrance examination at the École Polytechnique, to which he was admitted first in his class in 1798.

On his arrival; in Paris, fresh from the provinces, Poisson had to adapt himself in several ways to a radically new life for which he had been little prepared. His easy success in his studies left him time to make this adjustment much more quickly than is usual in such cases. Lagrange, who had just begun his courses on analytic functions, found in Poisson an attentive student always capable of contributing pertinent remarks in class; and Laplace was even more impressed by Poisson's ability to assimilate difficult material. The reputation that Poisson enjoyed among his fellow students is mentioned in an article on him by Arago, whom he preceded by five years at Polytechnique. This comment is certainly an echo of direct testimony, and there is no reason to doubt Arago's assertion that during these years Poisson evinced a lively interest in the theater and in other aspects of cultural life. An openness to every new experience and a passion for learning allowed him to circumvent the difficulties that he might otherwise have encountered on account of his limited early education. However, his teachers were apparently unable to correct his innate clumsiness, for he could never learn to draft acceptable diagrams. This deficiency prevented him from advancing in descriptive geometry, which subject Monge had made a central element of the new school's curriculum and which contributed greatly to its reputation. On the other hand, Poisson possessed undoubted ability in mathematical analysis and displayed this gift in 1799–1800 in a paper on the theory of equations and on Bezout's theorem. At a time when it was difficult to recruit suitably qualified teaching personnel, this asset was sufficient to gain him nomination as *répétiteur* at Polytechnique immediately after his graduation in 1800.

Poisson owed this appointment principally to the backing of Laplace, who unwaveringly supported him throughout a career qualified as "easy" by [Victor Cousin](#). The main reason that "easy" is the right word was that Poisson was enabled to stay in Paris in the milieu of Polytechnique. He was named deputy professor in 1802 and four years later replaced Fourier as titular professor. Thereupon, Poisson had to wait only a short time to obtain supplementary posts outside the school. In 1808 he was appointed astronomer at the Bureau des Longitudes and in, 1809 professor of mechanics at the Faculty of Sciences. It would appear that he never disdained worldly connections or the advantages to be gained through the salons but it would be completely unjust to assume that he systematically cultivated these means of social advancement. As a participant in the Société Philomathique from 1803 and later in the Société d' Arcueil, he had no intellectual reservations about the idealism that animated Polytechnique; and if his friendships were useful, he did not cultivate them out of self-serving motives.

On 23 March 1812 Poisson was elected to the physics section of the Institute (to the place left vacant by Malus' death), and by 14 April his nomination had received the imperial approbation. This rapid confirmation shows that the authorities had not forgotten the acquiescent attitude that he had adopted in 1804. In that year Poisson prevented the students of the École Polytechnique from publishing a petition against the proclamation of the empire. He had taken this step, however, primarily to avoid a crisis at the institution to which he was devoted. He felt no genuine allegiance to the Napoleonic regime and easily accommodated himself to its overthrow. The restoration of [Louis XVIII](#) in 1814 caused no hiatus in his career, and he continued to accumulate official responsibilities. To the responsibilities he already exercised he added that of examiner at the Ecole Militaire in 1815 and of examiner of graduating students at Polytechnique the following year.

Within this pattern of continuity, however, Poisson's life and career entered upon a new phase at about this time. In 1817 he was married to Nancy de Bardi, an orphan born in England to émigré parents. The marriage constrained his life severely, leaving no time for anything but family, research, and professional obligations. His nomination in 1820 to the Conseil Royal de l' Université introduced him into the national educational system at the highest administrative level—at the very moment when the government's conservative general political stance was issuing in a campaign against the scientific programs and policies adopted during the Revolutionary and Napoleonic periods. Enlisting the aid of colleagues, notably Ampère, he managed to resist this pressure. His efforts in defense of science which continued until the end of his life, constitute a considerable achievement

While assuming these weighty pedagogical responsibilities, Poisson was becoming steadily more influential within the Academy of Sciences. Following Laplace's death in 1827 he felt it to be his mission to build upon the latter's scientific legacy; and Cauchy's exile in 1830 contributed still further to casting Poisson in the role of France's leading mathematician. It was primarily against him that [Evariste Galois](#) directed his celebrated criticism of French mathematics, and historians have been too eager to accept its validity at face value. Circumstances thrust upon Poisson more responsibility than any one man could have borne. It is all the more remarkable then, that he published virtually all of his books during the last ten years of his life. To be sure, none of them manifests a profound originality. His many articles and memoirs (of which he himself prepared a list) must therefore be considered in order to arrive at a just assessment of his oeuvre. It must also be acknowledged that his books exhibit an uncommon gift for clear exposition and constitute an ambitious project for the instruction of future generations of students. He exhausted himself in the attempt to realize this project and died regretting that he had left it unfinished. Accordingly, any final judgment of his work and influence must give considerable weight to his role as educator.

It was precisely this aspect of Poisson's contribution that was ignored in the unsigned article on him published in Larousse's Grand dictionnaire on *universel du XIX siècle* (1874). The author wrote: "The reputation of a mathematician really depends on extraordinary powers of analysis. The experimental scientist, whose physical discoveries the analyst formulates in incomprehensible expressions, is generally incapable of verifying the quality of the help that he is getting. To the layman, what is most striking in the procedures of mathematicians, even though it really has almost no merit, is the art of making transformations. Poisson possessed that skill in a high degree. He amazed people and was taken for a great man. But, in order to be remembered, a scientist needs to have ideas, and Poisson had only those of others. Moreover, when he had to choose, as between two opposing ideas, the one that he would dignify with an application of his analysis, he generally made the wrong choice."

Although this criticism is so hostile as to amount to denigration it does serve to bring out several of Poisson's characteristic traits. First, it stresses that the tireless manipulation of mathematical equations was his special province. His zeal for extending as far as possible the type of activity for which he had a gift was natural; and the limitations or deficiencies of his results are less interesting than what they reveal about the historical context, which was one of intense scientific activity.

Two authors who were very close to Poisson during the last decade of his life have left accounts that substantially agree as to his position within French science. Their gratitude for his help, moreover, did not lead them to abstain from criticism, and they provide several details that are not available elsewhere.

In 1840 Guillaume Libri, who had not yet become notorious for his bizarre administration of the French national archives, wrote an *éloge* of Poisson in which he affirmed, "Surely no one would dare to say that Poisson lacked inventiveness, but dare to say that Poisson lacked inventiveness, but he especially liked unresolved questions that had been treated by others or areas in which there was still work to be done." In a note he added that Poisson, who refused to attend to two matters at once had a small wallet for papers on which he jotted down information and recorded observations of subjects to be examined later. Libri, who evidently had the document in his possession, stated by way of example that Poisson considered research on algebraic equations and definite integrals to be hopeless. By contrast, he seems to have found it more important to pursue Euler's works on problem of *géométrie dépendant des différences mêlées*, that is on, problems of mathematical physics involving partial differential equations.

Cournot, who owed his university career to Poisson and replaced him in 1839 as chairman of the jury d' Agrégation in mathematics, recorded in his *Souvenirs* that "the abundance, adaptability, and resourcefulness" that his benefactor displayed to a greater degree than anyone else "in involved calculation" [*dans les hunts calculs*] was combined with an eagerness to examine "all questions and preferably... Despite that, or because of it," Cournot adds with discernment, "he did not enjoy the rare good fortune of developing one of the those completely new fame of their innovator in the history of science. He proceeded steadily along his path rather than crossing into any new domain."

Although this judgment must be slightly modified, as will be seen below, it epitomizes the essential aspects of Poisson's mathematical work. Poisson was succeeded on the Conseil Royal de l' Université in 1840 by Poincaré, with whose scientific personality Cournot compared Poisson's in a felicitous Poisson: he stuck to a few simple, ingenious ideas that were completely his own. He considered them and reconsidered them at his leisure, without worrying about producing a great deal and even (let us speak plainly) without possessing much knowledge," Although Cournot adds only that "it would take too long to explain to someone who is not a member of the profession all the different ways in which mathematics can be cultivated," Cournot obviously preferred Poincaré's originality by laziness. It is also clear that Cournot was more impressed by the "abundance" of Poisson's works than he was concerned to submit them to a genuine historical critique

Although Poisson's list of his own published has greatly impressed posterity by its length, it has not science adequate critical interest. Historians of science are aware that in citing his memoirs Poisson also listed the extracts derived from them, *philomathique de Paris* and in the *Annales de chimie et de physique* Accordingly, they have eliminated from the list those works thought to be duplicate or even triplicate entries and have been content to wonder at the remainder, which consists of nearly 300 original titles. But the classification by chronological order and by subject matter obliges the historian, precisely because of the lack of exact bibliographical data, to undertake laborious research in order to reconstruct the actual conditions under which Poisson produced his work during the Empire and the Restoration. The delay in publication of official periodicals such as the *Journal de l' École polytechnique* and the *Mémoires de l' Académie des sciences* deprived them of much of their importance as disseminators of new knowledge. Other private publications, originating in various scientific circles and learned Societies, were more effective from this point of view; but they too experienced difficult periods, during which they substituted for each other. One reason for the abundance of Poisson's titles, therefore, lies in the special circumstances of the scientific life of the age.

Before considering Poisson's scientific work, let us try to complete a portrait of its author. At the end of his article Arago delivered a parting shot to the effect that Poisson had the habit of saying "Life is good for only two things: to study mathematics and to teach it." Since we cannot assume that the permanent secretary of the Academy of Sciences wrote this purely and simply for the pleasure of coining bon mot, it is reasonable to believe that it contains an element of truth. Here again, Cournot furnishes the necessary context.

Cournot was told that Poisson wished to receive a visit from him, but the young *docteur ès sciences* postponed the visit for several years, fearing that he would be recruited as a *lycée* professor. Thus, Cournot's image of Poisson was very much like Arago's sketch that of a narrow mathematical zealot. Cournot's change of opinion after several years of direct contact may therefore be accepted with confidence.

Cournot was eager to elaborate on his first impression of Poisson. Alluding to both his appearance and his style of a straightforward man of the common people, he reports that "With a formal and even elaborate manner Poisson combined great intellectual subtlety and a large store of commonsense and forbearance that disposed him toward conservative ideas. Given the distinguished reputation that he enjoyed in the scientific world [in 1815], the royalist party were quite willing to take his conservative outlook for royalism, and he went along with this!... when the Revolution of 1830 occurred, Poisson saw no grounds for withholding his support from a government that aimed at being sensible and moderate; and he resolved to tolerate Cousin's philosophical declaration, as he had tolerated to its tents personally and that in public life, as in mathematics, things would run their course." This portrait is very probably close to the truth. When in 1837 Poisson accepted elevation to the nobility (he became a baron), even as Laplace had done, he was displaying not so much a political conformism as the desire to neglect no measures likely to be useful in promoting the interests of science.

Reporting further on his contacts with Poisson Cournot wrote that "At the very end of his life, when it had become painful for him to speak, I saw him almost weep from the chagrin that he had experienced as chairman of a competitive examination [for the *agrégation*], for he had become convinced that our young teachers were concerned solely with obtaining a post and possessed no love for science at all." Perhaps it was when confronted with such an audience and sensing its debased motivation that the aging teacher adopted the habit of bluntly uttering the aphorism cited by Arago. Poisson undoubtedly had forgotten how fully he himself had been protected in his youth from the cares borne by senior colleagues of the scientific establishment. Still, he had always scrupulously fulfilled all his official duties, without ever taking time from his own exhausting program of research. This conscientiousness is proof that Poisson, an unbeliever in religion, had found an ideal to which he had become increasingly willing to sacrifice even his health. Such conduct can evoke only respect.

The number and variety of the subjects that Poisson treated make it impractical to offer an exhaustive account of his scientific work. Accordingly, we shall attempt only to outline its most important aspects. In Poisson's case, a classification by subject matter would be less helpful than one by chronological order, which reveals the shifts in his interests during his long career.

The early stages of Poisson's research can be studied in the eleventh through fourteenth *cahiers* of the *Journal de l' École polytechnique* and in the second series of the *Bulletin de la Société philomathique de Paris* In each instance the year 1807 is seen to be the *terminus ad quem* of the period in which he completed his mathematical training and was clearly seeking out subjects that readily offered scope for further development Libri's remarks, cited above, are quite illuminating for this period. They indicate that he was particularly drawn to the integration of differential and of partial differential equation and to their possible application in the study of the oscillations of a pendulum in a resisting medium and in the theory of sound. Naturally coming in the wake of the great works of the preceding century, the choice of these topics did not demonstrate any

particular originality. Still this initial selection was decisive for his career, since it continued to guide all his subsequent research.

From 1808 to 1814 Poisson clearly set out to make an original contribution. At the beginning of this period the Academy of Sciences was emerging from an administrative crisis and was preparing to resume its role and assure the publication of its periodicals. This auspicious situation provided Poisson with a favorable opportunity for advanced Fourier's first memoir on the theory of heat, read to the Institute in December 1807, was not favored with a report; and it was only through Poisson's efforts that it appeared three months later, in abridged form, in the *Bulletin de la Société Philomathique de Paris* of which journal Poisson soon assumed editorial direction. This fact is especially noteworthy. Poisson himself was not impeded by the obstacles encountered in scientific publishing, at their worst around 1810. He submitted three major papers to the Academy, all of which received flattering reports from committees that included Laplace: "Sur les inégalités des moyens mouvements des planètes" (20 June 1808), "Sur le mouvement de rotation de la terre" (20 March 1809), and "Sur la variation des constantes arbitraires dans les question de mécanique" (15 October 1809). Poisson's career was thus off to a most promising start, and his election to the Academy came just at the moment when Biot was scheduled to report on his fourth major memoirs "Sur la distribution de l'électricité à la surface des corps conducteurs" (9 March 1812). This report was never delivered since Poisson himself had become one of the judges. During this period he composed two other works, a new edition of Clairaut's *Théorie de la figure de la terre* (1808) and the two-volume *Traité de mécanique* (1811), a textbook. Yet these, taken together with the four memoirs do not seem to constitute a body of work of sufficient significance to justify such a rapid rise.

Nevertheless the memoirs are not without a certain importance. In the first Poisson simply pursued problems raised and mathematically formulated by Laplace and Lagrange regarding the perturbations of planetary motions with respect to Kepler's solution of the problem of motion for two bodies. Even here, however, he improved the demonstration of the stability of the major axes of the orbits and of the mean motions. In finding approximate solution by means of various series expansions he also showed that determination of the possibility of secular inequalities required the inclusion of high order terms in the calculations. Finally he simplified the notation and disposition of the terms representing the perturbing function.

Poisson's remark so impressed Lagrange that he was led to reconsider the bases of his improved the analysis methods of integrating the differential equations of motion subject to perturbing forces which appeared in the supplement to Laplace's *Mécanique céleste* Book VIII (1808) where it is known as the variation of arbitrary constants. Lagrange presented his results to the Institute and they gave rise in turn to Poisson's memoir of October 1809. In order to grasp fully what was involved in this affair, a certain amount of technical detail is necessary. The terminology and notation in the following account have been somewhat modernized for ease of comprehension.

Consider a material system described by k coordinates q_i satisfying k second-order differential equation (1) of the Lagrange type. From these equations, obtain a second set (2) by adding to the second members of (1) the partial derivatives $\delta\Omega/\delta q_i$ of a function that yields the perturbing force. It is known that every integral of the first set of equations (1) whether or not it can be completely expressed in an analytic form, depends on $2k$ arbitrary constants a_r . The method called variation of constants consists in asking which functions of time can be substituted for a_r in order that the integrals of (1), corresponding to a_r , can satisfy (2).

Lagrange's contribution was to develop fully the implications Laplace's observation that since the number of a_r is twice the number of the equation (1) or (2), these constants can be submitted to k well-chosen conditions. Lagrange's choice results in showing that the derivatives of the sought-for functions a_r with respect to time are the solutions of a linear system in which the coefficients of the unknowns are independent of time.

This result, simple in the formulation just cited, is what seemed to Poisson to call for a more direct derivation, which he arrived at thanks to his ingenious idea of modifying the very nature of the general problem. He introduced, besides the parameters q_i new variables with T being half the live force, a homogeneous quadratic form with respect to the time derivatives \dot{q}_i . The systems of equation (1) and (2) then take "the simplest form that can be given to them," namely which form subsists on introduction of the perturbation function Ω . One obtains a system of $2k$ first-order differential equations with respect to the $2k$ unknowns q_i, \dot{q}_i , as a function of time. Let (1') be those that correspond to (1). The a_r previously considered can be identified with the first integrals of (1'): $a_r = f(t; q_i, \dot{q}_i)$, and Poisson directly expressed the values of da_r/dt needed to satisfy (2'). These values linear system proposed by Lagrange, the only difference lying in the calculations and of the result.

Accordingly, there is nothing surprising about Lagrange's reaction, as expressed in the second edition of his *Mécanique analytique* (1811; Part Two, Section 8). Poisson, he writes, "has produced a fine memoir," but "perhaps he never would have thought of writing it... had he not been assured in advance of the nature of the results. Lagrange adds, moreover, that the advantage of the new calculations is only apparent. For he contends that the form of the constant coefficients that he himself had given, is more practical, since first priority should be given to all the problems in which the integration of (1') yields expressions of q_i and \dot{q}_i as a function of a_r and of time.

This was perhaps the first occasion on which Lagrange expressed dissatisfaction with Poisson's work, but his criticism was not totally justified. The property that he called singular, namely the constancy of the coefficients $[a_1, a_2]$ undoubtedly the same in the cases of Poisson's coefficients: although one should not be too hasty in minimizing the importance of the changes in form.

Considering a_r as first integrals, rather than thus designating the constants of integration, opened a new perspective on the problem. For if it is assumed that there are two functions of the that remain constant because of (1) can parenthesis also yields a function that remains constant. It may be asked if is not a third third first integrals α, β of the system (1'), that is to say, that two functions of the q_i, u_i , and t that remain constant because of (1') can be determined, then the algorithm of Poisson's parenthesis (α, β) also yields a function that remains constant. It may be asked if (α, β) is not a third first integral. As Joseph Bertrand perceptively observed in 1853, (α, β) must not, in being constant, reduce to an identity because of the nature of the functions α, β . This condition limits the scope of "Poisson's theorem"; but to limit is not to destroy, and the theorem retains some interest.

At the end of his memoir Poisson noted that this method of computation immediately clarifies the identify of the mathematical problem involved when passing from the perturbations of the rotation of solids to those of a material point attracted to a center. Lacroix, who was assigned to report on the memoir to the Academy, was much impressed by his fact, but apparently few other members shared his enthusiasm. Hamilton, and then Jacobi, later derived inspiration from Poisson's calculations in creating the mathematical techniques that underlay the great developments in theoretical physics up to the start of the twentieth century. With a bit of patriotic exaggeration Bertrand stated that Jacobi, upon discovering Poisson's "parenthesis" around 1841, found it "prodigious The truth is more modest. In *Vorlesungen iiber Dynamik* (1842) Jacobi proclaimed that Poisson made the most important advance in the transformation of equations of motion since the first version of *Mécanique analytique* This is a fair judgement. It must simply be added that, in the works preceding his election to the Academy, Poisson displayed something more than skilled calculation. His sense of formalization led him to discover analogies, to unify problems and topics previously considered distinct, and to extend definitively "the domain of the calculus" (*l' emprise du calcul*)

This expression was not peculiar to Poisson, although he used it often. In his efforts to extend the use of mathematics in the treatment of physical he limited himself to brief outlines, one of which, that concerning the potential in the interior of attracting masses (1813), later gave rise to important results in electrostatics. He was more ambitious in the memoirs on electricity and magnetism and on the theory of electrostatics. He was more ambitious in the memoris on electricity and magnetism and on the theory of elastic surfaces. The second of these, read to the Academy on 1 August 1814, represented his attempt to block the progress of [Sophie Germain](#), who he knew was competing under the guidance of Legendre for the prize in physics.

Political events once again interrupted French scientific life, and several years elapsed before the delays in publication of specialized journals were reduced to the point where they no longer hindered the exchange of ideas. The period 1814–1827, from the fall of the Empire to the death Laplace, constitutes the third period of Poisson's career. He worked closely with Laplace, under whose influence he investigated a number of topics. One of these was the speed of sound in gases; here Laplace in 1816 had simply stated, without demonstration, a correction to Newton's formula. In other studies Poisson considered the propagation of heat, elastic vibration theory, and what was later called potential. He took up ideas that Laplace had proposed and at the same time conducted a vast amount of his own research. It is somewhat difficult to gain a clear view of all this varied activity, but the many debates in which Poisson participated offer considerable insight into his thinking and reveal the beginnings of an ambitious plan.

Poisson's relations with Fourier began to deteriorate in 1815 with Poisson's publication in the *Bulletin de la Société philomathique de Paris and Journal de physique* of sketches concerning the theory of heat. Fourier wrote on this occasion: "Poisson has too much talent to apply it to the work of others. To use it to discover what is already known would be to waste it.... He adds, it is true, that his method differs from mine and that it is the only valid one. But I do not agree with these two propositions. With regard to the second claim, which is unheard of in mathematics, if Poisson wants others to accept it, he must eliminate from his memoir the part [that I indicated] and take care never to eliminate from below the exponential or trigonometric signs the quantities of which the [absolute value](#) is not infinitely small" (manuscript notes written for Laplace, Bibliothèque Nationale, MS Fr.22525, fol. 91). Fourier's slashing criticism was justified, and Poisson accepted it in three memoirs (1820–1821), which he published in the eighteenth (1820) and nineteenth (1823) *cahiers* of the *Journal de l' École polytechnique* (1820–1821). In a note to one of these papers. moreover, he admitted that he had been to the office of the secretary of the Academy to consult Fourier's prize manuscript of 1812. Nevertheless, in preceding Fourier in publication, he involved himself in an unfortunate enterprise. In dealing with "the manner of expressing functions by series of periodic quantities and the use of this transformation," he proved to be less adroit than in the formalization of mechanics. He scarcely improved either the manner or the use, and his contribution amounted to no more than an emphasis on the necessity of deepening the notion of convergence for the series under consideration.

Furthermore, it was not only with respect to integrals of the differential equations of the propagation of heat that Poisson felt uncomfortable at the kinds of calculation he was encountering. He recurred to a remark that Laplace had made in 1809 to the effect that Fourier's method involved the serious mistake of equating two infinitesimals of different orders. Poisson expended much effort in modifying the infinitesimal derivation of the fundamental equation of the motion of heat, claiming that what he cared about was rigor.

In fact, however, Poisson had misunderstood Fourier's point of view. Where Fourier was considering the flow of heat across a surface, he was thinking—like Laplace before him—of bodies being heated by intercorporeal action. He held the caloric model of heat to be indispensable to theory. Particles of caloric combine with particles of ponderable matter engendering repulsive forces between the latter at distances too small to be detected. Inasmuch as he substituted this mechanistic model for Fourier's analysis in order to derive the same type of equations as his rival, his reasoning is artificial—not to say byzantine—in its complexity. No more than Fourier did he, at least at this juncture, reckon with the difficulties arising from the nature and

physical dimensions of the constants introduced in the course of calculation. Later on, to be sure, he made some headway on that score. We shall come back to that problem.

The physics of heat, which was attracting a growing number of researchers during this period, consists of phenomena other than those that, like radiation, suggest the adoption of a hypothesis concerning events occurring on an infinitesimal, molecular scale. Poisson made worthwhile contributions to the understanding of these other areas, largely through his insistent amassing of theoretical and experimental data.

In “Sur la chaleur des gaz et des vapeurs,” published in August 1823 in *Annales de chimie et de physique*, Poisson developed ideas published four months before by Laplace in Book XII of *Mécanique céleste*. Poisson introduced all the precautions needed to render the confused notion of quantity of heat susceptible to mathematical analysis. He called quantity of heat the magnitude that characterizes the transition of a given mass of gas from an arbitrary initial state of temperature and pressure to another state. This definition makes more abstract the quantitative aspect that naturally follows from the concept of heat as a caloric fluid. Poisson could thus deal comfortably with this magnitude, since for him it is simply a function q of p , ρ , and θ (pressure, density, and temperature). The equation of state $p = ap(1 + \alpha\theta)$ was already classic, and the growing acceptance of the notions of specific heats, at constant pressure and constant volume, allowed him to write the simple partial differential equation of which should be the integral. He also showed that independently of any additional hypothesis, and whatever the arbitrary function used in the integration, the adiabatic transformations (the term did not yet exist) correspond to the formulas $p \cdot p^{\gamma} = \text{constant}$ and $(\theta + 266.67) \cdot \rho^{\gamma} = \text{constant}$, γ being the ratio of the specific heats, assumed constant.

[Sadi Carnot](#), who read this article, unfortunately took no interest in these formulas. He devoted his attention in these formulas. He devoted his attention solely to the calculations in which Poisson’s choice of the arbitrary function of integration was based on a hypothesis taken from Laplace. These calculations led Poisson to propose formulas for the saturated steam pressure H at temperature θ and for the quantity of heat of a given mass of steam. Although Carnot did not pick out from this memoir the results that could have been of greatest help to him, he did discover an interesting numerical fact in another article by Poisson published in the same volume of *Annales de chimie et de physique*

In this brief note on the relative velocity of sound, Poisson discussed the formula that Laplace had proposed to correct Newton’s treatment. The new formulation took account of the of variations in temperature produced by vibrations of the air. He reported that a comparison of this formula with observations led him to conclude that atmospheric air subjected to a sudden compression of 1/116 is heated 1°C. This “indirect” manner of investigating the physics of gases agrees “very closely” with the interpretation of the experiments of Clement and Desormes. Carnot was their disciple, and mention of them in the article was probably what attracted of them in the article was probably what attracted his interest to it. Considering the place of this numerical value in *Réflexions sur la puissance motrice du feu* (1824), it is important to explain both the origin and the nature of Carnot’s concern with this aspect of the subject especially as the explanation challenged simplistic notions of scientific progress. We must, however, resist the temptation to study all such examples offered by Poisson’s activity. These may clearly be seen in the records of his debates with Fresnel on light and with [Sophie Germain](#) and Navier on the vibration of elastic surfaces. Poisson continued to depend on the notion of the corpuscle and the force to which it is subject as his fundamental conceptual tool in studying all areas of physics. Accordingly, it is not surprising that while he became increasingly interested in vibratory he never took the final step of admitting transverse vibrations—or, indeed, any oscillation occurring in a direction different from that of the pressures constituting the corpuscular equilibrium. For this reason he was incapable of providing creative solutions to problems arising in this domain.

Poisson was justly accused by his opponents of having failed to cite contemporary scientists in his writings. But this oversight—which, in any case, was never complete—can readily be repaired by internal criticism. For example Poisson’s work reflects Sophie Germain’s critical remarks, which were as cutting as Fourier’s concerning his method for mathematizing the treatment of elasticity. Specifically, he added to his molecular hypotheses a coefficient for determining the macroscopic relationship between lengthening or extension of elastic materials and variation in thickness. Similarly, his dispute with Navier over the general laws of elasticity led Poisson to demonstrate the importance of the notion of the normal state. On the same occasion, he showed that in calculating the expressed by definite integrals that have a value of zero in the normal state and that remain approximately showed real tact in “subjecting” physical phenomena to mathematical calculation.

It is in this context that Poisson’s contribution to the mathematical treatment of attractive force must be mentioned. His interest in this subject dates from his analysis in 1812–1813 of James Ivory’s work; and here again Laplace was the source for his examination of the integral V of the inverse of the distance function, which function was first called potential by [George Green](#). Late in 1813 Poisson pointed out that in addition to the equation for the attraction exerted by a mass on an external point namely.

the equation $\Delta V = -4\pi\varrho$ must also be considered, where ϱ is the x, y, z , when this point is inside the attracting mass. The demonstration provoked several objections, and Poisson attempted to prove, it notably in two memoris of 1824 published in the *mémoires de l’Académie des sciences* (5 [1821–1822/1826], 247–338, 488–533), In 1826 he stated the triple equation.

$$\Delta V = 0, \quad = 2\pi\varrho, \quad = -4\pi\varrho$$

depending on whether the point x, y, z , is internal to, on the surface of, or external to the attracting mass.

“Mathematicians have long known of the first of these cases,” he added. “An investigation several years ago led me to the third. To it I now add the second, thus completing this equation. Its importance in a great number of questions is well known (“Sur la théorie du magnétisme en mouvement,” *ibid* 6 [1823/1827], 463).

Actually, it was Green who saw how to exploit Poisson’s formulation. Green seized on the importance of the above memoirs in his *Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism* (1828), although he showed little interest in resolving the analytical difficulties still inherent in them. In 1839 Gauss emphasized the need for an improved demonstration of the intermediate equation $\Delta V = -2\pi\rho$, which was not given until 1876 (Riemann). Poisson apparently never knew that he had inspired Green’s work. Indeed, up until 1828 the discussions that we have summarized above were continued mainly in relation to an ambitious project for promulgating a charter for applied mathematics.

In the preface to the long “Mémoire sur l’équilibre et le mouvement des corps élastiques” (14 April 1828), the hints yield to explicit declaration. In applying mathematics to physics, Poisson stated, it was necessary at first to employ abstraction and “in this regard, Lagrange has gone as far as possible in replacing physical ties by equations between coordinates.” Now, however, “along with this admirable conception,” it is necessary to “construct physical mechanics, the principle of which is to reduce everything to molecular actions.” In other words, the death of Laplace the previous year enabled Poisson to move boldly ahead with his long range plans and to present himself as Laplace’s successor.

The last period of Poisson’s life was dominated by his feeling of being the chosen leader of French science—at a time when he already felt called upon to guide the future of the French university system. Since it was during this last period that he decided to publish books, this sense of a twofold mission determined the form that they took; and these works are therefore treatises, concerned primarily with pedagogical matters. Although ably written and including clear historical accounts of the topics treated, they did little to advance contemporary research. Poisson was always reticent about the contributions of his contemporaries. He even omitted Poinsot’s name at the very place in the second edition of *Traité de mécanique* (1833) where he was expounding the new mechanics of solids. Nevertheless, Poisson did have a genuine talent for summarizing the state of knowledge and the theoretical situation in areas of active research. This skill is evident in the report that he presented to the Academy on Jacobi’s *Fundamenta nova theoriae functionum ellipticarum*, which was published in 1831.

It would, however, be unfair to reduce Poisson’s work during the last period merely to an intelligent reformulation of data commingling his own results with those from other sources. In *Théorie mathématique de la chaleur* (1835), reprinted in 1837 with an important supplement, he offered evidence of his own originality in his treatment of the integration of the auxiliary differential equation

which is encountered in the problem of heat distribution inside bodies. He showed that the same integral series that is derived from Bessel’s work has the sum

i being such that $m=i(i+1)$.

Although he was concerned only with those cases in which this equation leads to simple results in finite form, the expression for the Bessel function that it gives rise to is often—and justly—called Poisson’s integral.

Similarly, as [Gaston Bachelard](#) observed, Poisson scored a point in this work by demonstrating how the conductivity of heat in the interior of bodies, far from being contained in the notion of flux as Fourier had held, must be derived from an absorption coefficient that restores a neglected functional dimension. It was in this area that the contribution mentioned in connection with Poisson’s mechanical model for conduction of heat was the most fruitful. That conception enabled Poisson to understand on the molecular scale that the complete and correct equation for radiation of heat is

where u is the temperature at the point x,y,z , as a function of time t : C is the [specific heat](#); and K is the coefficient of conductivity. Fourier’s method, on the other hand, led to the omission of the factor dK/du .

Whatever the value of Poisson’s argument in this regard, the admirable program of physical mechanics, as he conceived it, did not provide a model of natural phenomena that could successfully withstand the passage of time.

Poisson believed that he had corrected an error in Laplace’s treatment of capillarity by introducing a variation of the liquid density to characterize the action of the container walls, but he was as mistaken in this area as he was in his handling of the radiant heat of these treatise, one would not guess that they were intended as the first draft of a course on mathematical physics.

Toward the end of his life, Poisson turned his attention to other subjects, producing two works of considerable repute. The first, *Recherches sur la probabilité des jugements en matière criminelle et en matière civile* (1837), is significant for the author’s participation in an important contemporary debate. The legitimacy of the application of the calculus to areas relating to the moral order, that is to say within the broad area of what is now called the humanistic sciences, was bitterly disputed beginning in 1820 in politically conservative circles as well as by Saint-Simonians and by such philosophers as [Auguste Comte](#). Poisson was bold enough to take pen in hand to defend the universality of the probabilistic thesis and to demonstrate

the conformability to the order of nature of the regularities that the calculus of probability, without recourse to hidden causes, reveals when things are subjected to a great number of observations. It is to Poisson that we owe the term “Law of large numbers.” He improved Laplace’s work by relating it explicitly to Jacob Bernoulli’s fundamental theorem and by showing that the invariance in the prior probabilities of mutually exclusive events is not a necessary condition for calculating the approximate probabilities. It is also from Poisson that we derive the study of a problem that Laplace had passed over, the case of great asymmetry between opposite events, such that the prior probability of either event is very small. The formula for evaluation that he proposed for this case, which is to be substituted for Laplace’s general formula, was not recognized or used until the end of the nineteenth century. The formula states that $P = \sum_1^n \omega^n / n! e^{-\omega}$ expresses the probability that an event will not occur more than n times in a large number, of trials, when the probability that it will occur in any one trial equals the very small fraction ω . In fact, Poisson’s work in this area was not accepted or applied by his contemporaries except in Russia under Chebyshev; it seemed, rather, an echo of Laplace’s and was barely accorded the attention appropriate to an excellent popularization. It was many years before the importance of the [Poisson distribution](#) was recognized. The formula states that expresses the probability that an event will not occur more than n times in a large number, μ , of trials, when the probability that it will occur in any one trial equals the very small fraction ω . In fact, Poisson’s work in this area was not accepted or applied by his contemporaries except in Russia under Chebyshev; it seemed, rather, an echo of Laplace’s and was barely accorded the attention appropriate to an excellent popularization. It was many years before the importance of the [Poisson distribution](#) was recognized.

The second work in this category *Recherches sur le mouvement des projectiles dans l’air* (1839), was far better known in its day. It is the first work to deal with the subject by taking into account the rotation of the earth and the complementary acceleration resulting from the motion of the system of reference. A decade after its publication it inspired Foucault’s famous experiment demonstrating the earth’s rotation. Poisson, who had supervised Coriolis’ doctoral research, recognized the importance of his invention of a term to correct for the deviations from the law of motion that arise in a rotating reference system. Unfortunately, Poisson did not consider himself obliged to cite the name of the actual inventor of the term.

The very multiplicity of Poisson’s undertakings might be considered a reason for his failure to enjoy the good fortune of which Cournot spoke. He constantly exploited the ideas of other scientists, often in an unscrupulous manner. Still, he was frequently the first to show their full significance, and he did much to disseminate them. In the last analysis, however, what was the scientific value of his own contribution? It is tempting to reply that he merely displayed a singular aptitude for the operations of mathematical analysis. And it is no accident that he rejected the view of Lagrange and Laplace that Fermat was the real creator of the integral and differential calculus. To be sure, he waited until he was free to voice his disagreement; for it was not until 1831, in “Mémoire sur le calcul des variations,” that ignoring French chauvinism, he dared to assert “This [integral and differential] calculus consists in a collection of rules... rather than in the use of infinitely small quantities...and in this regard its creation does not predate Leibniz, the author of the algorithm and of the notation that has generally prevailed.” In discussing this question in its historical context, Poisson clearly revealed that aspect to which he was most sensitive and for which he was most gifted

Is this all that can be said? We do not think so. Having reproached Poisson for being unable or unwilling to understand him, Galois wrote, perhaps thinking of him; “The analysts try in vain to conceal the fact that they do not deduce; they combine, they compose... when they do arrive at the truth, they stumble over it after groping their way along,” But—and this is precisely the point Poisson was not one of those analyst who attempt to obscure the way in which they really work. He did combine a great deal, compose a great deal, and stumble frequently—and often guess right. He had a tremendous zeal for changing the manner of treating problems, for fashioning and refashioning formulas, and for taking from the experiments of others that material to which his mathematical techniques would be applied. This zeal occasioned criticism and ironic comments, but it was nevertheless rooted in genuine ability, one capable of motivating an experimental approach

Poisson was certainly not a genius. Yet, just as surely, he was one of those without whom progress in French science in the early nineteenth century would not have occurred.

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