Thales | Encyclopedia.com

18-22 minutes

(*b*. Miletus, Ionia, 625 B.C. [?] *d*. 547 B.C. [?])

natural philosophy.

Thales is considered by Aristotle to be the "founder" (ἀρχηγός) of Ionian natural philosophy.[1] He was the son of Examyes and Cleobuline, who were, according to some authorities, of Phoenician origin. But the majority opinion considered him a true Milesian by descent (ἰυαγενής Μιλήσιος), and of a distinguished family. This latter view is probably the correct one since his father's name seems to be Carian rather than Semitic, and the Carians had at this time been almost completely assimilated by the Ionians. According to Diogenes Laèrtius, Apollodorus put Thales' birth in Olympiad 35.1 (640 B.C.) and his death at the age of 78 in Olympiad 58 (548–545 B.C.). There is a discrepancy in the figures here; probably 35.1 is a mistake for 39.1 (624), since the confusion of ϵ̄ and ῡ is a very common one. Apollodorus would in that case characteristically have made Thales' death correspond with the date of the fall of Sardis, his *floruit* coincide with the eclipse of the sun dated at 585 B.C.–which he is alleged to have predicted–and assumed his birth to be the conventional forty years before his prime.[2]

Even in antiquity there was considerable doubt concerning Thales' written works. It seems clear that Aristotle did not have access to any book by him, at least none on cosmological matters. Some authorities declare categorically that he left no book behind. Others, however, credit him with the authorship of a work on navigation entitled "The Nautical Star Guide," but in spite of a tradition suggesting that Thales defined the Little Bear and recommended its navigational usefulness to Milesian sailors,[3] it is extremely doubtful that he was the actual author of this work, since Diogenes Laërtius informs us that this book was also attributed to a certain Phokos of Samos. It is most unlikely that a work of Thales would have been ascribed to someone of comparative obscurity, but not the converse.

Much evidence of practical activities associated with Thales has survived, testifying to his versatility as statesman, tycoon, engineer, mathematician, and astronomer. In the century after his death he became an epitome of practical ingenuity.[4] Herodotus records the stories that Thales advised the Ionians to establish a single deliberative chamber at Teos and that he diverted the river Halys so that Croesus' army might be able to cross. (Herodotus is skeptical about the latter explanation.)[5] Aristotle preserves another anecdote that credits Thales with considerable practical knowledge. According to this account, Thales, when reproached for his impracticality, used his skill in astronomy to forecast a glut in the olive crop, went out and cornered the market in the presses, and thereby made a large profit. Aristotle disbelieves the story and comments that this was a common commercial procedure that men attributed to Thales on account of his wisdom.[6] Plato, on the other hand, whose purpose is to show that philosophy is above mere utilitarian considerations, tells the conflicting anecdote that Thales, while stargazing, fell into a well and was mocked by a pretty Thracian servant girl for trying to find out what was going on in the heavens when he could not even see what was at his feet.[7] It is clear that these stories stem from separate traditions–the one seeking to represent the philosopher as an eminently practical man of affairs and the other as an unworldly dreamer.

Thales achieved his fame as a scientist for having predicted an eclipse of the sun. Herodotus, who is our oldest source for this story, tells us that the eclipse (which must have been total or very nearly so) occurred in the sixth year of the war between the Lydians under Alyattes and the Medes under Cyaxares, and that Thales predicted it to the Ionians, fixing as its term the year in which it actually took place.[8] This eclipse is now generally agreed to have occurred on 28 May 585 B.C. (–584 by astronomical reckoning). It has been widely accepted that Thales was able to perform this striking astronomical feat by using the so-called "Babylonian saros," a cycle of 223 lunar months (18 years, 10 days, 8 hours), after which eclipses both of the sun and moon repeat themselves with very little change. Neugebauer, however, has convincingly demonstrated that the "Babylonian saros" was, in fact, the invention of the English astronomer Edmond Halley in rather a weak moment.[9] The Babylonians did not use cycles to predict solar eclipses but computed them from observations of the latitude of the moon made shortly before the expected syzygy. As Neugebauer says,

. . . there exists no cycle for solar eclipses visible at a given place; all modern cycles concern the earth as a whole. No Babylonian theory for predicting a solar eclipse existed at 600 B.C., as one can see from the very unsatisfactory situation 400 years later, nor did the Babylonians ever develop any theory which took the influence of geographical latitude into account.[10]

Accrodingly, it must be assumed that if Thales did predict the eclipse he made an extremely lucky guess and did not do so upon a scientific basis, since he had no conception of geographical latitude and no means of determining whether a solar eclipse would be visible in a particular locality. He could only have said that an eclipse was possible somewhere at some time in the (chronological) year that ended in 585 B.C. But a more likely explanation seems to be simply that Thales happened to be the *savant* around at the time when this striking astronomical phenomenon occurred and the assumption was made that as a savant he *must* have been able to predict it. There is a situation closely parallel to this one in the next century. In 468–467 B.C.

a huge meteorite fell at Aegospotami. This event made a considerable impact, and two sources preserve the absurd report that the fall was predicted by Anaxagoras, who was the Ionian *savant* around at that time.[11]

The Greeks themselves claim to have derived their mathematics from Egypt.[12] Eudemus, the author of the history of mathematics written as part of the systematization of knowledge that went on in the Lyceum, is more explicit. He tells us that it was "Thales who, after a visit to Egypt, first brought this study to Greece" and adds "not only did he make numerous discoveries himself, but he laid the foundations for many other discoveries on the part of his successors, attacking some problems with greater generality and others more empirically." Proclus preserves for us some of the discoveries that Eudemus ascribed to Thales, namely, that the circle is bisected by its diameter,[13] that the base angles of an isosceles triangle are equal,[14] and that vertically opposed angles are equal.[15] In addition he informs us that the theorem that two triangles are equal in every respect if they have two angles and one side respectively equal was referred by Eudemus to Thales with the comment that the latter's measuring the distance of ships out at sea necessarily involved the use of this theorem.[16]

From the above it can be seen that Eudemus credited Thales with full knowledge of the theory behind his discoveries. He also held that Thales introduced geometry into Greece from Egypt. Our surviving sources of information about the nature of Egyptian mathematics, however, give us no evidence to suggest that Egyptian geometry had advanced beyond certain rule–of–thumb techniques of practical mensuration. Nowhere do we find any attempt to discover why these techniques worked, nor anything resembling a general and theoretical mathematics. It seems most unlikely, then, that the Greeks derived their mathematics from the Egyptians. But could Thales have been the founder of theoretical mathematics in Greece, as Eudemus claimed? Here again the answer must be negative. The first three discoveries attributed to him by the Peripatetic most probably represent "just the neatest abstract solutions of particular problems associated with Thales."[17] Heath points out that the first of these propositions is not even proved in Euclid.[18] As for the last of them, Thales could very easily have made use of a primitive angle–measurer and solved the problem in one of several ways without necessarily formulating an explicit theory about the principles involved.

Van der Waerden, on the other hand, believes that Thales did develop a logical structure for geometry and introduced into this study the idea of proof.[19] He also seeks to derive Greek mathematics from Babylon. This is a very doubtful standpoint. Although Babylonian mathematics, with its sexagesimal place–value system, had certainly developed beyond the primitive level reached by the Egyptians, here too we find nowhere any attempt at proof. Our evidence suggests that the Greeks were influenced by Babylonian mathematics, but that this influence occurred at a date considerably later than the sixth century B.C. If the Greeks had derived their mathematics from Babylonian sources, one would have expected them to have adopted the much more highly developed place-value system. Moreover, the Greeks themselves, who are extremely generous, indeed overgenerous, in acknowledging their scientific debts to other peoples, give no hint of a Babylonian source for their mathematics.

Our knowledge of Thales' cosmology is virtually dependent on two passages in Aristotle. In the *Metaphysics* (A3, 983b6) Aristotle, who patently has no more information beyond what is given here, is of the opinion that Thales considered water to be the material constituent of things, and in the *De caelo* (B13, 294a28), where Aristotle expressly declares his information to be indirect, we are told that Thales held that the earth floats on water. Seneca provides the additional information (*Naturales quaestiones*, **III** , 14) that Thales used the idea of a floating earth to explain earthquakes. If we can trust this evidence, which seems to stemultimately from Theophrastus via a Posidonian source, the implication is that Thales displays an attitude of mind strikingly different from anything that had gone before. Homer and Hesiod had explained that earthquakes were due to the activity of the god Poseidon, who frequently bears the epic epithet "Earth Shaker," Thales, by contrast, instead of invoking any such supernatural agency, employs a simple, natural explanation to account for this phenomenon. Cherniss, however, has claimed that Aristotle's knowledge of Thales' belief that the earth floats on water would have been sufficient to induce him to infer that Thales also held water to be his material substrate.[20] But it is impossible to believe that Aristotle could have been so disingenuous as to make this inference and then make explicit conjectures as to why Thales held water to be his ἀρχή. Aristotle's conjectured reasons for the importance attached by Thales to water as the ultimate constituent of things are mainly physiological. He suggests that Thales might have been led to this conception by the observation that nutriment and semen are always moist and that the very warmth of life is a dampwarmth. Burnet has rejected these conjectures by Aristotle on the ground that in the sixth century interests were meteorological rather than physiological.[21] But, as Baldry has pointed out, an interest in birth and other phenomena connected with sex is a regular feature even of primitive societies long before other aspects of biology are thought of.[22] However this may be, it is noteworthy that, in view of the parallels to be found between Thales' cosmology and certain Near Eastern mythological cosmogonies,[23] there exists the possibility that Thales' emphasis upon water and his theory that the earth floats on water were derived from some such source, and that he conceived of water as a "remote ancestor" rather than as a persistent substrate. But even it Thales was influenced by mythological precedents[24] and failed to approximate to anything like the Aristotelian material cause, our evidence, sparse and controversial though it is, nevertheless seems sufficient to justify the claim that Thales was the first philosopher. This evidence suggests that Thales' thought shared certain basic characteristics with that of his Ionian successors. These Milesian philosophers, abandoning mythopoeic forms of thought, sought to explain the world about them in terms of its visible constitutents. Natural explanations were introduced by them, which took the place of supernatural and mystical ones.[25] Like their mythopoeic predecessors, the Milesians firmly believed that there was an orderliness inherent in the world around them. Again like their predecessors, they attempted to explain the world by showing how it had come to be what it is. But, instead of invoking the agency of supernatural powers, they sought for a unifying hypothesis to account for this order and, to a greater or lesser extent, proceeded to deduce their natural explanations of the various phenomena from it. Two elements, then, characterize early Greek philosophy, the search for natural as opposed to supernatural and mystical explanations, and secondly, the search for a unifying hypothesis. Both of these elements proved influential in paving the way for the development of the sciences, and it is in the light of this innovation that Thales's true importance in the history of science must be assessed.

NOTES

1.*Metaphysics*, A3, 983b17 ff. (DK, 11A12).

2. These datings are now approximately in accordance with the figures given by Demetrius of Phalerum, who placed the canonization of the Seven Sages (of whom Thales was universally regarded as a member) in the archonship of Damasias at Athens (582–581 B.C.).

3. Callimachus, *Iambus*, 1, 52 f. 191 Pfeiffer (DK, 11A3a).

4. See Aristophanes, *Birds* 1009; *Clouds* 180.

5. Herodotus, I, 170; I, 75 (DK, 11A4, 11A6).

6.*Politics*, A11, 1259a6 (DK, 11A10).

7.*Theaetetus*, 174A (DK, 11A9). It is odd that Plato should have applied this story to someone as notoriously practical in his interests as Thales. It makes one think that there may be at least a grain of truth in the story. See my review of Moraux's Budé edition of the *De caelo*, in *Classical Review*, n.s., **20** (1970), 174, and M. Landmann and J. O. Fleckenstein, "Tagesbeobachtung von Sternen in Altertum." in *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, **88** (1943), 98, notwithstanding Dicks' scornful dismissal of their suggestion. Certainly the motive for this story is clear, but it could have been Thales' practice that determined its form. In general Dicks is far too skeptical in his treatment of the stories told of Thales and relegates them to the status of "the famous story of the First World War about the Russians marching through England with 'snow on their boots.'" But on this latter story see Margo Lawrence, *Shadow of Swords* (London, 1971), in which she reveals that soldiers from Russia, wearing Russian uniform, carrying balalaikas, and singing Slavonic songs, did in fact disembark in 1916 at Newcastle upon Tyne. Admittedly the snow on their boots must be left to folklore.

8. I, 74 (DK, 11A5).

9. O. Neugebauer, The Exact Sciences in Antiquity, 141.

10.*Ibid*., 142.

11. See Diogenes Laërtius, II, 10 (DK, 59A1), and Pliny, *Historia naturalis*, **II** , 149 (DK, 59A11), See also Cicero, *Dedivinatione*, **I** .50.112 (DK, 12A5a), and Pliny, *ibid*., **II** , 191, for a sixth–century parallel, where Anaximander is alleged to have predicted an earthquake.

12. See Herodotus, **II** , 109, who believes that geometry originated from the recurrent need to remeasure land periodically flooded by the Nile: Aristotle, *Metaphysics*, A3, 98Ib20–25, who believes that mathematics evolved in a highly theoretical way as the invention of a leisured class of Egyptian priests: and Eudemus, who, in spite of being a Peripatetic, sides with Herodotus rather than with Aristotle (see Proclus, *Commentary on Euclid's Elements*, I, 64, 16 "Friedlein")

13.*Commentary on Euclid's Elements*, 157.10 (DK, 11A20)

14.*Ibid*., 250.20.

15.*Ibid*., 299.1.

16.*Ibid*., 352.14.

17. G. S. Kirk, *The Presocratic Philosophers*, 84.

18. T. L. Heath, *Greek Mathematics*, **I** , 131.

19. B. L. van der Waerden, *Science Awakening*, 89.

20. H. Cherniss, "The Characteristics and Effects of Presocratic Philosophy," in *Journal of the History of Ideas*, **12** (1951), 321.

21. J. Burnet, *Early Greek Philosophy*, 48.

22. H. C. Baldry, "Embryological Analogies in Early Greek Philosophy," in *Classical Quarterly*, **26** (1932), 28.

23. For an excellent account of Egyptian and Mesopotamian cosmogonies, see H. Frankfort, ed., *Before Philosophy* (Penguin Books, London, 1949), pub. orig. as *The Intellectual Adventure of Ancient Man* (Chicago, 1946).

24. Aristotle, it may be noted, cites the parallel in Greek mythology of Oceanus and Tethys, the parents of generation (*Metaphysics*, A3, 983b27ff. [DK,1B10]). But the Greek myth may itself be derived from an oriental source.

25. The gods of whom Thales thought everything was full (see Aristotle. *De anima*, A5, 411a7 [DK, 11A22]) are manifestly different from the personal divinities of traditional mythology.

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28-35 minutes

(*b*. Samos, ca. 560 b.c.; *d*.Metapontum, ca. 480 b.c.)

mathematics, theory of music, astronomy.

Most of the sources concerning Pythagoras' life, activities, and doctrines date from the third and fourth centuries a.d., while the few more nearly contemporary (fourth and fifth centuries b.c.) records of him are often contradictory, due in large part to the split that developed among his followers soon after his death. Contemporary references, moreover, scarcely touch upon the points of Pythagoras' career that are of interest to the historian of science, although a number of facts can be ascertained or surmised with a reasonable degree of certainty.

It is, for example, known that in his earlier years Pythagoras traveled widely in Egypt and Babylonia, where he is said to have become acquainted with Egyptian and Babylonian mathematics. In 530 b.c. (or, according to another tradition, 520 b.c.) he left Samos to settle in Croton, in southern Italy, perhaps because of his opposition to the tyrant Polycrates. At Croton he founded a religious and philosophical society that soon came to exert considerable political influence throughout the Greek cities of southern Italy. Pythagoras' hierarchical views at first pleased the local aristocracies, which found in them a support against the rising tide of democracy, but he later met strong opposition from the same quarter. He was forced to leave Croton about 500 b.c., and retired to Metapontum, where he died. During the violent democratic revolution that occurred in Magna Greacia in about 450 b.c., Pythagroas' disciples were set upon, and Pythagorean meetinghouses were destroyed. Many Pythagoreans thereupon fled to the Greek mainland, where they found a new center for their activities at Phleius; others went to Tarentum, where they continued as a political power until the middle of the fourth century B.C.

The political vicissitudes of Pythagoras and his followers are significant in the reconstruction of their scientific activities. True to his hierarchical principles, Pythagoras seems to have divided his adherents into two groups, the *ἀκουσματικοί*, or "listeners" who were enjoined to silence, in which they memorized the master's words, and the *μαθηματικοί*, who, after a long period of training, were allowed to ask questions and express opinions of their own. (The term *μαθηματικοί* originally meant merely those who had attained a somewhat advanced degree of knowledge, although it later came to imply "scientist" or "mathematician.")

A few decades after Pythagoras' death, these two groups evolved into sharp factions and began a controversy over which of them was most truly Pythagorean. The *ἀκουσματικοί* based their claim on their literal adherence to Pythagoras' own words (αὐτὸς ἔφα, "he himself has spoken"); the *ἀκουσματικοί*, on the other hand, seem to have developed Pythagoras' ideas to such an extent that they were no longer in complete agreement with their originals. The matter was further complicated because, according to ancient tradition, Pythagoras chose to reveal his teachings clearly and completely to only his most advanced disciples, so that the ἀκουσματικοί received only cryptic, or even mysterious, hints. The later Pythagorean tradition thus includes a number of strange prescriptions and doctrines, which the ἀκουσματικοί interpreted with absolute literalness; the more rationalistic group (led at one time by Aristoxenus, who was also a disciple of Plato and Aristotle) preferred a symbolic and allegorical interpretation.

This obscurity concerning Pythagoras' intent has led historians of science into differences of opinion as to whether Pythagoras could really be considered a scientist or even an initiator of scientific ideas. It is further debatable whether those ancient authors who made real contributions to mathematics, astronomy, and the theory of music can be considered to have been true Pythagoreans, or even to have been influenced by authentically Pythagorean ideas. Nonetheless, apart from the theory of metempsychosis (which is mentioned by his contemporaries), ancient tradition assigns one doctrine to Pythagoras and the early Pythagoreans that can hardly have failed to influence the development of mathematics. This is the broad generalization, based on rather restricted observation (a procedure common in early Greek science), that all things are numbers.

Pythagoras' number theory was based on three observations. The first of these was the mathematical relationships of musical harmonies—that is, that when the ratio of lengths of sound-producing instruments (such as strings or flutes) is extended to other instruments in which one-dimensional relations are involved, the same musical harmonies result. Secondly, the Pythagoreans noted that any triangle formed of three sticks in the ratio 3:4:5 is always a right triangle, whatever the length of its segments. Their third important observation derived from the fixed numerical relations of the movements of heavenly bodies. It was thereby apparent to them that since the same musical harmonies and geometric shapes can be produced in different media and sizes by the same combination of numbers, the numbers themselves must express the harmonies and shapes and even the things having those harmonies and shapes. It could thus be said that these things—or, as they were later called, the essences (*οὐσύμ*) of these things—actually were numbers. The groups of numbers that embodied the essence of a thing, and by which it might be reproduced, were called *λόγοι* ("words"), a term that later came to mean "ratio."

The translation of philosophical speculation into mathematics is thus clear. This speculation about numbers as essences was extended in several directions; as late as the end of the fifth century b.c., philosophers and mathematicians were still seeking the number of justice, or marriage, or even of a specific man or horse. (Attempts were made to discover the number of, for example, a horse, by determining the number of small stones necessary to produce something like the outline of it.) By this time, however, the Pythagoreans had split into a set of groups holding highly differing viewpoints, so it would be inaccurate to assume that all Pythagorean speculations about numbers were of this primitive, unscientific kind.

The theory of special types of numbers, which lay somewhere between these mystical speculations and true science, was developed by the Pythagoreans during the fifth century B.C. The two aspects of the theory are apparent in that the Pythagoreans distinguished between two types of "perfect" numbers. The number ten was the only example of the first group, and its perfection derived from its fundamental role in the decimal system and in its being composed of the sum of the first four numbers, 1 + 2 + 3 + 4 = 10. Because of this second quality it was called the tetractys, and represented by the figure ; it was considered holy, and the Pythagoreans swore by it. The second type of perfect numbers consisted of those equal to the sum of their factors, as, for example, six (1 + 2 + 3) or twenty-eight (1 + 2 + 4 + 7 + 14). Euclid, in the *Elements* (IX. 36) gave the general theory for this numerical phenomenon, stating that if $2^n - 1$ is a prime number, then $(2^n - 1)2^{n-1}$ is a perfect number.

Similar speculations prompted the search for "amicable" numbers—that is, numbers of which each equals the sum of the factors of the other—and for integers satisfying the Pythagorean formula $a^2 + b^2 = c^2$ (as, for example, $3^2 + 4^2 = 5^2$, or $5^2 + 12^2 = 13^2$). Only one pair of amicable numbers, 284 and 220, was known by the end of antiquity, and its discovery is attributed by lamblichus to Pythagoras himself, who is said to have derived it from the saying that a friend is an alter ego. Proclus (in Friedlein's edition of Euclid's *Elements*, p. 426) also attributes to Pythagoras himself the general formula by which any number of integers satisfying the equation $a^2 + b^2 = c^2$ may be found,

where *n* is an odd number. If this tradition is correct (and it is doubtful), Pythagoras must have learned the formula in Babylonia, where it was known, according to O. Neugebauer and A. Sachs (in *Mathematical Cuneiform Texts* [New Haven, 1945], p. 38).

Figured numbers were of particular significance in Pythagorean arithmetic. These included triangular numbers, square numbers, and pentagonal numbers, as well as *heteromeke* numbers (numbers forming rectangles with unequal sides), stereometric numbers (pyramidal numbers forming pyramids with triangular or square bases), cubic numbers, and altar numbers (stereometric numbers corresponding to *heteromeke* numbers). These numbers were represented by points, with., , for example, for the triangular numbers, and for the square numbers. The triangular numbers thus occur in the series 1, 3, 6, 10, 15, …, which can be expressed by the formula $n(n + 1)/2$, while square numbers have the value n^2 and pentagonal numbers may be given the value $n(3n - 1)/2$. *Hetromeke* numbers may be expressed as $n(n + 1)$, $n(n + 2)$, and so on; pyramidal numbers with triangular bases are formed by the successive sums of the triangular numbers 1, 4, 10, 20, 35, …. Pythagorean authors gave only examples of the various kinds of figured numbers until the second century A.D., and it was only in the third century a.d. that Diophantus developed a systematic mathematical theory based upon Pythagorean speculations.

The theory of *μεσότητες*, or "means," is also undoubtedly Pythagorean and probably of considerable antiquity. Iamblichus asserts (in Pistelli's edition of lamblichus' commentary on Nicomachus' *Introductio arithmetica*, p. 118) that Pythagoras learned of arithmetic means during his travels in Babylonia, but this cannot be definitely proved. The theory was at first concerned with three means; the arithmetic, of the form a — b = b — c; the geometric, of the form $a:b = b:c$; and the harmonic, of the form $(a - b):a = (b - c):c$. Other means were added at later dates, particularly by the Pythagorean Archytas of Tarentum, in the first half of the fourth century b.c.

It would seem likely, as O. Becker (in *Quellen und Studien zur Geschichte der Mathematik*, III B, pp. 534 ff.) and B. L. van der Waerden have pointed out, that Euclid took the whole complex of theorems and proofs that are based upon the distinction between odd and even numbers from the Pythagoreans, and that these reflect the Pythagorean interest in perfect numbers. This adaptation would seem to be particularly apparent in the *Elements*, IX. 30, and IX. 34, which lay the groundwork for the proof of IX. 36, the general Euclidean formula for perfect numbers. The proofs given in Euclid are strictly deductive and scientific, however, and would indicate that one group of Pythagoreans had quite early progressed from mysticism to true scientific method.

Although the contributions made by Pythagoras and his early successors to arithmetic and number theory can be determined with some accuracy, their contributions to geometry remain problematic. O. Neugebauer has shown (in *Mathematiker Keilschrifttexte*, I, 180, and II, 53) that the so-called Pythagorean theorem had been known in Babylonia at the time of Hammurabi, and it is possible that Pythagoras had learned it there. It is not known whether the theorem was proved during Pythagoras' lifetime, or shortly thereafter. The pentagram, which played an important role in Pythagorean circles in the early fifth century b.c., was also known in Babylonia, and may have been imported from there. This figure, a regular pentagon with its sides extended to intersect in the form of a five-pointed star, has the interesting property that its sides and diagonals intersect everywhere according to the golden section; the Pythagoreans used it as a symbol by which they recognized each other.

Of the mathematical discoveries attributed by ancient tradition to the Pythagoreans, the most important remains that of incommensurability. According to Plato's *Theaetetus*, this discovery cannot have been made later than the third quarter of the fifth century b.c., and although there has been some scholarly debate concerning the accuracy of this assertion, there is no reason to believe that it is not accurate. It is certain that the Pythagorean doctrine that all things are numbers would have been a strong incentive for the investigation of the hidden numbers that constitute the essences of the isosceles right-angled triangle or of the regular pentagon; if, as the Pythagoreans knew, it was always possible to construct a right triangle given sides in the ratio 3 : 4 : 5, then it should by analogy be possible to determine the numbers by which a right-angled isosceles triangle could be constructed.

The Babylonians had known approximations to the ratio of the side of a square to its diagonal, but the early Greek philosophers characteristically wished to know it exactly. This ratio cannot be expressed precisely in integers, as the early Pythagoreans discovered. They chose to approach the problem by seeking the greatest common measure—and hence the numerical ratio of two lengths—through mutual subtraction. In the case of the regular pentagon, it may easily be shown that the mutual subtraction of its diagonals and sides can be continued through an infinite number of operations, and that its ratio is therefore incommensurable. Ancient tradition credits this discovery to Hippasus of Metapontum, who was living at the period in which the discovery must have been made, and who could easily have made it by the method described. (Hippasus, one of the *μαθηματικοί*, who is said to have been set apart from the other Pythagoreans by his liberal political views, is also supposed to have been concerned with the "sphere composed by regular pentagons," that is, the regular dodecahedron.)

An appendix to book X of the *Elements* incorporates a proof of the incommensurability of the diagonal of a square with its side. This proof appears to be out of its proper order, and is apparently much older than the rest of the theorems contained in book X; it is based upon the distinction between odd and even numbers and closely resembles the theorems and proofs of book IX. Like the proofs of book IX, the proof offered in book X is related to the Pythagorean theory of perfect numbers; an ancient tradition states that it is Pythagorean in origin, and it would be gratuitous to reject this attribution simply because other members of the same sect were involved in nonscientific speculations about numbers. It is further possible that the ingenious process by which the theory of proportions (which had been conceived in the form of ratios of integers) had been applied to incommensurables—that is, by making the process of mutual subtraction itself the criterion of proportionality—was also a Pythagorean invention. But it is clear that the later elaboration of the theory of incommensurability and irrationality was the work of mathematicians who no longer had any close ties to the Pythagorean sect.

Pythagoras (or, according to another tradition, Hippasus) is also credited with knowing how to construct three of the five regular solids, specifically the pyramid, the cube, and the dodecahedron. Although these constructions can hardly have been identical to the ones in book XIII of the *Elements*, which a credible tradition attributes to Theaetetus, it is altogether likely that these forms, particularly the dodecahedron, had been of interest not only to Hippasus (as has been noted) but also to even earlier Pythagoreans. Their curiosity must have been aroused by both its geometrical properties (since it is made up of regular pentagons) and its occurrence in nature, since iron pyrite crystals of this form are found in Italy. An artifact in the form of a carved stone dodecahedron, moreover, dates from the tenth century b.c., and would seem to have played some part in an Etruscan cult.

The notion that all things are numbers is also fundamental to Pythagorean music theory. Early Pythagorean music theory would seem to have initially been of the same speculative sort as early Pythagorean mathematical theory. It was based upon observations drawn from the lyre and the flute, the most widely used instruments; from these observations it was concluded that the most beautiful musical harmonies corresponded to the most beautiful (because simplest) ratios or combinations of numbers, namely the octave (2:1), the fifth (3:2), and the fourth (4:3). It was thus possible to assign the numbers 6, 8, 9, and 12 to the four fixed strings of the lyre, and to determine the intervals of the diatonic scale as 9:8, 9:8, and 256:243. From these observations and speculations the Pythagoreans built up, as van der Waerden has pointed out, a deductive system of musical theory based on postulates or “axioms” (a term that has a function similar to its use in mathematics). The dependence of musical intervals on mathematical ratios was thus established.

The early music theory was later tested and extended in a number of ways. Hippasus, perhaps continuing work begun by the musician Lasos of Hermione, is said to have experimented with empty and partially filled glass vessels and with metal discs of varying thicknesses to determine whether the same ratios would produce the same harmonies with these instruments. (Contrary to ancient tradition, it would have been impossible to achieve sufficient accuracy by these means for him to have been altogether successful in this effort.) The systematic deductive theory was later enlarged to encompass the major and minor third (5:4 and 6:5), as well as the diminished minor third (7:6) and the augmented whole tone (8:7). The foundation for the enharmonic and chromatic scales was thus laid, which led to the more complex theory of music developed by Archytas of Tarentum in the first half of the fourth century b.c.

In addition to its specifically Pythagorean elements, Pythagorean astronomy would seem to have comprised both Babylonian observations and theories (presumably brought back by Pythagoras from his travels) and certain theories developed by Anaximander of Miletus, whose disciple Pythagoras is said to have been. It is not known precisely when Babylonian astronomy had begun, or what state it had reached at the time of Pythagoras, although ancient documents indicate that regular observations of the appearances of the planet Venus had been made as early as the reign of King Amisadaqa (about 1975 b.c.). The *mul apin* texts of about 700 b.c. give a summary of Babylonian astronomy up to that time, moreover, and contain divisions of the heavens into “roads of the fixed stars” (similar to the divisions of the zodiac), statements on the courses of the planets, and data on the risings and settings of stars that are obviously based on observations carried out over a considerable period of time. In addition, Ptolemy stated that regular observations of eclipses had been recorded since the time of King Nabonassar, about 747 b.c. The Babylonians of this time also knew that lunar eclipses occur only at full moon and solar eclipses at new moon, and that lunar eclipses occur at intervals of approximately six months; they knew seven “planets” (including the sun and moon), and therefore must have known the morning and evening star to be identical.

The Babylonians also knew that the independent motions of the planets occur in a plane that intersects the equator of the heavenly sphere at an angle. Greek tradition attributes the determination of this angle as 24° to Pythagoras, although the computation was actually made by Oenopides of Chios, in the second half of the fifth century b.c. Oenopides was not a Pythagorean, but he obviously drew upon Pythagorean mathematics and astronomy, just as the Pythagoreans drew upon the body of Babylonian knowledge.

Anaxirnander’s contributions to Pythagorean astronomy were less direct. The Pythagoreans rejected his chief theory, whereby the stars were in fact rings of fire that encircled the entire universe; according to Anaximander these fiery rings were obscured by “dark air,” so that they were visible only through the holes through which they breathed. Pythagoras and his adherents, on the other hand, accepted the Babylonian notion of the stars as heavenly bodies of divine origin. They did, however, make use of Anaximander’s assumption that the planets (or, rather, the rings in which they appear) are at different distances from the earth, or at any rate are nearer to the earth than are the fixed stars. This idea became an important part of Pythagorean astronomy (see Heiberg’s edition, Eudemus of Rhodes in Simplicius’ *Commentary* on Aristotle’s *De caelo*, p. 471, and Diels and Kranz’s edition of *Die Fragmente der Vorsokratiker*, sec. 12, 19).

Their knowledge of the periodicity of the movements of the stars undoubtedly strengthened the Pythagoreans in their belief that all things are numbers. They attempted to develop astronomical theory by combining it with this general principle, among others (including the principle of beauty that had figured in their axiomatic foundations of the theory of music). Their concern with musical intervals led them to try to determine the sequence of the planets in relation to the position of the earth (compare Eudemus, in the work cited, and Ptolemy, *Syniaxis*, IX, I). According to their theory, probably the earliest of its kind, the order of the planets, in regard to their increasing distance from the earth, was the moon. Mercury, Venus, the sun, Mars, Jupiter, and Saturn—a sequence that was later refined by placing Mercury and Venus above the sun, since no solar transits of these bodies had been observed.

Further theories by which the distances and periods of revolution of the heavenly bodies are correlated with musical intervals are greatly various, if not actually contradictory. Indeed, according to van der Waerden (in "Die Astronomie der Pythagoreer" pp. 34 ff.), a number of them make very little sense in the context of musical theory. It is almost impossible to tell what the original astronomical-musical theory on which these variants are based actually was, although it was almost certainly of considerable antiquity. It may be assumed, however, that in any original theory the celestial spheres were likened to the seven strings of a lyre, and were thought to produce a celestial harmony called the music of the spheres. Ordinary mortals could not hear this music (Aristotle suggested that this was because they had been exposed to it continuously since the moment of their birth), but later Pythagoreans said that it was audible to Pythagoras himself.

Another mystical notion, this one adopted from the Babylonians, was that of the great year. This concept, which was used by the Pythagoreans and probably by Pythagoras, held that since the periods of revolution of all heavenly bodies were in integral ratio, a least common multiple must exist, so that exactly the same constellation of all stars must recur after some definite period of time (the "great year" itself). It thereupon followed that all things that have occurred will recur in precisely the same way; Eudemus is reported to have said in a lecture (not without irony) that "then I shall sit here again with this pointer in my hand and tell you such strange things."

Pythagorean ideas of beauty required that the stars move in the simplest curves. This principle thus demanded that all celestial bodies move in circles, the circle being the most beautiful curve, a notion that held the utmost importance for the development of ancient astronomy. If van der Waerden's ingenious interpretation of the difficult ancient texts on this subject is correct (in "Die Astronomie der Pythagoreer," pp. 42 ff.), there may have been—even before Plato asked the non-Pythagorean mathematician Eudoxus to create a model showing the circular movements of all celestial bodies—a Pythagorean theory that explained the movements of Mercury and Venus as epicycles around the sun, and thus represented the first step toward a heliocentric system.

Ancient tradition also refers to an entirely different celestial system, in which the earth does not rest in the center of the universe (as in the theories of Anaximander, the Babylonians of the fifth century b.c., and the other Pythagoreans), but rather revolves around a central fire. This fire is invisible to men, because the inhabited side of the earth is always turned away from it. According to this theory, there is also a counter-earth on the opposite side of the fire. Pythagorean principles of beauty and of a hierarchical order in nature are here fundamental; fire, being more noble than earth, must therefore occupy a more noble position in the universe, its center (compare Aristotle, *De caelo*, II, 13). This theory is sometimes attributed to Philolaus, a Pythagorean of the late fifth century b.c., and he may have derived the epicyclic theory from it, although the surviving fragments of Philolaus' work indicate him to have been a man of only modest intellectual capacities, and unlikely to have been the inventor of such an ingenious system. Other ancient sources name Hicetas of Syracuse, a Pythagorean of whom almost nothing else is known, as its author.

The decisive influence of this theory in the history of astronomy lies in its explanation of the chief movements of the celestial bodies as being merely apparent. The assumption that the solid earth, on which man lives, does not stand still but moves with great speed (since some Pythagoreans according to Aristotle explained the phenomenon of day and night by the movement of the earth around the central fire) was a bold one, although the paucity and vagueness of ancient records make it impossible to determine with any certainty how far this notion was applied to other celestial phenomena. Further details on this theory are also difficult to ascertain; van der Waerden (in "Die Astronomie der Pythagoreer," pp. 49. ff.) discusses the problem at length. It is nevertheless clear that this daring, and somewhat unscientific, speculation was a giant step toward the development of a heliocentric system. Once the idea of an unmoving earth at the center of the universe had been overcome, the Pythagorean Ecphantus and Plato's disciple Heraclides were, in about 350 b.c., able to teach that the earth revolves about its own axis (*Aetius*, III, 13). A fully heliocentric system was then presented by Aristarchus of Samos, in about 260 b.c., although it was later abandoned by Ptolemy because its circular orbits did not sufficiently agree with his careful observations.

It is thus apparent that the tendency of some modern scholars to reject the unanimous and plausible ancient tradition concerning the Pythagoreans and their discoveries—and to attribute these accomplishments instead to a number of unknown, cautious, and pedestrian observers and calculators—obscures one of the most interesting aspects of the early development of Greek science.

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