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9-12 minutes

(more properly **Salah al-Dīn Mūsā Pasha**)

(*b*. Bursa, Turkey, ca. 1364; *d. Samarkand, Uzbekistan, ca*. 1436)

mathematics, astronomy.

Most historians of science have erred concerning Qāṭī Zāda. A. Sédillot, for example, called him Hassan Tchéleb'; and Montucla, in his history of mathematics, said that he was a Greek convert to Islam. Montucla may have been deceived by the surname Rūmī, for the peoples who lived in Asia Minor were called Rūm, meaning Roman (not Greek), because Asia Minor was once Roman. Qāṭī Zāda means "son of the judge."

After completing his secondary education at Bursa, Qāṭī Zāda became a student of the theologian and encyclopedist Mullā Shams al-īn Muḥammad al-Fanāri (1350–1431), who taught him geometry and astronomy. Sensing the great talent of his student, al-Fanārī advised him to go to Transoxiana, then a great cultural center, to continue his training in mathematics and astronomy. According to several historians, al-Fanārī gave Qāṭī Zāda letters of recommendation and one of his works (*Enmuzeğ al-ulum* "Types of Sciences") to present to the scholars of Khurasan and Transoxiana.

The year of Qāṭī Zāda's departure from Bursa is not known. It must have been after 1383, however, for in that year, still at Bursa, he had composed a work on arithmetic, *Risāla fi ʾ-ḥisāb*. When he presented himself to Ulugh Beg in Samarkand (*ca*. 1410)[1], he had spent some time in Iran, Jurjan, and Khorasan. At Jurjan he had met the philosopher-theologian Seyyid al-Sharif al-Jurjānī. Therefore, the year of departure from Bursa probably falls between 1405 and 1408.

In 1421 Ulugh Beg ordered the construction of a university at Samarkand and named Qāḍī Zāda as its rector. Serving in addition as professor of mathematics and astronomy, Qatjf Zada frequently had Ulugh Beg as a student in his classes. Also in 1421, under the direction of the young Persian astronomer and mathematician al-Kāshi, the construction of the observatory at Samarkand was completed. Astronomical observations had been made and the composing of astronomical tables had been begun. These tables were composed to correct and complete the tables of Naṣir alDin al-Ṭūsī. Ulugh Beg made al-Kāshi director of the observatory; and after his death in 1429, Qāḍī Zāda took his place. Qāḍī Zāda died before the astronomical tables were completed. ʻAlī Kūshjī succeeded him, and was director when the Jurjanian Tables were published.

Qāḍī Zāda married in Samarkand and had a son named Shams al-Dīn Muḥammad, who married a daughter of ʻAll Kūshjī; from this marriage was born Qutb al-Dīn, father of the Turkish mathematician Miram Chclebi.

One of Qāḍī Zāda's calculations is presented below as an example of his approach to a geometrical-algebraic problem. It concerns determining the value of sin 1°. The example is drawn from the *Dastūr al-ʿamal wa-taṣīṣ al-jadwal* ("Practical Formula and Correction of the Table") of Miram Chelebi, who states that Qāḍī Zāda wrote it.

Qāḍī Zāda, finding al-KashPs work on the approximate determination of the value of sin 1° to be very precise, commented upon it and gave further explanation in his *Risāla fi ʾl-jayb*[2].

Like a-lKāshi, Qāḍī Zāda supposed that an arc ABCD taken on a circle with center O and diameter is divided by the points *C* and *B* into three equal parts. Also like al-Kāshī, he was well aware of the impossibility of geometrically dividing an arc into three equal parts in order to obtain the chords and . Applying Ptolemy's theorem to the inscribed quadrilateral *ABCD* thus constructed, he wrote the following equation:

Considering the equalities

and

according to the hypothesis, equation (1) becomes

Qāḍī Zāda also supposed that arc ABCD is equal to 6°; therefore the chords and will belong to arcs of 2°.

Qāḍī Zāda next applied the iterative method of alKāshi to the determination of the chord belonging to the arc of 2°. That is, he algebraically divided the are of 6°, the chord of which was known, into three equal parts. In taking as unknown (as a function of the parts of the radius) the chord belonging to the arc of 2° (that is, = x) he obtained the equation

which is the equivalent of (2).

Since the chord belongs to the arc of 6°, Qāḍī Zāda (like al-Kāshi) obtained the following value of (in the sexagesimal system):

6^p.16.49.07.59.08.56.29.40

(where). This value of (in both authors) was determined by means of the arc of 72° and 60°, the chords of which were known geometrically. Knowing the chords of the arcs of 72° (the side of a regular pentagon) and of 60° (the side of a regular hexagon), they obtained the chord belonging to the arc of 72° - 60° = 12°. Then, applying the formula that gives the value of the chord belonging to half of an arc of which the chord is known, Qāḍī Zāda obtained the value of . To find the value of , which appears on the right-hand side of (1), he used the theorem discussed below (already utilized by al-Kāshi in his *Risāla al-muḥiṭiyya* ["Treatise on the Circumference"] for determining the value of π).

The theorem is that the ratio of the difference between the diameter of a circle and the chord belonging to any arc (when the arc is taken on this circle and one of its extremities passes through the extremity of the diameter) to the chord belonging to half of the supplement of this arc is equal to the ratio of this same chord to the radius of the circle.

Taking A'C as the arc, the theorem can be written in the following manner:

In equation (4)[3.] one first separates , then for the value of obtains (replacing with R and with 2R):

From the right triangle ACA' one obtains

and in equating the two values of given by (5) and (6), one arrives at the equation

Placing this value of in equation (3), one obtains (after simplifications) the equation

The value of having been determined, the above equation can be written in the sexagesimal system as

$3x = (6^p.16.49.07.59.08.56.29.40) + x^3$.

Qāḍī Zāda, applying al-Kāshi' iterative method to this last equation, found for the unknown x (the value of the chord relative to 2°) the value

$x = 2^p$.05.39.26.22.19.28.32.52.33.

Half of this value would yield approximately the sine of 1°:

1^p.02.49.43.11.14.44.16.26.16.30.

This value is equal to that found by al-Kāshi. The value of sin 1° in the decimal system thus would be 0.017452406437, which is exact to within 10sup(12), a ° degree of precision also achieved by al-Kāshl.[4.]

NOTES

1. At this time Ulugh Beg was seventeen years old and was governor of Samarkand. He had appointed Qāḍī Zāda as private professor of mathematics and astronomy. *Qdmfts al- a'lam*, 2, 1023.

2. The full title of this work by*Qāḍī Zāda is Risala fi istikhraj jayb daraja wahida bi'l-a 'mal al-nm assasa ala qawd(id hisabiyya wa-handaiyya* ("Treatise on the Determination of sin 1°, With the Aid of the Rules of Computation and of Geometry"). It is in the Dastūr al-'amal wa-taị̄ al-jadwal of Mīram Chelcbi: Süleymaniyye Library, Istanbul, Hassan Hüsnü 1284; and in the same library, Tchorloulou Ali Pasha 342. Also in the Bibliothèque Nationale, in Paris, MSS Thevenol, Anc. Fonds 171.

3. Although the equation (4) that expresses the theorem is not in Euclid's *Elements*, it can readily be verified. From the obvious equalities and , one immediately derives the relations and Demonstrating the theorem thus reduces to verifying

On the other hand, from the triangles (see Figure 2) *AMB* and *OAM* one can write the equations

Then, from the fact that one obtains,

which, after simplification, gives equation (4’).

4. Abu’ Wafā had found sin 1° ∾ m 0.017452414587, which is exact to within $10^{-7.}$ Bibliothéque Nationale Paris, MS 1138.

BIBLIOGRAPHY

I. Original Works. Risäla fiʿl-lusdb (“Treatise on Arithmetic”), which covers arithmetic, algebra, and measures, was written at Bursa in 1383. It is now in Šehid Ali Paša, Šehzada Ğarni, Istanbul.

Sharḥ al-Mulaḥḥas fi’l-haya, a commentary on the Mulaḥ#x1E25;as (“Compendium”) of the astronomer ’Umar al-Jaghmin (d. 1444/1445), was written at Samarkand in 1412–1413 for Ulugh Beg. Several copies of the work are at Istanbul and in various European libraries. It has been printed at Delhi, Lucknow, and Teheran.

Sharh Ashkāl al-taʾsis, a commentary on the *Ashkāl al-ta’sis* (“Fundamental Theorems”) of Shams al-Dīn al-Samarqandi, was written at Samarkand in 1412. It contains 35 propositions from Euclid’Elements. It is now in the Ayasofya (Süleymaniyya) Library, Istanbul, MS 2712. An author’s autograph copy is at Bursa, Haradji-oglou Library, no. 21. A Turkish trans, of *Ashkāl al-taʿsis* by Muftu Zāda ʾAbd al-Raḥim Effendi (1795) is in the library of the Technical University of Istanbul, No. (4316–5075).

Risāla fiʾl-hayʾa wa ʾl-handasa (“Treatise on Astronomy and Geometry”), in an author’s autograph copy, is in Ine Bey Library, Bursa, MS 25.

Risala fi’l samt al-qibla (“Treatise on the Azimuth of Qibla”), dealing with facing Mecca during prayer, is in Bursa, Ine Béy Library, MS 12. It also contains a short work by Miram Chelebi, Risāla fi tahqiq samt al-qibla wa-barāhiniha (“Treatise on the Verification and Proof of the Azimuth of Qibla”).

Risālat al-jayb (“Treatise on the Sine”), Qāḍī Zāda” most original work, was written at Samarkand at the time of al-Kāshi. See Mlram Chelebi, *Dastūr al-’amal wa-taṣḥuh al-jadwal*, Suleymaniyya Library, Istanbul, Hassan Husnu 1284 and Tchorloulou Ali Pasha 342.

II. Secondary Literature. See Adnan Adivar, Osmanli Tūrklarinde Ilim (Istanbul, 1943), 4; B. A. Rozenfeld and A. P. Youschkevitch, “Primechania k traktatu Kazi-Zade Ar-Rūmi” (“Notes, on a Treatise of Qāḍī Zāda”), in*Istoriko-matematicheskie issledovania*, **13** (1960), 552–556; Sūheyl Ünver, *Ali Kuschdju* (Istanbul, 1948), 73; and Sälih Zeki, Assār-i Bāqiyya, I (Istanbul, 1913), 186.

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