Chin Chiu-shao | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
24-30 minutes

(*b*. Szechuan, China, ca. 1202; *d*. Kwangtung, China, c. 1261),

mathematics.

Ch'in Chiu-chiu-shao (literary name Tao-ku) has been described by George Sarton as "one of the greatest mathematicians of his race, of his time, and indeed of all times". A genius in mathematics and accomplished in poetry, archery, fencing, riding, music, and architecture, Chi'in has often been judged an intriguing and unprincipled character, reminding one of the sixteenth-century mathematician Girolamo Cardano. In love affairs he had a reputation similar to Ibn Sina's. Liu K'e-chuang, in a petition to the emperor, described him as a person "as violent as a tiger or a wolf, and as poisonous as a viper or a scorpion." He was also described as an ill-disciplined youth. During a banquest given by his father, a commotion was creatd when a stone suddenly landed among the guests; investigation disclosed that the missile had come from the direction of Chi'in, who was showing a *fille de joie* how to use a bow as a sling to hurl projectiles. Chou Mi, in his supplementary volume to the *Kuei-yu tsa-chih* tells us how Chi'in deceived his friend Wu Chi'en in order to acquire a plot of his land, how he punished a female member of his household by confinement and starvation, and how he became notorious for being inclined to poison those he found disagreeable. We are also told that on his dismissal from the governorship of Ch'ung-chou in 1258 he returned home with immense wealth-after having been in office for just over a hundred days.

According to a recent study by Ch'ien Pao-tsung and others Chi'in was born in the city of P'u-chou (now An-yueh) in Szechuan province. Ch'in called himself a native of Lu-chun, in Shantung province; but he was simply referring to the place his ancestors came from rather than to his palce of birth. His father, Ch'in Chi-yu (literary name Hung-fu), was a civil servant. In 1219 Ch'in joined the army as the head of a unit of territorial volunteers and participated in curbing a rebellion staged by Chang Fu and Mo Chin. In 1224–1225 he follwed his father when the latter was transferrd to the Sung capital, Chung-tu(now Hangchow). There he had the opportunity to study astronomy at the astronomical bureau under the guidance of the official astronomers, Shortly afterward, however, his father was sent to the prefecture of Tung-Ch'üan (now San-t'ai in Szechuan province), and Ch'in had to leave the capital. About 1233 he served as a sheriff in one of the subprefectures in Szechuan.

The Mongols invaded Szechuan in 1236, and Ch'in fled to the east, where he first became a vice administrator (*t'ung-p'an*) in Ch'i-chou prefecture (now Ch'i-ch'un in Hupeh province) and the governor of Ho-chou (now Ho-hsien in Anhwei province). In the latter part of 1244 Ch'in was appointed one of the vice-administrators of the superior prefecture of Chien-k'ang-fu (now Nanking), but some three months later he relinquished this post because of his mother's death. He returned to Hu'chou (now Wuhsing in Chekiang province), and it was probably there that he wrote his celebrated mathematical treatise *Shu-shu chiu-chang* ("Mathematical Treatise in Nine section"), which appeared in 1247. In the preface of this book Ch'in mentions that he learned mathematics from a certain recluse scholar, but he does not give his identity.

In 1254 Ch'in returned to Chung-tu to reenter civil service, but for some unknown reason he soon resigned and went back to his native home. He paid a visit to Chia Shih-tao, an influential minister at that time, and was appointed governor of Ch'iung-chou (in modern Hainan) in 1258. A few months later, however, Ch'in was dismissed because of charges of bribery and corruption. Nevertheless, he managed to find another job as a civil aide to an intimate friend of his, Wu Ch'ien (literary name Li-chai), who was then in charge of marine affirs in the district of Yin (near modern Ningpo in Chekiang province). Wu ch'ien eventually became a minister, but in 1260 he lost favor and was given a lesser assignment in south China. Ch'in followed his friend to Kwangtung province and received an appointment in Mei-chou (now Mei-hsien), where he died shortly afterward. The year of Ch'in's death has been estimated as 1261, for there was an edict in the following year banning Wu Ch'ien and his associates from the civil service.

The title of Ch'in's *Shu-shu chiu-chang* has given rise to some confusion. According to Ch'en Chen-sun, a contmporary of Ch'in Chiu-shao's and owner of a copy of the work, the original title was *Shu shu* ("Mathematical Treatise") However, in his *Chih chai shu lu chieh it* he gives the title as *Shu-shu ta lueh* ("outline of Mathematical Methods") During the thirteenth century this treatise was referred to as the *Shu-shu ta-lueh* or the *Shu-hsueh la-lueh* ("outline of Mathematics"), while during the Ming period (1368–1644) it was known under the names *Shu-shu chiu-chang* and *Shu-hsueh chiu-chang* This has led Sarton to conclude that the *Shu-shu chiu-chng* and the *shu hsueh ta-lueh* were separate works. The treatise is now popularly known as the *Shu-shu chiu-chang*.

It appears that the *Shu-shu chiu-chang* existed only in manuscript form for several centuries. It was copied and included in the great early fifteenth-century encyclopedia *Yung-lo ta-tien* under the title *Shu hsüeh chiu-chang* this version was revised and included in the seventeenth-century imperial collection *Ssu-K'u ch'uan-shu*, and later a commentary was added to it by the Ch'ing mathmatician Li jui. There was also a handwritten copy of the *Shu-shu chiu-chang* during the early seventeenth century. A copy from the text belonging to the Wen-yuan K'o library was first made by Wang Ying-lin. And in 1616 Chao Ch'i-mei wrote that he had made a copy of the text that he borrowed from wang Ying-lin and had added a new table of contents to it. Toward the beginning of the nineteenth century this handwritten copy came into the possession of the mathematician Chang Tun-jen and attracted much attention during the time when interest in traditional Chinese mathematics was revived. Many copies were reproduced from the text owned by Chang Tun-jen. It seems that blocks were also made for the printing of the book, but it is not certin whether it was actually printed, also in the early nineteenth century Shen Ch'in-p'ei began to make a textual collation of the *Shu-shu chiu-chang*, but he died before his work was finished. One of his disciples, Sung Ching-ch'ang, completed it; and the result appeared in the *Shu-shu chiu-chang cha-chi* ("Notes on the Mathematical Treatise in Nine section"). In 1842 both the *Shu-shu chiu-chang* and the *Shu-shu chiu-chang cha-chi* were published and included in the *I-chia-t'ang ts'ung shu* collection. Later editions of both these books, such as those included in the *Ku-chin suan-hsüeh is'üng shu* the *Kuo-hsüeh chi-pen ts'üng shu*, and the *Ts'üng shu chi-ch'eng* collections, are based on the version in the *i-chia-t'ang ts'üng-shu* collection.

Each of the nine sections in the *Shu-shu chiu-chang* includes two chapters made up of nine problems. These sections do not correspond in any way to the nine sections of the *Chiu-hang suan-shu* of Liu Hui. They consist of (1)*ta yen ch'iu i shu*, or indeterminate analysis;(2)*t'ien shih* which involves astronomical, calendrical, and meteorologicl calculations; (3) *t'ien yu*, or land measurement; (4) *ts'e wang* (5)*fu i*, or land tax and state service; (6)*ch'ien ku*, or money and grains;(7)*ying chin*, or structural works; (8) *chun hi*, or military matters; and (9)*shih wu*, dealing with barter and purchase. The complete text has not yet been translated for investigated in full, although some individual problems have been studied.

With Ch'in's *Shu-shu chiu-chang* the study of interminate analysis in China reached its height. It had first appeared in Chinese mathematical texts about the fourth century in a problem in the *Sun-tzu suan ching;*

There is an unknown number of things, when counted in threes, they leave a remainder of two; when counted by fives, thy leave a remainder of three; and when counted by sevens, they leave a remainder of two. Fin the number of things.

The problem can be expressed in the modern form

$N \equiv 2 \pmod 3 \equiv 3 \pmod 5 \equiv 2 \pmod 7$,

Where the least integer for N is required. The *Suntzu suan ching* gives the following solution:

$N = 2 \times 70 + 3 \times 21 + 2 \times 15 - 2 \times 105 = 23$.

There is no explanation of the mathematical method in general, but the algorithmical procedure is given as follows:

If you count by threes and have the remainder 2, put 140.

If you count by fives and have the remainder 3, put 63.

If you count by sevens and have the remainder 2, put 30.

Add these numbers, and you get 233.

From this subtract 210, and you have the result.

A brief explanation of the procedure follows:

For each 1 as a remainder, when counting by threes, put 70.

For each 1 as a remainder, when counting by fives, put 21.

For each 1 as a remainder, when counting by sevens, put 15.

If the sum is 106 or more, subtract 105 from it, and you have the result.

(The number 105 to be subtracted is, of course, derived from the product of 3, 5 and 7.)

The next example of indeterminate problem appeared in that of the "hundred fowls," found in the fifth-century mathematical manual *Chang Ch'iu-chien suan ching*. It gives three different possible answers but no general solution. Since it is in the form of two simultaneous linear equations of three unknowns, i.e.,

$ax + by + cz = 100$

$a'x + b'y + c'z = 100$,

this problem is not of the same nature as that given in the *Sun'tzu suan ching*.

As early as the middle of the third century. Chinese calendar experts had taken as their starting point a certin date and time in the past known as the Grand Cycle *(Shang yuan)*, which was the last time that the winter solstice fell exactly at midnight on the first day of the eleventh month, which also happened to be the first day (*chia-tzu*, cyclical day) of a sixty-day cycle. If a denotes the tropical year, R1 the cyclical-day number of the winter solstice (i.e., the number of days in the sixty-day cycle between winter solstice and the last *chia-tzu* preceding it), *b* the synodic month, and R_2 the number of days between the first day of the eleventh month and the winter solstice, then *N*, the number of years since the Grand Cycle, can be found from the expression.

$aN \equiv R_1 \pmod{60} \equiv R_2 \pmod{b}$.

For several hundred years, calendar experts in China had been working out the Grand cycle fron new astronomical data as it became available. None, however, has passed on the method of computation. The earliest elucidation of the method available to us comes from Chi'in Chiu-shar. Problem 12 in his *Shu shu chiu-chang* deals exactly with the above and may be stated in the modern form

$6,172,608\ N \equiv 193,440 \pmod{60 \times 16,900}$
$\equiv 16,377 \pmod{499,067}$,

Chi'in's method of solving indeterminate analysis may be explained in the modern form as follows:

Given $N \equiv R_1 \pmod{a_1} \equiv R_2 \pmod{a_2} \equiv R_3 \pmod{a_3} \equiv \ldots \equiv R_n \pmod{a_n}$ where $a_1, a_2, a_3, \ldots a_n$ have no common factors.

If $k_1, k_2, k_3, \ldots k_n$ are factors such that

$k_1a_2a_3 \ldots a_n \equiv 1 \pmod{a_1}$

$k_2a_3 \ldots . a_na_1 \equiv 1 \pmod{a_2}$

$k_3a_1a_2a_4 \ldots a_n \equiv 1 \pmod{a_3}$, and

$k_na_1a_2a_3 \ldots a_{n-1} \equiv 1 \pmod{a_n}$,

then $N \equiv (R_1k_1a_2a_3 \ldots a_n) + (R_2k_2a_3 \ldots a_na_1) + (R_3k_3a_1a_2a_4 \ldots a_n) + \ldots + (R_nk_na_1a_2a_3 \ldots a_{n-1}) \pmod{a_1a_2a_3 \ldots a_n}$,

or, putting it more generally,

Where $M = a_1a_2a_3 \ldots a_n$, i.e., the least common multiple and *p* is the integer that yields the lowest value for *N*.

The rule is given in a German manuscript from Göttingen (ca. 1550), but it was not rediscovered in Europe before Lebesque (1859) and stieltjes (1890). The idntity of the Chinese rule with Gauss's formula has also been pointed out by Matthiessen in the last century, after chi'in's study of indeterminate analysis was first brought to the attntion of the west by Alexander Wylie.

The *Shu-shu chiu-chang* also deals with numbers which have common factors among them, in other words the more general form

$N \equiv R_i \pmod{A_i}$

Where $i = 1,2,3,\ldots,n$ and where A_i has common factors with A_j, A_k and so on.

The method involves choosing $A_1, A_2, \ldots . A_n$, which are relaive primes in pairs such that each A_i, divides the corresponding a_i and that further the LCM of $A_1, A_2 \ldots A_n$ equals that of $a_1\ a_2 \ldots a_n$ Then every solution of

$N \equiv R_1 \pmod{A_1} \equiv R_2 \pmod{A_2}$

$\equiv \ldots \equiv R_n \pmod{A_n}$

also satisfies

$N \equiv R_1 \pmod{a_1} \equiv R_2 \pmod{a_2}$

$\equiv \ldots \equiv R_n \pmod{a_n}$.

The above is valid only under the condition that each difference R_i-R_j is divisble by d, the GCD of the corresponding moduli Ai and Aj, i.e., R_i-R_j=O (mod d). The Chinese text does not mention this condition, but it is fulfilled in all the examples given by Ch'in Chiu-chao. Chi'in would go about searching for the least integral value of a multiple K_i such that

This is an important intermediate stage in the process of solving problems of indeterminate analysis; hence the Chinese term *chi'iu i shu* ("method of searching for unity") for indeterminate analysis. Over time the process became known as the *ta yen ch'iu i shu* ("the great Extension method of searching for unity"). The term *ta yen* ("Grat Extension") came from an obscure statement in the Book of Changes In an ancient method of divination, fifty yarrow stalks were taken, and one was set aside before the remaining forty-nine were divided into two random heaps. The Book of Changes then says:

The numbers of the Great Extension [multiplied together] make fifty, of which [only] forty-nine are used [in divination]. [The stalks representing these] are divided into two heaps to represent the two [emblematic lines, or heaven and earth] and placed [between the little finger and ring finger of the left hand], that there may thus be symbolized the three [powers of heaven, earth, and man]. [The heaps on both sides] are manipulated by fours to represent the four seasons…

Ch'in sought to explain the term *ta yen ch'iu i shu* in the first problem of his book by introducing the so-called "Great Extension number" 50 and the number 49 and showing how they could be arrived at from numbers 1,2,3, and 4-as mentioned above.

Since Ch'in also many technical terms used in conjunction with inderminate analysis, it will be worthwhile to follow, step by step, the actual process he used in working out the problem that may be expressed as $N = 1 \pmod 1 = 1 \pmod 2 = 1 \pmod 3 = 1 \pmod 4$.

Ch'in first arranged the given numbers 1, 2, 3, and 4, known as *yuan-shu* ("original number"), in a vertical column. He placed the number 1 to the left of each of these numbers, as in Fig. 1.

This he called the t'ten-yuan("celestial monad" or "celestial element"). Next, he cross-multiplied each celestial monad by the original numbers not pertaining to it, thus obtaining the yen-shu("operation numbers"), which were then placed to the left of the corresponding original numbers, as in Fig. 2.

He next removed all the common factors in the original numbers, retaining the only one of each. Thus the original numbers became prime to one another and were known by the term *ting-mu* ("definite base numbers"), as in Fig. 3. Each celestial monad was cross-multiplied by the definite base numbers not pertaining to it, giving another set of operation numbers, which were then placed to the left of the corresponding definite base numbers as in Fig.4.

Ch'in then took the definite base numbers as moduli and formed the congruences with their respective operation numbers:

$12 \equiv 1 \pmod 1$

$12 \equiv 1 \pmod 1$

$4 \equiv 1 \pmod 3$

$3 \equiv 3 \pmod 4$.

For the residues *(ch'i shu)* that were unity, the corresponding multipliers *(ch'eng lii)* were taken as unity. A residue that was not unity was place in the upper-right space on the counting board, with the corresponding definite base number below it. To the left of this residue Ch'in placed unity as the celestial monad, as in Fig.5.

Dividing the definite base number by the residue yielded unity in this case. Ch'in next multiplied the celestial monad by unity and placed the result(in this case, also unity) at the bottom left. This he called the kuei-shu ("reduced number"). The space for the definite base number was then filled by the residue of the base number, as shown in Fig. 6.

Next the residue of the operation number in the upper right-hand corner was divided by the residue of the base number so that a quotient could be found to give a remainder of unity. If a quotient could not be found, then the process had to be repeated, taking the number in the lower right-hand corner and that in the upper right-hand corner alternately until such a quotient was found. In this case, however, a quotient of 2 would give a remainder of unity, as in Fig.7.

Multiplying the quotient of 2 by the reduced number and adding the result to the celestial monad gave 3, the corresponding multiplier, as in Fig.8.

Having found all the multipliers, Ch'in then arranged them side by side with their operation numbers and definite base numbers, as in Fig. 9.

Then he multiplied the operation numbers by their corresponding multipliers and found the so-called "reduced use numbers" *(fan-yung-shu)*. These were placed to the left of the corresponding definite base numbers, as in Fig.10.

The operation modulus *(yen-mu)*, or the least common multiple, was obtained by multiplying all the definite base numbers together. If common factors had been removed, they had to be restored at this stage. The products of these factors and the corresponding definite base numbers and the reduced use numbers were restored to their original numbers, and the reduced use numbers became the definite use numbers *(ting-yung-shu)*, as shown in Fig. 11.

In this particular problem Ch'in tried to explain that the sum of the operation numbers amounted to 50, while that of the definite use numbers came to 49. The former is the *ta yen* number, and the latter is the number that was put into use as stated in the *Book of Changes*. To obtain, *N*, the definite use numbers were multiplied by the respective remainders given in the problem. Their sum, after it had been diminished repeatedly by the least common factor, would ultimately give the required answer.

A list of the technical terms used by Ch'in for indeterminate analysis is given in Fig. 12.

It should be pointed out that Ch'in's use of the celestial monad or celestial element *(t'ien-yuan)* differs from the method of celestial element *(t'ienyuan-shu)* that was employed by his contemporary Li Chih and that later was known in Japan as the *tengen jutsu* In the former, the celestial element denotes a known number, while in the latter it represents an unknown algebraic quantity.

Ch'in represented algebraic equations by placing calculating rods on the countingboard so that the absolute term appeared on the top in a vertical column; immediately below it was the unknown quantity, followed by increasing powers of the unknown quantity. Originally, Ch'in used red and black counting rods to denote positive and negative quantities, respectively; but in the text, negative quantities are denoted by an extra rod placed obliquely over the first figure of the number concerned. The *Shu-shu chiuchang* also is the oldest extant Chinese mathematical text to contain the zero symbol. For example, the equation $-x^4 + 763{,}200x^2 - 40{,}642{,}560{,}000 = 0$ is represented by calculating rods placed on a counting board as in Fig. 13, and can be expressed in Arabic numerals as in Fig. 14

More than twenty problems in the *Shu-shu chiuchang* involve the setting up numerical equations. Some examples are given below

$4.608x^3 - 3{,}000{,}000{,}000 \text{ X } 30 \text{ X } 800 = 0$

$-x^4 + 15{,}245x^2 - 6{,}262{,}506.25 = 0$

$-x^4 + 1{,}534{,}464x^2 - 526{,}727{,}577{,}600 = 0$

$400x^4 - 2{,}930{,}000 = 0$

$x^{10} + 15x^8 + 72x^6 - 864x^4 - 11{,}664x^2 - 34{,}992 = 0$

Ch'in always arranged his equations so that the absolute term was negative. Sarton has pointed out that this is equivalent to Thomas Harriot's practice (1631) of writing algebraic equations so that the absolute term would stand alone in one member. Ch'in used a method called the *ling lung k'ai fang*, generally known by the translation "harmoniously alternating evolution," by which he could solve numerical equations of any degree. The method is identical to that rediscovered by Paolo Ruffini about 1805 and by William George Horner in 1819. It is doubtful that Ch'in was the originator of this method of solving numerical equations of higher degrees, since his contemporary Li Chih was also capable of solving similar equations, and some two decades later Yang Hui also described a similar method without mentioning Ch'in or Li, referring instead to several Chinese mathematicians of the twelfth century. Wang Ling and Joseph Needham have indicated that if the text of the *Chiu-changsuan-ching* (first century) is very carefully followed, the essentials of the method are there.

In the *Shu-shu chiu-chang* various values for are used. In one place we find the old value $\pi = 3$, in another we come across what Ch'in called the "accurate value" $\pi = 22/7$, and in yet another instance the value $\pi =$ is given. This last value was first mentioned by Chang Heng in the second century.

Formulas giving the areas of various types of geometrical figures are also mentioned in the *Shu-shu chiu-chang* although some of them are not very accurate. The area, A, of a scalene triangle with sides *a*, *b*, and *c* is obtained from the expression

That is,

The area, A, of a quadrangle with two pairs of equal sides, a and b, with c the diagonal the figure into two isosceles triangles (see Fig.15) is given by

the expression

$-A^4 + 2(X + Y)A^2 - (Y - X)^2 = 0,$

The area, A, of a so-called “banana-leaf-shaped” farm (see Fig. 16) formed by two equal circular arcs with

a common chord, c, and a common sagitta, b, is incorrectly given by the expression

This formula is put in another form by Mikami but has been wrongly represented by Li Yen. An earlier expression for the area of a segment (Fig. 17) given

the chord, c, and the arc sagitta, s, is given in the *Chiu-chang suan-shu* in the form

This has been in use in China for 1,000 years. Ch’in’s formula is a departure from the above, but it is not known how he arrived at it. In 1261 another formula was given by Yang Hui in the form

$-(2A)^2 + 4Ab^2 + 4db^3 - 5b^4 = 0$

where d is the diameter of the circle.

Sometimes Ch’in made his process unusually complicated. For example, a problem in chapter 8 says:

Given a circular walled city of unknown diameter with four gates, one at each of the four cardinal points. A tree lies three *li* north of the northern gate. If one turns and walks eastward for nine *li* immediately leaving the southern gate, the tree becomes just visible. Find the circumference and the diameter of the city wall.

If x is the diameter of the circular wall, c the distance of the tree from the northern gate, and b the distance to be traveled eastward from the southern gate before the tree becomes visible (as shown in Fig. 18),

then Ch’in obtains the diameter from the following equation of the tenth degree:

$y^{10} + 5cy^8 + 8c^2y^6 - 4c(b^2 - c^2)y^4 - 16c^2b^2y^2 - 16c^3b^3 = 0,$

where $x = y^2$.

It is interesting to compare this with an equivalent but simpler expression given by Ch’in’s contemporary Li Chili in the form

$x^3 + cx^2 - 4cb^2 = 0.$

In one of the problems in chapter 4, Ch’in intends to find the height of rainwater that would be collected on the level ground when the rain gauge is a basin with a larger diameter at the opening than at the base. The diameters of the opening and the base, a and b; the height, h, of rainwater collected in the basin; and the height, H, of the basin are given. The height of rain collected on level ground, h' is given by the formula h' =

The Shu-shu chiu-chang also concerns itself with series, such as

One of the problems in chapter 13 deals with finite difference, a subject that had attracted considerable attention from Chinese mathematicians and calendar makers.

Linear simultaneous equations are also discussed in Ch’in’s book. The numbers are set up in vertical columns. For example, the simultaneous equations

(1) $140x + 88y + 15z = 58{,}800$

(2) $792x + 568y + 815z = 392{,}000$

(3) $64x + 30y + 75z = 29{,}400$

are set up on the countingboard as follows:

29,400	392,000	58,800
64	792	140
30	568	88
75	815	15

These equations are solved by first eliminating z in equations (1) and (3), and then in (2) and (3). From these x and y are obtained. Finally z is found by substituting the values of x and y in equation (3).

Ch'ien Pao-tsung has pointed out that from the *Shu-shu chiu-chang* one can gather much information on the sociological problems in thirteenth-century China, from finance and commerce to the levy of taxes. The book gives information not only on the merchandise imported from overseas but also on the use of the rain gauge by the Sung government, which was greatly concerned with rainfall because of the importance of agriculture.

BIBLIOGRAPHY

Further information on Ch'in Chiu-shao and his work may be found in Ch'ien Pao-tsung, *Ku-suan kao-yuan* ("Origin of Ancient Chinese Mathematics") (Shanghai, 1935) pp. 7, 43–66; and *Chung-kuo shu-hsüeh-shih* ("History of Chinese Mathematics") (Peking, 1964) pp. 157–167, **206–209** ; Ch'ien Paotsung *et al., Sung Yuan shu-hsüeh-shih lun-wên-chi* ("Collected Essays on Sung and Yuan Chinese MAthematics") Peking, 1966) pp. 60–103; Hsu- Shun-fang, *Chung-suan-chia li tai-shu-hsueh yen-chiu* ("Study of Algebra by Chinese Mathematicians") (Peking, 1951) pp. 16–21; Li Yen, *Chung-kuo shu-hsüeh ta-kang* ("Outline of Chinese Mathematics") I (Shanghai, 1931) 117–136, and *Chung-kuo suan-hsueh-shih* ("History of Chinese Mathematics") (Shanghai, 1937; repr. 1955) pp. 98–99, 128–132; Li Yen and Tu Shih-jan, *Chung-kuo ku-tai shu-hsueh chien-shih* ("A Short History of Ancient Chinese Mathematics") I (Peking, 1963) II (Peking, 1964) 49–151, 210–216; Yoshio Mikami, *The Development of Mathematics in China and Japan* (Leipzig, 1913; repr. New York) pp. 63–78; Joseph Needham, *Science and Civilisation in China*, III Cambridge, 1959) 40–45, 119–122, 472, 577–578; George Sarton, *Introduction to the History of Science*, III (Baltimore, 1947) 626–627; D. E. Smith and Yoshio Mikami, *A History of Japanese Mathematics* (Chicago, 1914) pp.48–50, 63; and Alexander Wylie, *Chinese Researches* (Shanghai, 1897; repr. Peking, 1936; Taipei, 1966) pp. 175–180.

Ho Peng-yoke