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(*b*. Castelfranco, near Treviso, Italy, 11 January 1707; *d*. Treviso, 17 January 1775)

mathematics.

Riccati was the second son of the mathematician Jacopo Francesco Riccati. He received his early education at home and under the auspices of the Jesuits, whose order he entered on 20 December 1726. In 1728 he went to teach literature in the Jesuit college in Piacenza; the following year he was sent to the college in Padua, where he remained until he was transferred to Parma in 1734. At some subsequent date he went to Rome to study theology, then, in 1739, returned to Bologna, where for the next thirty years he taught mathematics in the College of San Francesco Saverio. Like his father, Riccati was also skilled in hydraulic engineering and, under government commissions, carried out flood control projects along the Reno, Po, Adige, and Brenta rivers. He was much honored for this work, which saved the Venetian and Bolognan regions from disastrous flooding, and was made one of the first members of the Società dei Quarante. When Pope Clement XIV suppressed the Society of Jesus in 1773, Riccati retired to his family home in Treviso, where he died two years later.

Riccati further followed his father's example in studying the integration of differential equations, including some derived from geometrical problems. He, too, was well informed concerning pre-Eulerian mathematical analysis, and took his topics from other eminent mathematicians. Thus, a memoir by Johann I Bernoulli led him to consider the relationship between the lengths of two curves and a treatise by Jakob Hermann prompted him to suggest some methods whereby the conic equations of Cartesian coordinates might be discussed. He was also concerned with the rectification of conic sections and studied elliptic integrals as an introduction to the theory of elliptic functions.

Riccati's principal works in mathematics and physics were published in his two-volume *Opusculorum ad res physicas et mathematicas pertinentium* (1757–1762). He here introduced the use of hyperbolic functions to obtain the roots of certain types of algebraic equations, particularly cubic equations. He discussed this method further in the three-volume *Institutiones analyticae* (1765–1767), which he wrote in collaboration with Girolamo Saladini. In the latter work Riccati for the first time used the term "trigonometric lines" to indicate circular functions. He demonstrated that just as in a circle of radius 1, the coordinates of the extremity of an arc of ϕ length may be considered to be functions of twice the area of the sector determined by the arc $x = \cos \Phi$, $y = \sin \Phi$ so in an equilateral hyperbola, the coordinates of a point may be expressed as a function of twice the area of a hyperbolic sector w. He thus was able to make use of hyperbolic functions possessing properties similar to those of circular functions, obtaining (in modern notation)

from which the relations $\cosh^2 w - \sinh^2 w = 1$; $\sinh w + \cosh w = e^w$; and $\cosh 0 = 1$; $\sinh 0 = 0$.

Riccati and Saladini (who published a commentary on the process in Italian) then went on, in the second volume, to establish the formulas for the addition and subtraction of hyperbolic functions as well as the general formulas $2 \sinh nw = (\cosh w + \sinh w)^n - (\cosh w - \sinh w)^n$; $2 \cosh nw = (\cosh w + \sinh w)^n (\cosh w - \sinh w)^n$. They were then able to calculate the derivatives of $\sinh w$ and $\cosh w$, which they deduced from the geometric properties of the hyperbola. Riccati and Saladini thus anticipated Lambert in his study of hyperbolic functions, although Lambert, who published his findings in 1770, is often cited as having been the first to mention them.

Riccati and Saladini also considered the principle of the substitution of infinitesimals in the *Institutiones analyticae*, together with the application of the series of integral calculus and the rules of integration for certain classes of circular and hyperbolic functions. Their work may thus be considered to be the first extensive treatise on integral calculus, predating that of Euler. Although both Newton and Leibniz had recognized that integration and derivation are inverse operations, they had defined the integral of a function as a second function from which the former function is derived; Riccati and Saladini, on the other hand, considered differentiation to be the division of a quantity into its elements and integration to be the addition of these elements and offered examples of direct integrations.

Riccati's geometrical work includes a study, published in 1755 as "De natura et proprietatibus quarundam curvarum quae simul cum tractrice generantur, quaeque proinde syntractoriae nominabuntur," in which he examined the location of the points that divide the tangents of a tractrix in a certain relationship. Leibniz and Huygens also studied this curve, which may be defined as the locus of points so taken that the segment of the tangent between the point of contact and the intersection with a fixed straight line will be of constant length. Thus, given a cone of revolution with vertex V and a generator g, let t be a tangent perpendicular to g; each plane π passing through t will cut the cone in a conic, Γ, of which F_1 and F_2 are the foci. Rotating the plane π plane around the tangent t produces a curve called the strophoid, which had been discovered in France, possibly by

Roberval. This curve was further studied by De Moivre, in 1715, and later by Gregorio Casali; Riccati and Saladini discussed it in the first volume of *Institutiones analyticae*.

Riccati and Saladini also considered the figure of the four-leaf rose, introduced by Guido Grandi, and further discussed the problem posed by Ibn al-Haytham in which given two points, *A* and *B*, it is required to find on a circular mirror a point *C* so located that a ray of light starting from *A* and reflected by the mirror at *C* passes through *B*. Ibn al-Haytham himself offered only a tortuous and confused solution to this problem, but in 1676 a simple geometrical solution was stated by Huygens, whose result Riccati and Saladini refined and further simplified. In the second volume of their work, they generalized a problem that was proposed to Descartes by Debeaune, then solved by Johann I Bernoulli and by L'Hospital.

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II. Secondary Literature. On Riccati and his work, see Amedeo Agostini, "Riccati," in *Enciclopedia italiana*, XXIX (1936), 241; L. Berzolari, G. Vivanti, and D. Gigli, eds., *Enciclopedia delle matematiche elementari*, I, pt. 2 (Milan, 1932), 389, 478, 491; III, pt. 2 (1950), 826; and Gino Loria, *Curve piane speciali algebriche e trascendenti* (Milan, 1930), I, 72, 231, 427 and II, 153; and *Storia delle matematiche*, 2nd ed. (Milan, 1950), 663, 681, 706, 725.

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