Richard, Jules Antoine | Encyclopedia.com

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(*b*. Blet, Cher, France, 12 August 1862; *d*. Châteauroux, Indre, France, 14 October 1956)

mathematics.

Richard taught in several provincial lycées, including those at Tours, Dijon, and Châteauroux. He defended a doctoral thesis, on the surface of Fresnel waves, at the Faculté des Sciences of Paris on 22 November 1901. Of an eminently philosophical cast of mind, Richard published a work on the philosophy of mathematics at Paris in 1903. He collaborated on several scientific journals, most notably *Enseignement mathématique* (1905–1909), in which he was able to give free reign to his critical mind.

In an article published in *Enseignement mathématique*, "Sur une manière d'exposer la géométrie projective" (1905), Richard cited Staudt, David Hilbert, and Charles Méray. He based his exposition on the theorem of homological triangles, that is, on an implicit axiomatics very close to that of Staudt.

In a philosophical and mathematical article, "Sur la nature des axiomes de la géométrie" (1908), Richard distinguished four attitudes displayed by theoreticians and submitted them successively to critical analysis: (1) Geometry is founded upon arbitrarily chosen axioms or hypotheses; there are an infinite number of equally true geometries; (2) Experience Provides the axioms; the basis of science is experimental, and its development is deductive; (3) Axioms are definitions —this third point of view is totally different from the first; (4) Axioms are neither experimental nor arbitrary; they force themselves upon us because without them experience would be impossible (this is the Kantian position). Richard found something unacceptable in each of these attitudes. He observed that the notions of the identity of two objects or of an invariable object are vague and that it is essential to make them precise; it is the role of axioms to do this. "Axioms are propositions the task of which is to make precise the notion of identity of two objects preexisting in our mind." Further on he asserted, "To explain the material universe is the goal of science."

Utilizing the group of anallagmatic spatial transformations and taking a subgroup that leaves a sphere invariant, Richard later remarks in the article that for a real sphere the subgroup is Lobachevskian, for a point sphere it is Euclidean, and for an imaginary sphere it is Riemannian. "One sees from this that, having admitted the notion of angle, one is free to choose the notion of the straight line in such a way that one or another of the three geometries is true." Hence, for Richard, difficulties persist, since "to study these groups we are obliged to assume that ordinary geometry has in fact been established." This article gave rise to several polemics, and Richard, having received a letter from Giuseppe Peano, returned to the question the following year.

In an article on mechanics, Richard took a mild swipe at Poincaré: "The consistent relativist will say not only that it is convenient to suppose that the earth revolves; he will say that it is convenient to suppose that the earth is round, that it has an invariable shape, and that it is greater than a billiard ball not contained in its interior."

“Richard’s paradox or antinomy” was first stated in 1905 in a letter to Louis Olivier, director of the *Revue générale des sciences pures et appliquées*. Richard wrote, in substance:

The Revue has pointed out certain contradictions encountered in the general theory of sets.

It is not necessary to go as far as the theory of ordinal numbers to find such contradiction. Let *E* be the set of real numbers that can be defined by a finite number of words. This set is denumerable. One can form a number not belonging to this set.

“Let *p* be the *n*th decimal of the *n*th number of the set *E*; we form a number *N* having zero for the integral part and *p* + 1 for the *n*th decimal, if *p* is not equal to either 8 nor 9, and unity in the contrary case.” This number does not belong to the set *E*. If it were the *n*th number of this set, its *n*th cipher would be the *n*th decimal numeral of this number, which it is not. I call *G* the group of letters in quotation marks [above]. The number *N* is defined by the words of the group *G*, that is to say by a finite number of words. It should therefore belong to the set *E*. That is the contradiction.

Richard then attempted to remove the contradiction by noting that *N* is not defined until after the construction of the set *E*. After having received some comments from Peano, he returned to the problem for the last time in 1907. Richard never presented his antinomy in any other form, although certain variants and simplifications falsely bearing his name are found in the literature.

BIBLIOGRAPHY

On Richard’s paradox, see “Les principes des mathématiques et le problème des ensembles,” in *Revue générale des sciences pures et appliquées*, **16** , no. 12 (30 June 1905), 541–543, which includes Richard’s letter and Oliver’s comments. The letter alone is reproduced in *Acta mathematica*, **30** (1906), 295–296. Richard returned to the question in “Sur un paradoxe de la théorie des ensembles et sur l’axiome de Zermelo,” in *Enseignement mathématique*, **9** (1907), 94–98.

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Jean Itard