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(*b*. Györ, Hungary, 22 January 1880; *d*. Budapest, Hungary, 28 February 1956)

mathematics.

Riesz's father, Ignacz, was a physician; and his younger brother Marcel was also a distinguished mathematician. He studied at the Polytechnic in Zurich and then at Budapest and Göttingen before taking his doctorate at Budapest. After further study at Paris and Göttingen and teaching school in Hungary, he was appointed to the University of Kolozsvàr in 1911. In 1920 the university was moved to Szeged, where, in collaboration with A. Haar, Riesz created the János Bolyai Mathematical Institute and its journal, *Acta scientiartun mathematicarum*. In 1946 he went to the University of Budapest, where he died ten years later after a long illness.

Riesz's output is most easily judged from the 1,600-page edition of his writings (cited in the text as Works). He concentrated on abstract and general theories connected with mathematical analysis, especially functional analysis. One of the theorems for which he is best remembered is the Riesz-Fischer theorem (1907), so called because it was discovered at the same time by Emil Fischer. Riesz formulated it as follows (Works, 378–381; cf. 389–395). Let $\{\phi_i(x)\}$ be a set of orthogonal functions over [a, b] of which each member is summable and square-summable. Associate with each ϕ_i, a real number a_i. Then is convergent if and only if there exists a function *f* such that

In this form the theorem implies that the $\{a_i\}$ are the coefficients of the expansion off *f* in terms of the (ϕ_i) and that *f* itself is square-summable. This result, the converse of Parseval's theorem, immediately attracted great interest and soon was being re-proved.

Riesz had been motivated to discover his theorem by Hilbert's work on integral equations. Under the influence of Maurice Fréchet's abstract approach to function spaces, such studies became associated with the new subject of functional analysis. Riesz made significant contributions to this field, concentrating on the space of L^p functions (functions *f* for which f_p1, part; is Lebesgue integrable). He provided much of the groundwork for Banach spaces (*Works*, esp. 441–489) and later applied functional analysis to ergodic theory.

Riesz's best-known result in functional analysis has become known as the Riesz representation theorem. He formulated it in 1909, as follows (*Works*, 400–402). Let *A* be a linear (distributive, continuous) functional, mapping real-valued continuous functions *f* over [0,1] onto the real numbers. Then *A* is bounded, and can be represented by the Stieltjes integral

where α is a function of bounded variation. The theorem was a landmark in the subject and has proved susceptible to extensive generalizations and applications.

Another implication of Hilbert's work on integral equations that Riesz studied was its close connection with infinite matrices. In *Les systèmes d'équations linéaires à une infinité d'inconnues* (1913; *Works*, 829–1016), Riesz tried not only to systematize the results then known into a general theory but also to apply them to bilinear and quadratic forms, trigonometric series, and certain kinds of differential and integral equations.

Functional analysis and its ramifications were Riesz's most consistent interests; and in 1952 he published his other book, a collaboration with his student B. Szökefnalvy-Nagy, *LeÇons d'analyse fonctionnelle*. A classic survey of the subject, it appeared in later French editions and in German and English translations.

In much of his work Riesz relied on the Lebesgue integral, and during the 1920's he reformulated the theory itself in a "constructive" manner independent of the theory of measure (*Works*, 200–214). He required only the idea of a set of measure zero and built up the integral from "simple functions" (effectively step functions) to more general kinds. He also re-proved some of the basic theorems of the Lebesgue theory.

In the topics so far discussed, Riesz was a significant contributor in fields that had already been developed. But a topic he created was subharmonic functions. A function *f* of two or more variables is subharmonic if it is bounded above in an open domain *D*; is continuous almost everywhere in *D*; and, on the boundary of any subdomain *D'* of *D*, is not greater than any function *F* that is continuous there and harmonic within. The definition is valuable for domains in which the Dirichlet problem is solvable and F is unique, for then $f \leq F$ within D and $f = F$ on its boundary. By means of a criterion for subharmonicity given by

where r is the radius and (x_0, y_0) center of a small circle within D, Riesz was able to construct a systematized theory (see esp. *Works*, 685–739) incorporating applications to the theory of functions and to potential theory.

Among Riesz's other mathematical interests, some early work dealt with projective geometry. Soon afterward he took up matters in point set topology, such as the definition of continuity and the classification of order-types. He also worked in complex variables and approximation theory.

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