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(*b*. Györ, Hungary, 16 November 1886; *d*. Lund, Sweden, 4 September 1969)

mathematics.

Marcel Riesz, the younger brother of Frigyes Riesz and the son of Ignácz Riesz, a physician, showed his talent for mathematics early by winning the Lor-ánd Eötvös competition in 1904. After studying at the University of Budapest, he worked on trigonometric series under the influence of Lip³t (Leopold) Fejér, One of Fejér's theorems states that the first order Cesáro sums of the Fourier series of a continuous function tend to *f(x)* at every point *x*. Fejér also pointed out that a trigonometric series can be (*C*, l)-summable to zero at each point $x \neq 2k\,\pi$ ($k = 0, \pm 1, \ldots$) without the coefficients being all zero. Riesz generalized Fejér's theorem, replacing (*C*, l)-summability with (C,α)-summability with any $\alpha > 0$. In his doctoral dissertation, "Über sum-mierbare trigonometrische Reihen" (1912), Riesz gave a condition implying that if a trigonometric series is (*C*, 1)-summable to zero everywhere, then all its coefficients are zero. This result was later sharpened by A. Rajchman, Antoni Zygmund, and S. Verblunsky.

Pierre Fatou's dissertation, "Séries trigonomét-riques et séries de Taylor" (1906), influenced both Riesz brothers. One of Fatou's theorems asserts that if the power series represents a holo-morphic function *f(z)* in the unit disk $|z| < 1$ and then the power series converges at each point of the circle $|z| = 1$ where *f(z)* is regular. Riesz improved the result in two directions: first, if *f(z)* is regular on a closed arc, then the power series converges uniformly on the arc; second, if then (*C, k*)-summability replaces convergence.

It was the typical means (now also called Riesz means) that made Riesz internationally known. Given a sequence $0 < \lambda_0 < \lambda_1 \ldots < \lambda_n < \ldots$, the means of type ($\lambda_n$) and order *k* of the series $\sum u_n$, are given by and they are tailormade for the Dirichlet series . Anexposition can be found in a Cambridge Tract written jointly with G. H. Hardy.

Like most young Hungarian mathematicians of the period, Riesz visited Göttingen regularly; he also spent the year 1910 to 1911 in Paris, There he received an invitation from Gustav Mittag-Leffler, whom he had met at the 1908 International Congress of Mathematicians in Rome, to deliver three lectures in Stockholm. Riesz accepted, and spent the rest of his life in Sweden. Some of the most distinguished Swedish mathematicians were his doctoral students.

After his arrival in Sweden, Riesz proved an interpolation formula from which S. N. Bernstein's inequality between a polynomial and its derivative follows. During World War 1 he wrote his only joint paper with his brother, on the boundary behavior of an analytic function. Their theorem, with its many variants and generalizations, became central in several branches of mathematics.

In the 1920's Riesz's interests, which until then had been concentrated on classical analysis, broadened. Under the influence of his brother, he turned to functional analysis. In three notes on the moment problem of Thomas Stieltjes and Hans Hamburger, he proved and applied a result on the extension of positive linear operators similar to the Hahn-Banach theorem. He also introduced a class of orthogonal polynomials to investigate the existence and the uniqueness of the solutions of the moment problem. He never published detailed proofs because others became interested in these questions, and he did not wish to compete with them.

In 1927 Riesz published his two most often quoted results: his theorem on conjugate functions and his convexity theorem. The former states that if the function *f* belongs to $L^p(-\infty, \infty)$ and $1 < p < \infty$, then the conjugate

also belongs to $L^p(-\infty, \infty)$, and the map that associates $\bar{f}$ with *f* is continuous. Conjugate functions originated in the theory of Fourier series, and Riesz's theorem solved a problem that had been open for some time. The generalization of his result to several variables by A. P. Calderón and Antoni Zygmund led to singular integrals and pseudo-differential operators, which play a fundamental role in the theory of partial differential equations.

The convexity theorem asserts that if a linear map is continuous from $L^p{}_0$ to $L^q{}_0$ and from $L^p{}_1$ to $L^q{}_1$, then it is also continuous from L^p to L^q where $p^{-1} = (1 - \theta)p_0^{-1}, + \theta p_1^{-1}$, $q^{-1} = (1 - \theta)q_0^{-1} + \theta q_1^{-1}$, $0 \leq \theta \leq 1$. The theorem has earlier results of Felix Hausdorff and William H. Young on Fourier series and integrals, and of Frigyes Riesz on orthonormal series as a consequence. A simple proof was later found by Riesz's student Olof Thorin, and the theorem is now attributed to both. It became the starting point of abstract "interpolation theorems" developed mainly by E. M. Stein, A. P. Calderon. J. L. Lions, and J. Peetre.

In 1926 Riesz obtained a professorship at the University of Lund, where he became interested in partial differential equations, mathematical physics, number theory, and (through the *Moderne algebra* of B. L. van der Waerden) abstract algebra. At the Oslo International Congress of Mathematicians in 1936 he presented four short communications: one on mixed volumes in the theory of modules, one on reciprocal modules, and two on generalizations of the Riemann-Liouville integral.

The Riemann-Liouville integral, which generalizes differentiation and integration on the real line to fractional orders, figured in the theory of typical means. Its analogue, defined by Riesz on n -dimensional Euclidean space—the Riesz potential of fractional order—has properties that, among other things, prove that the kernel of the Newtonian potential is a positive function. This observation, used in the 1935 dissertation of Riesz's student Otto Frostman led to a renewal of potential theory by M. Brelot, H. Cartan, J. Deny, G. Choquet, and others.

The Riemann-Liouville integral defined on an n -dimensional space with the Lorentz metric yields a new approach to the Cauchy problem for the wave equation and, more generally, for hyperbolic partial differential equations with variable coefficients in which Jacques Hadamard's concept of finite parts of integrals is replaced by analytic continuation. Riesz found these results between 1933 and 1936, but his monumental paper did not appear until 1949.

Riesz was a visiting professor at the University of Chicago in 1947 and 1948. After his retirement in 1952, he spent much time in the United States, mainly at the Courant Institute in New York, the University of Washington, Stanford University, the University of Maryland, and Indiana University. In 1960 illness forced him to return to Lund. Riesz was a member of the Swedish Academy of Sciences and in 1950 received an honorary doctorate from the University of Copenhagen.

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