Rudolff (or Rudolf), Christoff | Encyclopedia.com

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(*b*. Jauer, Silesia [now Jawor, Poland], end of the fifteenth century; d. Vienna, Austria, first half of the sixteenth century)

mathematics.

Virtually nothing is known about Rudolff's life.[1] It was formerly thought that he was born in 1499 and died in 1545, but these dates are not confirmed by any documentary evidence.[2] The earliest reliable information attests his presence in Vienna in 1525, the year in which he dedicated his *Coss* to the bishop of Brixen (now Bressanone, Italy). This book was the first comprehensive work in German on algebra, or *Coss*, as it was then called. Rudolff learned the subject from Grammateus, who taught at the University of Vienna from 1517 to 1521. In 1521 Grammateus went to Nuremberg and then Erfurt, not returning to Vienna until the summer semester of 1525.[3] Consequently, Rudolff must have been in Vienna before 1521. He supported himself by giving private lessons; and although lie was not affiliated with the university, he was able to use its library. Some critics accused him of stealing the examples for his *Coss* from the Vienna Library, an accusation against which he was defended by Michael Stifel in the preface to the new edition of the *Coss*. In 1526 Rudolff published an arithmetic book entitled *Künstliche Rechnung mit der Ziffer und mit den Zahlpfennigen*, and in 1530 an *Exempelbüchlin*; which was reprinted as "exempelbüchle" in later editions of the Künstliche *Rechnung*. He stated his intention to publish an improved, Latin version of his *Coss* containing new examples, but he never did so.[4]

The *Coss* is divided into two parts. In the first, Rudolff devotes twelve chapters to the topics that the reader must master before taking up the study of algebra (the solution of equations). In chapters 1–4 he presents the basic operations and the rule of three, giving examples with whole numbers and fractions, and then treats the extraction of square and cube roots. In the section "Progredieren" he states in the style of recipes the summation formulas of arithmetic and geometric series. By relating the geometric series, the "Progredieren in Proportz" (*proportio dupla, tripla*, and so forth), to the series of natural numbers, he obtains the configuration

This procedure enables him to determine an arbitrarily high member of the geometric series.

In chapters 5 and 6 Rudolff carries out the four operations and the rule of three on algebraic polynomials,[5] after first setting forth the names and symbols of the powers of the unknowns. The schema

0	1	2	3	4	5	6	7	8	9
	r	*z*	*c*	*zz*	β	*zc*	bβ	*zzz*	*cc*

shows that he considers the proper designation of a member without *x* to be $x^0 = 1$. (Grammateus still used *N* (for *numerus*) in such cases instead of 0.) Chapters 7–11 are devoted to roots, binomials, and residues. Rudolff distinguishes three types of roots: rational, irrational, and communicant. Two roots are communicant if they have a common rational factor: for example, and . If a factor is brought under the radicals, then the "denominierte" number is formed; for example, gives rise to Rudolff computes using letters instead of numbers.[6] The first part of the book concludes with a short explanation of the five types of "proportioned" numbers (*multiple, super-particular*, and so forth).

In part two of the *Coss* (which is divided into three sections) Rudolff discusses firstand second-degree equations and their variations of higher degree. He assumes the existence of only eight distinct *equationen* or "rules of the coss," not the twenty-four distinguished by earlier cossists. In his presentation of the sixth rule ($ax^2 + c = bx$) he deliberately admits only the one solution that fits the conditions of the problem under study. Later he recognized his error.[7] The second section offers rules (*cautelae*) for solving equations, and the third is a collection of problems containing over 400 examples. Some of the problems involve abstract numbers; others, taken from daily life, are presented in fantastic forms similar to those of the *Enigmata* of recreational mathematics. In some of the problems Rudolff introduces a second unknown, q (for *quantitas*). If there are more unknowns than equations, the problem is considered indeterminate. For several such problems concerned with "splitting the bill" (*Zechenaufgaben*) Rudolff supplied all the possible solutions.[8]

The *Coss* ends with three cubic problems. Rudolff does not work out their solutions because, as he stated, he wanted to stimulate further algebraic research.[9] In an "addendum" he presents still another "verbal computation" and a drawing of a cube with edge According to Stifel, with this illustration Rudolff sought to hint at the solution of the cubic equation, which was then unknown.[10]

Rudolff's other major work, *Künstliche Rechnung nut der Ziffer und mit den Zahlpfennigen*, consists of three parts: a "Grundbüchlein," in which the beginner is introduced to computing with whole numbers and fractions; a "Regelbüchlein," which treats the rule of three and the "Welsh practice"; and an "Exempelbüchlein," some 300 problems that vividly evoke the commerce and manufacturing of the period. Rudolff also includes several *Enigmata*, termed "amusing calculations." Examples of this type are the rule of Ta-Yen, hound and hare, or the "horse sale," in which the price of thirty-two horseshoe nails is expressed in a geometric sequence. As in the *Coss*, the two number sequences are related to each other, with the first sequence again beginning with 0.[11]

The *Exempelbüchlin* that Rudolff published in 1530 contains 293 problems, as well as tables of measurements for many regions, a list of symbols used in gauging, and numerous hints for solving problems. Decimal fractions appear in the computation of compound interest.[12]

Rudolff's importance in the history of mathematics lies in his having written the first comprehensive book on algebra in German. In this work he went far beyond his teacher Grammateus, especially concerning calculation with rational and irrational polynomials. Rudolff was aware of the double root of the equation $ax^2b = cx$ and gave all the solutions to indeterminate first-degree equations. His writings are remarkable both for the occasional appearance of decimal fractions and for improvements in symbolism. Adding a diagonal stroke to the points used by earlier cossists, Rudolff introduced the signs √, for the second, third, and fourth roots.[13] His work also gives a hint of the beginnings of exponential arithmetic and the fundamental idea of logarithms—that is, setting x^0 equal to 1. His methodical hints on using the *Coss* are worth noting as well. In brief, Rudolff's role in the development of mathematical studies in Germany was analogous to that of Fibonacci in Italy.[14]

Rudolff learned arithmetic from early printed books on the subject, some of which he cited.[15] It is obvious that he thoroughly studied Johannes Widman's arithmetic book (1489).[16] He also obtained information from other writings on algebra. He mentioned earlier books in which the solutions to problems are introduced by the words "ponatur una res,"[17] and in which the second power of the unknown is designated by the word "substantia" instead of "zensus." These remarks indicate Robert of Chester's translation of the *Algebra* of al-Khwārizmī, which exists in a fourteenthcentury manuscript.[18] In particular, Rudolff used an algebraic treatise included in a volume compiled at Vienna by Johann Vögelin.[19] Further, he was acquainted, directly or indirectly, with the Regensburg algebra (1461), from which he took a problem involving computation of compound interest; this problem, the solution to which was was earlier used by Widman in his arithmetic book of 1489.[20]

Rudolff's *Künstliche Rechnung* was widely read and was reprinted as late as 1588, at Augsburg. The importance of the *Coss* was recognized by Gemma Frisius and Stifel, but it soon went out of print. In 1553 Stifel brought out a new edition of the *Coss* containing supplementary material.

NOTES

1. J. E. Scheibel, who, like Rudolff, came from Silesia, sought information about his compatriot. See *Einleitung zur mathematischen Bücherkenntnis*, **I** (Breslau, 1769–1775), 313. As late as 1850 nothing was known in Vienna concerning the circumstances of Rudolff's life. See *Zeitschrift für Slawistik*, **1** (1956), 132.

2. R. Wolf, *Geschichte der Astronomie* (Munich, 1877), 340.

3. On Grammateus see W. Kaunzner, "über die Algebra bei Heinrich Schreyber," in *Verhandlungen des Historischen Vereins für Oberpfalz und Regensburg*, **110** (1970), 227–239; and *Neue deutsche Biographic*, VI (1964), 738–739.

4. See *Coss*, A_{11}r and *Künstliche Rechnung*, fol. S_4v.

5. The division by a polynomial was only an attempt at this operation; see *Coss*, fol. E_1r.

6. See *Coss*, fols. G_{11}r, P_7v, P_8r.

7. On the ambiguous wording of the sixth rule see *Künstliche Rechnung*, fol. S_4v f.

8. See *Coss*, R_{VI}r f.

9. The third of these problems—cited by Cantor (*Vorlesungen über Geschichte der Mathematik*, II, pt. 2, 426) and Smith (*History of Mathematics*, II, 458)—and the suggested method for solving it both derive from Stifel.

10. The drawing is in P. Treutlein, p. 89.

11. See *Künstliche Rechnung*, fol. s_2v.

12. The decimal fractions are separated by a stroke; for example: 393|75. See *Exempelbüchlin*, fol. x_1v.

13. See H. E. Wappler, “Zur Geschichte der deutschen Mathematik im 15. Jahrhundert,” p. 13, n. 1.

14. On this point see M. Terquem, “Christophe Rudolff,” p. 326.

15. See *Coss*, C_vr.

16. The problem of “splitting the bill” (*Zechenaufgabe*) that Rudolff found in “an arithmetic book”—see *Coss*, R_{VI}v—appeared earlier in the work by Widman. A comparison between Rudolff’s problems and Widman’s is given by Treutlein, pp. 120 f.

17. The solutions to problems in both the Regensburg algebra of 1461 and the Latin algebra of Codex Dresdensis C 80 begin with words such as “Pono quod lucrum sit una res” or “Pono quod A 1 r‘em’ habeat.” See, respectively, M. Curtze, “Ein Beitrag zur Geschichte der Algebra in Deutschland im fünfzehnten Jahrhundert,” pp. 58 ff.; and Wappler, *op. cit*., p. 19.

18. Vienna, Cod. Vind. 4770; see L. C. Karpinski, *Robert of Chester’s Latin Translation of the Algebra of al-Khowarizmi* (Ann Arbor, Mich., 1915), repr. as pt. I of Karpinski and J. G. Winter, *Contributions to the History of Science* (Ann Arbor, 1930).

19. This volume is Cod. Vind. 5277. See Wappler, *op. cit*., p. 3, n. 2. Cod. Lat. Mon. 19691 is a copy of Cod. Vind. 5277. The marginal notes in the Vienna MS were incorporated into the text of the Munich MS. They both contain the 24 old rules of the cossists, as did Widman’s Latin algebra. Widman’s work had many problems in common with Cod. Lat. Mon. 26639. The relationships among all these texts have not yet determined; on this question see W. Kaunzner, *über Christoff Rudolff and seine Coss*, p. 2 and n. 28.

20. Problem 16 of the “Fünfften regl” in Rudolff’s Coss (fol. X_{VI}r) appears in the following works: Cod. Dresd. C 80, fol. 356v (see Wappler, *op. cit*., p. 21); the Regensburg algebra in Cod. Lat. Mon. 14908, fol. 149v (see Curtze, *op. cit*., p. 61); and Widman’s arithmetic of 1489, 127v.

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II. Secondary Literature. See *Allgemeine deutsche Biographie*, XXIX, 571–572; M. Cantor, *Vorlesungen über Geschichte der Mathematik*, II, 2nd ed. (Leipzig, 1913), 397–399, 425–429; M. Curtze, “Ein Beitrag zur Geschichte der Algebra in Deutschland im fünfzehnten Jahrhundert,” in *Abhandlungen zur Geschichte der Mathematik*, **7** (1895), 31–74; A. Drechsler, *Scholien zu Christoph Rudolphs Coss* (Dresden, 1851); C. J. Gerhardt, *Geschichte der Mathematik in Deutschland* (Munich, 1877), 54–59; W. Kaunzner, *über Christoff Rudolff und seine Coss*, no. 67 in Veröffentlichungen des Forschungsinstituts des Deutschen Museums für die Geschichte der Naturwissenschaften und der Technik, ser. A (Munich, 1970); and *Deutsche Mathematiker des 15. and 16. Jahrhunders und ihre Symbolik*, no. 90 in the same series (Munich, 1971); C. F. Müller, “Henricus Grammateus und *sein Algorisnms de integris*,” in *Programm Gymnasium Zwickau* (1896); D. E. Smith, *Rara arithmetica* (Boston–London, 1908), 151 ff.; and *History of Mathematics*, 2 vols. (New York, 1923–1925), **I** , 328 f., and II, 721; M. Terquem, “Christophe Rudolf,” in *Annali di scienze matematiche e fisiche*, **8** (1857), 325–338; P. Treutlein, “Die deutsche Coss,” in *Abhandlungen zur Geschichte der Mathematik*, **2** (1879), 15 ff., 44 ff.; F. Unger, *Die Methodik der praktischen Arithmetik* (Leipzig, 1888), 238; and H. E. Wappler, “Zur Geschichte der deutschen Mathematik im 15. Jahrhundert,” in *Programm Gymnasium Zwickau* (1887); and “Zur Geschichte der deutschen Algebra,” in *Abhandlungen zur Geschichte der Mathematik*, **9** (1899), 537–554.

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