Saccheri, (Giovanni) Girolamo | Encyclopedia.com

(*b*. San Remo, Italy, 5 September 1667; *d*. Milan, Italy, 25 October 1733)

mathematics.

Saccheri is sometimes confused with his Dominican namesake (1821–1894), a librarian at the Bibliotheca Casanatense of Rome. In 1685 Saccheri entered the Jesuit novitiate in Genoa and after two years taught at the Jesuit college in that city until 1690. Sent to Milan, he studied philosophy and theology at the Jesuit College of the Brera, and in March 1694 he was ordained a priest at Como. In the same year he was sent to teach philosophy first at Turin and, in 1697, at the Jesuit College of Pavia. In 1699 he began teaching philosophy at the university, where until his death he occupied the chair of mathematics.

One of Saccheri's teachers at the Brera was Tommaso Ceva, best known as a poet but also well versed in mathematics and mechanics. Through him Saccheri met his brother Giovanni, a mathematician living at the Gonzaga court in Mantua. This Ceva is known for his theorem in the geometry of triangles (1678). Under Ceva's influence Saccheri published his first book. *Quaesita geometrica* (1693), in which he solved a number of problems in elementary and coordinate geometry. Ceva sent this book to Vincenzo Viviani. one of the last surviving pupils of Galileo, who in 1692 (*Acta eruditorum*, 274–275) had challenged the learned world with the problem in analysis known as the window of Viviani. Although it had been solved by Leibniz and others, Viviani published his own solution and sent it to Saccheri in exchange for the *Quaesita*. Two letters from Saccheri to Viviani (1694) are preserved, one containing Saccheri's own solution (without proof).

While in Turin, Saccheri wrote *Logica demonstrativa* (1697), important because it treats questions relating to the compatibility of definitions. During his years at Pavia he wrote the *Neo-statica* (1708), inspired by and partly a polemic against T. Ceva's *De natura gravium* (Milan, 1669). This book seems of little importance now, being well within the bounds of Peripatetic statics. *Euclides ab omni naevo vindicatus* (1733), also written at Pavia, contains the classic text that made Saccheri a precursor of the discoverers of non-Euclidean geometry.

Saccheri's two most important books, the *Logica* and the *Euclides*, were virtually forgotten until they were rescued from oblivion—the *Euclides* by E. Beltrami in 1889 and the *Logica* by G. Vailati in 1903. They show that Euclid's fifth postulate (equivalent to the parallel axiom) intrigued Saccheri throughout his life. In the *Logica* it led him to investigate the nature of definitions and in the *Euclides* to an attempt to apply his logic to prove the correctness of the fifth postulate. Although the fallacy in this attempt is now apparent, much of Saccheri's logical and mathematical reasoning has become part of mathematical logic and non-Euclidean geometry.

The *Logica demonstrativa* is divided into four parts corresponding to Aristotle's *Analytica priora*. *Analytica posteriora, Topica*, and De *sophisticis Elenchis*. It is an attempt, probably the first in print, to explain the principles of logic *more geometrico*. Stress is placed on the distinction between *definitiones quid nominis* (nominal definitions), which simply define a concept, and *definitiones quid res* (real definitions), which are nominal definitions to which a postulate of existence is attached. But when we are concerned with existence is attached, the question arises whether one part of the definition is compatible with another part. This may be the case in what Saccheri called complex definities. In these discussions he was deeply influenced by Euclid's *Elements*, notably by the definition of parallelism of two lines. He warned against the definition, given by G. A. Borelli (*Euclides resitutus* [Pisa, 1658]), of parallels as equidistant straight lines. Thus Saccheri was one of the first to draw explicit attention to the question of consistency and compatibility of axioms.

To test whether a valid proposition is included in a definition, Saccheri proposed reasoning seemingly analogous to the classical *reductio ad absurdum*. using for his example *Elements* IX, 12; if 1, a_1, a_2,. . . a_x form a geometric progression and a_n has a prime factor *p*, then a_1 also contains this factor. There was a difference in Saccheri's proposal, however: his demonstration resulted from the fact that, reasoning from the negation, we obtain exactly the proposition to be proved, so that this proposition (an example of his reasoning is seen below). As Vailati observed, Saccheri's reasoning had much in common with that of Leibniz (see L. Couturat, *Opuscules et fragments inédits de Liebniz* [Paris, 1903b]); but whereas Leibniz's primary inspiration came from algebra and the calculus, Saccheri's came from geometry.

In the *Euclides* Saccheri applied his logical principle to three "blemishes" in the *Elements*. By far of *reductio ad absurdum* to Euclid's parallel axom. He took as true Euclid's first twenty-six propositions and then assumed that the fifth postulate was false. Among the consequences of this hypothesis he sought a proposition to test the postulate itself. He found it in what is now called the quadrilateral

of Saccheri, an isosceles birectangular quadrilateral consisting of side *AB* and two sides of equal length. *AD* and *BC* at right angles to *AB*. Then without the fifth postulate it cannot be proved that the angles at *C* and *D* are right. One can prove that they are equal, since if a line *MP* is drawn through the midpoint *M* of *AB* perpendicular to *AB*, it intersects *DC* at its midpoint *P*. Thus there are three possibilities, giving rise to three hypotheses:

1. that of the right angle: $\llcorner C = \llcorner D = 1$ right angle:

2. that of the obtuse angle: $\llcorner C = \llcorner D > 1$ right angle:

3. that of the acute angle: $\llcorner C = \llcorner D < 1$ right angle.

Saccheri proceeded to prove that when each of these hypotheses is true in only one case, it is true in only case, it is true in every other case. Thus in the first case the sum of the angles of a triangle is equal to, in the second it is greater than, and in the third case it is less than, two right angles.

For the proofs Saccheri needed the axiom of Archimedes and the principle of continuity. Then came the crucial point: he proved that for both the hypothesis of the right angle and that for both the hypothesis of the right angle and that of the obtuse angle the fifth postulate holds. But the fifth postulate implies the hypothesis of the right angle; hence the hypothesis of the obtuse angle is false. (This argument is not now cogent because in the case of the obtuse angle the existence of the finite length of lines is accepted). He could not dispose of the hypothesis of the acute angle in this way, but he was able to show that it leads to the existence of asymptotic straight lines, which, he concluded, was repugnant to the nature of the straight line. Saccheri thus thought that he had established the truth of the hypothesis of the right angle and, hence. of the fifth postulate anf of Euclidean geometry as a whole.

Several other theorems resulted from Saccheri's three hypotheses, some of which are now established as part of non-Euclidean geometry. The three types of quadrangles had already been studied by al-Khayyāmī and Nasīr-al-Dī al-Tūsī; the latter was cited by John Wallis (1693) in a book known to Saccheri.

Saccheri's *Euclides*, although it had little direct influence on the subsequent discovery of non-Euclidean geometry, was not so forgotten as is sometimes believed. (See Segre, below)

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