Shanks, William | Encyclopedia.com

(*b*. Corsenside, Northumberland, England, 25 January 1812; *d*, Houghton-le-Spring, Durham, England, 1882)

mathematics.

Shanks's contributions to mathematics lie entirely in the field of computation, in which he was influenced by William Rutherford of Edinburgh. From 1847 his life was spent in Houghton-le-Spring, a small town in the coal-mining area of County Durham. There he kept a boarding school, and carried out his laborious and generally reliable calculations, most of which concerned the constant π, the ratio of the circumference of a circle to its diameter.

Modern methods for the calculation of π rely mainly on the formula

discovered independently by James Gregory (1670) and Leibniz (1673). With $x = 1$ it yield

but the series converges too slowly to be of use, and more rapid processes may be obtained by using the Gregory series in formulas derived from the addition theorem

arc tan x + arc tan y = arc tan $\{(x+y)/(1-xy)\}$.

By repeated application of this theorem, John Machin (1706) found the convenient formula

and calculated π to 100 decimal places.

This and similar formulas encouraged more extended calculations; here it is enough to note that in 1853 Rutherford gave 440 decimal places; and in the same year Shanks, in conjunction with Rutherford, gave 530 places, which proved to be his most accurate value. Also in 1853 Shanks gave 607 places, and the value to 500 places was independently checked. Some errors were corrected in 1873; and by that year Shanks, using Machin's formula, carried his calculations to 707 decimal places. There the matter rested for a considerable period. Subsidiary calculations provided the natural logarithms of 2, 3, 5, 10 to 137 decimal places and the values of 2^n, with $n = 12m + 1$ for $m = 1, 2, ..., 60$. Shanks also computed the value of e and of Euler's constant gama; to a great many decimal places, and prepared a table of the prime numbers less than 60,000.

In 1944 D. F. Ferguson of the Royal Naval College, Dartmouth, attracted by the formula

proceeded to calculate π and compare his value with that given by Shanks. At the 528th decimal place there was a disagreement that was not reduced when Ferguson rechecked his own work, which he eventually carried to 710 decimal places. This discrepancy was communicated to R. C. Archibald, editor of *Mathematical Tables and Aids to Computation*, who suggested to J. W. Wrench, Jr., and L. B. Smith that they might recalculate π by Machin's formula; their value, to 808 decimal places confirmed Ferguson's result and his identification of two terms omitted by Shanks, which had caused the latter's errors. Modern computing machinery has carried the calculation of π to great lengths: in 1949 the first such determination, by ENIAC, went to 2,000 decimal places; by 1960 at least 100,000 places were known.

BIBLIOGRAPHY

I. Original Works. Shank's book was *Contributions to Mathematics, Comprising Chiefly the Rectification of the Circle...* (London—Cambridge—Durham, 1853). His papers are listed in Royal Society *Catalogue of Scientific Papers*, V 672; VIII, 941; and XI, 401; and include nine memoirs published in *Proceedings of the Royal Society*, **6–22** (1854–1874).

II. Secondary Literature. Poggendorff mentions Shanks briefly, in III, 1241; but his reference in IV 1390, to an obituary by J. C. Hoffmann in *Zeitschrift für mathematischen und naturwissnschaftlichen Unterricht***26** (1895), is misleading; this item merely reproduces Shanks's 1873 figures, with a little comment. Local sources could supply only the information that Shanks kept a school. A. Fletcher, J.C.P. Miller, and L. Rosenhead, *An Index of Mathematical Tables* 2nd ed. (London, 1962), gives full bibliographical details and critical notes. A concise history of the evaluation of π is given by E. W. Hobson, *Squaring the Circle* (London, 1913; repr. 1953).

Ferguson's two notes on his evaluation of π are in *Mathematical Gazette*, no. 289 (May 1946), 89–90; and no. 298 (Feb. 1948), 37. A note by R. C. Archibald, J. W. Wrench, Jr., L. B. Smith, and D. F. Ferguson, in *Mathematical Tables and Other Aids to Computation*, **2** (Apr. 1947), gives agreed figures (as in Ferguson's second note) and adds some details.

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