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(*b*. Beawar, Rajasthan. India, 24 November 1879; *d*. Wellington. New Zealand, 31 January 1934)

mathematics.

Sommerville, the son of Rev. James Sommerville of Jodhpur, India, was educated in Scotland, first at the Perth Academy, then at the University of Scotland. First at the perth Academy, then at the University of St. Andrews, where he was awarded Ramsay and Bruce .scholarships and in the mathematics department of which he served as lecturer from 1902 to 1914. During that time he met. and in 1912 married. Louisa Agnes Beveridge. Originally of Belfast, Ireland. From 1915 on Sommerville was professor of pure and applied mathematics at Victoria University College. Wellington. New Zealand. He was active in the Edinburgh mathematical Society, to Whose presidency he was elected in 1911, He helped to found the Royal nomieal Society of New Zealand and became first executive secretary. Somrnerville pi over the mathastronomical Society of New Zealend and became its first executive secretary. Sommerville presided over the mathematics section at the Adelaide section at the Adelaide meeting (1924) of the Australasian Associat the Advancement of Science. In 1928 the In Institute (Royal Society) of New Zealand awarded him its Hector Medal.

Although primarily a mathematician, Sommerville was interested in other sciences, particularly astronomy, anatomy, and chemistry. Crystallography held special appeal for him. and crystal forms doubtless motivated his investigation of repetitive space-filling geometric patterns. Also, his abstract conceptions called for the construction of clarifying models, which revealed an artistic skill that was even more evident in his many watercolors of New Zealand scenes.

Sommerville contributed to mathematics both as a teacher and as an original researcher. His biographer, H. W. Turnbull. who considered him(in 1935) Scotland’s leading geometer of the twentieth century, stated that his pedagogic style was schoolarly. unobtrusive, and much appreciated at St Andrews, One of his most distinguished pupils, A. C. Aitken revealed that when the New Zealand University of Otago was without a mathematics professor. Sommerville willingly provided a sort of “correspondence course” in higher mathematics Further evidence of his teaching ability is reflected in his four textbooks, which are models of deep,lucid exposition. Among them are *The ElementsNon-Euclidean Geometry* and *An Introduction to the Geometry of n Dimensions*, books whose indicate his two major research specialties whose contents develop geometric concept that Sommerville himself created. In addition to his texts, his *Bibliography of Non-Euclidean Geometry* is also a bibliography of *n*-dimensional geomtry.

Sommerville wrote over thirty original papers, almost all on geometric topics. Notable exce were his 1928 “Analysis of Preferential Voting” (geometrized, however, in his 1928 “Certain Hyperspaial Partitionings Connected With Preferential Voting”) and two 1906 papers that gave pure mathematical treatment to statistical questions arising from notions in Karl Pearson’s biometric research.

In his texts Sommerville explained how non-Euclidean geometries arose from the use of alternatives to Euclid’s parallel postulate. Thus, in the Lobachevskian or hyperbolic geometry, it is assumed that there exist two parallels to a given line through an outside point. In Riemannian or elliptic geometry, the assumption of no parallels is made. By suitable interpretation Klein. Cayley, and then Sommerville showed that Euclidean and non-Euclidean geometries can all be considered as sub-geometries of projective geometry. For Klein any geometry was the study of invariants under a particular transformation group. From his point of view, projective geometry is the invariant theory associated with (he group of linear fractional transformations. Those special plane projective transformations leaving invariant a specified conic section, Cayley’s “absolute,” constitute a subgroup of the plane projective group; and the corresponding geometry is hyperbolic, elliptic, or Euclidean according to whether the conic is real (an ellipse, for example), imaginary, or degenerate. This conception makes it possible in all three geometries to express distance and angle measure in terms of a cross ratio, the fundamental invariant under projective transformation.

Even in two of his earliest investigations, namely. “Networks of the Plane in Absolute Geometry” (1905) and “Semi-Regular Networks of the Plane in Absolute Geometry” (1906), Sommerville used the Cayley-Klein notion of non-Euclidean geometries, in particular the projective measurement of lengths and angles. These two papers Indicated a trend that he was to follow in much of his research, namely, the study of tesselations of Euclidean and non-Euclidean spaces, a theme sug-bested by the repetitive designs on wallpaper or textiles and by the arrangement of atoms in crystals. Sommerville showed that whereas there are only three regular tesselations in the Euclidean plane (its covering by congruent equilateral triangles, squares, or regular hexagons), there are five mosaics of congruent regular polygons of the same .kind in the elliptic plane, and an infinite number of the h patterns in the hyperbolic plane. In all cases the variety is greater if "semi-regular" networks of regular polygons of different kinds are permitted, moreover. as Sommervilie pointed out, still further fvariations are attainable because the regular patterns are topologically equivalent, if not aesthetically so, to nonregular designs. In several papers and in his text on *n*-dimensional geometry, he generalized his earlier results and methods to include honeycombs of polyhedrons in three-dimensional spaces and "honeycombs" of polyiopes in spaces (Euclidean and non-Euclidean) of 4, 5, ·, *n* dimensions.

Many of Sommerville' s geometric concepts have algebraic counterparts in the theory of groups Thus, since his repetitive patterns can be considered as the result of moving a single basic design to different positions, it is possible to asssociate with each tesselation or honeycomb one or more "crystallographic groups,",each a set of motions that displace a fundamental region so that it will cover an entire plane, space, or hyperspace. Thus, if a square (with sides horizontal and vertical) is the fundamental region in a Euclidean plane, one can cover that plane with duplications of the square by two basic motions or their inverses, namely translation of the square one side-length to the right, and a similar translation upward. Those two motions are said to "generate" a crystallographic group corresponding to the network of squares. For that same network a different crystallographic group is generated by three basic motions— the two reflections of the square in its vertical sides, and the translation of the square one side-length upward.

There are also associations with group theory in Sormmerville's "On Certain Projective Configurations in Space of *n* Dimensions and a Related Problem in Arrangements"(1906), in which he showed interrelationships between certain finite groups and the finite projective geometries of Veblen and Bussey. Such groups also played a role in his "On the Relation Between the Rotation-Groups of the Regular Polytopes and Permutation Groups" (1933).

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