Study, Eduard | Encyclopedia.com

(b. Coburg, Germany, 23 March 1862; *d*. Bonn, Germany, 6 January 1930)

mathematics

Study, the son of a Gymnasium teacher, studied mathematics and science, beginning in 1880, at the universities of Jena, Strasbourg, Leipzig, and Munich. One of his favorite subjects was biology, and even late in life he investigated entomological questions and assembled an imposing butterfly collection. He received the doctorate from the University of Munich in 1884 and the following year became a *Privatdozent* in mathematics at Leipzig, where he was influenced chiefly by Paul Gordan, an expert in invariant theory.

In 1888 Study left this post to take a similar one at Marburg. From July 1893 to May 1894 he lectured in the United States, mainly at the Johns Hopkins University. He was appointed extraordinary professor at Göttingen in 1894 and full professor at Greifswald in 1897. In 1904 he succeeded Lipschitz at Bonn, where he remained unitil his retirement in 1927; he died of cancer three years later.

Study was largely self-taught in mathematics, and his writings reflect a highly individual way of thinking. He worked in many areas of geometry but did not accept the geometric axiomatics that Pasch and Hilbert were then developing. (On this point see Study's remarks in his more philosophical writings [1, 2].) Study mastered Grassmann's *Ausdehnungslehre*, Lie's theory of continuous groups, and the calculus of invariant theory; he was highly skilled at employing related algebraic techniques in the solution of geometric questions.

It was then usual for geometers to state their findings with little concern for exactitude in individual aspects of problems, and many theorems were labeled simply "in general," without any indication of the scope of their validity. Questions concerning real numbers, for example, were not carefully distinguished from those concerning complex numbers. Many of Study's papers were addressed to drawing such distinctions. His objections, buttressed by counterexamples, to Schubert's principle of the conservation of number were particularly well known, and the principle was eventually firmly established with suitable restrictions on its range of applicability [3].

In his own work Study demonstrated what he considered to be a thorough treatment of a problem. Moreover, a number of the problems he chose to discuss–for example, Appolonius' tangent problem [4] and Lie's straight-line–sphere transformation [5]–had long been thought resolved. Study was the first to show how the totality of the conic sections of the plane–that is, the conic sections considered as unions of elements–can be mapped into a point set M_5 of P_{27}[6].

With Corrado Segre, Study was one of the leading pioneers in the geometry of complex numbers. He systematically constructed the analytic geometry of the complexly extended Euclidean spaces R_2 and R_3; and, with Fubini, he was the first to introduce metrics for these spaces [7]. His contributions to complex differential geometry include the first systematic studies of isotropic curves and the introduction of isotropic parameters [8].

Adept in the methods of invariant theory–which are almost completely forgotten today–Study, employing the identities of the theory, sought to demonstrate that geometric theorems are independent of coordinates. This undertaking was not a simple one, but he achieved a number of successes. In a long work [9] he derived the formulas of spherical trigonometry from a new point of view, and in the process created many links between trigonometry and other branches of mathematics. He wrote other works on invariant theory, but they provoked little response even at the time of their publication [10].

Study was the first to investigate systematically all algebras possessing up to four generators over *R* and *C* [11], including W. R. Hamilton's quaternions, which interested him chiefly because of their applications to geometry and Lie groups. In his long work *Geometrie der Dynamen* [12] Study made a particularly thorough examination of Euclidean kinematics and the related subject of the mechanics of rigid bodies. Unfortunately, because of its awkward style and surfeit of new concepts, this work has never found the public it merits.

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