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(*b* Geneva, Switzerland, 29 September 1803; *d*. Paris, France, 18 December 1855)

mathematics, physics.

Sturm's family, originally from Strasbourg, had lived in Geneva since the middle of the eighteenth century. He was the elder son of Jean-Henri Sturm, a teacher of arithmetic, and Jeanne-Louise-Henriette Gremay. Sturm at first studied classics, a field in which he displayed considerable ability. For example, at age sixteen he improvised Greek and Latin verses without the aid of a dictionary. In order to perfect his German, he attended the Lutheran church to hear sermons given in that language. In 1819, the year of his father's death, Sturm abandoned his literary studies and devoted himself to mathematics. At the Geneva Academy he attended the mathematics lectures of Simon L'Huillier and the physics lectures of Marc-Auguste Pictet and Pierre Prevost. L'Huillier, who in 1821 was preparing to retire, soon discovered Sturm's abilities; he encouraged Sturm, offered him advice, and lent him books. His influence, however, was less decisive than that of his successor, Jean-Jacques Schaub[1]. Sturm also attended a course in mathematics given by Baron Jean-Frédéric-Théodore Maurice and one in astronomy taught by Alfréde Gautier. Among Sturm's fellow students were Auguste de La Rive, Jean-Baptiste Dumas, and Daniel Colladon, his best friend[2].

Having completed his studies at the Academy, Sturm moved early in May 1823 to the château of Coppet, about fifteen kilometers from Geneva, as tutor to the youngest son of Mme de Staël[3]. About ten people lived at the château, including Duke Victor de Broglie, his wife, the former Albertine de Staël, and their three children. Sturm's duties as tutor left him sufficient free time to write his first articles on geometry, which were published immediately in *Annales de mathématiques pures et appliquées*, edited by J. D. Gergonne. Toward the end of the year, he accompanied the duke's family to Paris for a stay of approximately six months. Through de Broglie's assistance he was able to enter the capital's scientific circles.

During this period Sturm wrote to Colladon: "As for M. Arago, I have two or three times been among the group of scientists he invites been among the group of scientists he invites to his house every Thursday, and there I have seen the leading scientists, MM Laplace, Poisson, Fourier, Gay-Lussac, Ampère, etc. Mr de Humboldt, to whom I was recommended by Mr de Broglie, has shown an interest in me; it is he who brought me to this group. I often attend the meetings of the Institut that take place every Monday."[4]

In May 1824 Sturm returned to Coppet with the de Broglie family, but toward the end of that year he gave up teaching in order to devote himself to scientific research. With Colladon he undertook a study of the compression of liquids, which had just been set by the Paris Academy as the subject of the grand prize in mathematics and physics for the following year. They decided to measure the speed of sound in water–Lake Geneva was nearby–and then to seek the coefficient of compressibility of water, introduce this coefficient into Poisson's formula for the speed of sound, and compare their results with those predicted by the formula. The project did not, however, yield the desired results. In addition, Colladon seriously injured his hand during the tests.

On 20 December 1825 Sturm and Colladon left for Paris with the intention of attending physics courses and of finding the inSturments needed for the experiments that would enable them to complete their memoir. Arago often invited them to his house, and for a time Sturm gave mathematics lessons to his eldest son. In addition, Ampère offered them the use of his physics laboratory.

At the Sorbonne and at the Collège de France, Sturm and Colladon attended the lectures of Ampère and Gay-Lussac in physics and of Cauchy and Lacroix in mathematics. They also were present during the tests on steam engines that Arago and Dulong conducted near the Paris observatory. In addition they visited Fourier, who at this time was engaged in research on heat. Fourier asked Colladon to measure the thermal conductivity of various substances and, recognizing Sturm's inclination and talent for theoretical work, suggested that the latter make a thorough study of a certain procedure in analysis, later called harmonic analysis, that Fourier believed would be of great use in theoretical physics.

Sturm and Colladon finished their paper on the compression of liquids and submitted it to the Academy, which eventually decided that none of the memoirs it had received merited the prize and that the same subject would be set for the 1827 award. Meanwhile, Sturm and Colladon had been appointed assistants to Ampère, who suggested that they collaborate on a major treatise on experimental and theoretical physics (the project was never undertaken). In November 1826 Colladon returned to Geneva and measured the speed of sound in water between Thonon and Rolle, situated on opposite banks of Lake Geneva. He obtained a value of 1,435 meters per second. The agreement was good with the theoretical speed determined by Poisson's formula, which gave 1,437.8 meters per second. Upon his return to Paris, he and Sturm completed the new version of their memoir. This time it won the grand prize of 3,000 francs, a sum that enabled them to pay the costs of their experiments and to prolong their stay in Paris.

Henceforth their scientific careers diverged. Even in his physical research, however, Sturm continued to obtain interesting results in geometry, notably on the theory of caustic curves of reflection, the poles and polars of conic sections, Desagues's theorem, and involutions.

In 1829, through Ampère's influence, Sturm was appointed chief editor for mathematics of the *Bulletin des sciences et de l'industrie*. On 13 May of that year he presented to the Academy "Mémoire sur la résolution des équations numériques," containing the famous theorem that perhaps did more to assure his reputation than the rest of his writings together. The founder of the *Bulletin*, André Étienne, Baron d'Audebard de Férussac, invited his principal collaborators to assemble at his Paris residence once a week; and it is possible that Sturm met Niels Abel and Évariste Galois there, as well as Cournot, Coriolis, Duhamel, Hachette, and Lacroix.[5]

Sturm and Colladon wished to obtain posts in the state school system; but even though they had the backing of several influential members of the Academy, they were unsuccessful because they were foreigners and Protestants. The revolution of July 1830 proved beneficial of their cause: Arago was able to have Sturm named professor of *mathématiques spéciales* at the Collège Rollin and Colladon, professor of mechanics at the École Centrale des Arts et Manufactures. (Colladon returned to Geneva in 1839.) It is interesting that the minister of public education after the revolution was Duke Victor de Broglie.[6]

Sturm became increasingly interested in the theory of differential equations; and in September 1833, six months after he had acquired French citizenship, he read a memoir on this subject before the Academy. About this time the Geneva Academy considered offering him a post, and in October 1833 he received official notification through La Rive. But Sturm declined it, for his decision to remain in France was irrevocable. He also rejected an offer from the University of Ghent.

Upon the death of Ampère, a seat in the Académie des Sciences became vacant. On 28 November 1836 Sturm was nominated to it by Lacroix; the other candidates were Liouville, Duhamel, Lamé, and Jean-Louis Boucharlat. At the following meeting, it was announced that Liouville and Duhamel had withdrawn their names, considering it right that the seat go to Sturm; he was elected by forty-six of the fifty-two votes cast.

Sturm's career now progressed rapidly; in 1838 he was named *répétiteur* of analysis in Liouville's course at the École Polytechnique, where he became professor of analysis and mechanics in 1840. Also that year he assumed the chair of mechanics formerly held by Poisson at the Faculty of Sciences. In 1837 Sturm became *chevalier* of the Legion of Honor, and in 1840 he won the Copley Medal of the Royal Society and was elected a member of that body. He was already a member of the Berlin Academy (1835) and the Academy of St. Petersburg (1836).

Sturm was obliged to spend much time preparing his courses on differential and integral calculus and on rational mechanics. An excellent lecturer, he was admired for both his personal qualities and his knowledge. Sturm dedicated his remaining time to research. From analysis he turned to optics, particularly to vision, and to mechanics, in which, independently and by a new method, he derived one of Duhamel's theorems on the variation in *vis viva* resulting from a sudden change in the links of a moving system.

Around 1851 Sturm's deteriorating health obliged him to arrange for a substitute at the Sorbonne and at the École Polytechnique. He became obese, had a nervous breakdown, and no longer derived pleasure from intellectual work. His doctors ordered him to walk a great deal and to move to the country. Two years later Sturm resumed some of his teaching duties, but the illness returned–probably with other complications, the nature of which is not known–and it slowly took his life.

On 20 December 1855 a crowd of scientists, friends, and students accompanied Sturm's body to the cemetery of Montparnasse. Moving speeches were given by a Protestant minister and by Liouville, who called Sturm "a second Ampère: candid like him, indifferent to wealth and to the vanities of the world."[7]

Sturm's moral qualities, his innate sense of duty and of honor, and his devotion to the ideals of friendship brought him the esteem and affection of all who knew him. His life, like his writings, was a model of clarity and rigor. Favorable circumstances smoothed the way and permitted him to display his genius; but his long friendship with Colladon and the patronage of such highly placed persons as de Broglie, Arago, and Ampère are also inseparable from his career and should be taken into account in explaining hius success.

In the rest of the article we shall not consider Sturm's earliest works nor, in particular, his many articles on plane geometry-in each of which he made a valuable, original contribution. The essential features of his work in this area were incorporated in later works on geometry, often without mention of their origin. We shall, instead, examine rather closely three other important aspects of his work.

Sturm's Theorem . Although the problem of finding the number of real roots of the equation $f(x) = 0$ had already been encountered by Descartes and by Rolle, it was not investigated systematically until the mid-eighteenth century. Gua de Malves made the first significant attempts in 1741, and in 1767 Lagrange approached the problem by forming the transform with the squares of the differences of the zeroes of the polynomial. Later, Fourier considered the sequence formed by the first member of the equation and its successive derivatives. Poisson suggested the problem to Cauchy, who in 1813 sent three notes on the subject to the Academy and in 1815 discussed it at length in his "Mémoire sur la détermination du nombre de racines réelles dans les équations algébriques."[8] By successive eliminations, Cauchy established a system of rational functions of the coefficients of the given polynomial; and from the sign of these functions he deduced the number of zeroes. His was the first complete solution, but the calculations involved are so long and laborious that it was never adopted.

Sturm used Fourier's method, as well as some unpublished results that Fourier had communicated to him. (Sturm credited Fourier for these in the article published in *Bulletin des sciences et de l'industrie*.) But instead of working with the successive derivatives, he was able to develop his method by using only the first derivative. The essential part of the argument is as follows:[9]

Let $V = 0$ be an equation of arbitrary degree with distinct roots, and let V_1 be the derivative of V. One proceeds as in finding the greatest common divisor of V and V_1, the sole difference being that it is necessary to change the signs of all the remainders when they are used as divisors. Let $Q_1 . . ., Q_{r-1}$ be the quotients and $V_2, . . . , v_{r-1}$the remainders, V_r being a constant. One therefore has

$V = V_1Q_1 - V_2$

$V = V_2Q_2 - V_3$

. . . . . .

$V_{r-2} = V_{r-1}Q_{r-1} - V_r$

The statement of the theorem then reads:

Let us substitute two arbitrary numbers *a* and *b* positive or negative, for *x* in the sequence of functions $V, V_1, V_2, . . . , V_{r-1}, V_r$. If *a* smaller than *b*, the number of the variations in the sequence of the signs of these functions for $x = b$ will, at most, be equal to the number of the variations in the sequence of the signs of these same functions for $x = a$. And if it is less, the difference will be equal to the number of real roots of the equation $V = 0$ between *a* and *b*.

"Variation" in this statement means "change of sign." The demonstration, which includes an examination of two cases, a scholium, and two corrolaries, requires several pages. Sturm's discovery elicited great excitement, and he became famous as the mathematician who had filled a lacuna in algebra. It was not long, however, before voices were raised in support of Cauchy–that, for example, of Olry Terquem, editor of the *Nouvelles annales de mathématiques*, who accorded priority to Cauchy while recognizing that Sturm had found a simpler method. Cauchy himself later asserted his priority. As for Sturm, he was satisfied to speak of the "theorem of which I have the honor to bear the name." Charles Hermite made the following assessment: "Sturm's theorem had the good fortune of immediately becoming classic and of finding a place in teaching that it will hold forever. His demonstration, which utilizes only the most elementary considerations, is a rare example of simplicity and elegance."[10]

Cauchy subsequently found a way to determine the number of imaginary roots of an equation; but here, too, Sturm arrived at the same results by a shorter and more elementary method. The proof of this "Cauchy theorem" was published in *Journal de mathématiques pures et appliquées* for 1836 in an article signed by Sturm and Liouville.

The functions $V, V_1, . . . , V_r$ are called Sturm functions. J. J. Sylvester discussed them in two articles and expressed them by means of the roots of the given equation.[11]

Differential Equations and Infinitesimal Geometry . On 28 September 1833 Sturm presented a memoir on second-order differential equations to the Académie des Sciences, but it was not published until three years later, in *Journal de mathématiques*. In this work Sturm studied equations of the form.

Where *L, M*, and *N* are given functions of *x*, and *V* is the unknown function. The integration is, in general, impossible. Sturm's insight was to determine the properties of *V* without assigning it in advance to any class. Although used today, this method of proceeding was not at all common at that time. Sturm started by writing the given equation as

where *K* and *G* are new functions of *x* that can be determined subsequently. This type of differential equation is encountered in several problems of mathematical physics.

Liouville maintained a special interest in this area of Sturm's research, to which he himself made several important additions in two notes to the Academy in 1835 and 1836. Further, in his *Journal* he published a work written with Sturm on the expansion of functions in series; their paper begins with the differential equation

Maxime Bôcher, professor at Harvard University, gave a series of lectures at the Sorbonne in the winter of 1913–1914 on the use of Sturm's methods in the theory of differential equations.

In infinitesimal geometry, Sturm examined the problem of finding the surface of revolution that is at the same time a minimal surface. Delaunay had demonstrated that it is generated by the rotation of the curve described by the focus of an ellipse or of a hyperbola that rolls without sliding on a straight line. His method consisted in imposing on the differential equation of minimal surface the condition that it be a surface of revolution. Sturm handled the problem in another way. He began with an arbitrary surface of revolution; calculated its volume; and sought to determine, with the aid of the calculus of variations, in which cases this volume could become minimum. He thus arrived at the differential equation of the meridian and showed that it is indeed that of the curve described by the focus of a conic section. Furthermore, he demonstrated that in the case of the parabola, the meridian is a catenary curve. He then generalized the question and determined the curve that must be rolled on a straight line in order for a certain point of the plane of this curve to describe another curve the differential equation of which is known. Sturm's solution appeared in Liouville's *Journal* of 1841.

Experimental and Mathematical Physics. Sturm and Colladon's prizewinning "Mémoire sur la compression des liquides et la vitesse du son dans l'eau" consists of three parts. The first contains a description of the apparatus used to measure the compression of liquids, an account of the experiments concerning the compressibility of glass, and the tables of the results for mercury, pure water and water saturated with air, alcohol, sulfuric ether, ethyl chloride, acetic ester, nitrous ester, sulfuric acid, nitric acid, acetic acid, essence of turpentine, carbon disulfide, water partially saturated with ammonia gas, and seawater. The second part records the experiments to measure the heat emitted by liquids following the application of strong and sudden pressures, as well as tests made to determine the influence of mechanical compression on the electrical conductivity of several highly conductive liquids. The third part gives the details of Colladon's experiments on the propagation of sound in water and compares the values obtained experimentally with those resulting from the insertion of the measurements of compressibility in Poisson's formula.

Sturm also published many articles on mechanics and analytical mechanics. Three of the most important deal, respectively, with a theorem of Sadi Carnot's on the loss of *vis viva* in a system of which certain parts are inelastic and undergo sudden changes in velocity; with the motion, studied by Poinsot, of a solid about a fixed point; and with a way of shortening the calculations of W. R. Hamilton and Jacobi for integrating the equations of motion. Further, Sturm's *Cours de mécanique*, like his *Cours d'analyse*, was used by many students and remained a classic for half a century.

In addition to the memoir of 1838 on optics, Sturm earlier wrote many articles and notes on caustics and caustic surfaces. His studies on vision culminated in a long work that displayed a profound knowledge of physiology.

Fourier's influence on Sturm is reflected in a memoir of 1836 on a class of partial differential equations. In it Sturm considers the distribution of heat in a bar, either straight or curved, that is composed of a homogeneous or nonhomogeneous substance, and is of constant or variable thickness but of small dimensions. Under these conditions it may be assumed that all the points of a plane section perpendicular to the axis of the bar are at the same temperature at the same instant. In this work, one of his longest and most important, Sturm exhibits such a richness of ideas and skill in handling mathematics as an inSturment for solving a problem in theoretical physics that he may unhesitatingly be placed on the same level as his teacher Fourier.

Sturm, who was so adept at combining mathematics with physics in his work, appears today, by virtue of his modes of thinking, as a very modern scientist. Since 1900 there has been growing interest in his mathematical work, especially in the United States. His contribution to physics, on the other hand, has not yet received the examination it merits. There is still no thorough, full-scale study of his life and work based on the unpublished documents.

NOTES

1. Jean-Jacques Schaub (1773–1825) left MSS on the theory of numerical approximations and on the elementary concepts of the calculus of quaternions. His greatest importance for the history of mathematics is that he was the teacher and patron of Sturm, whose family found itself in financial difficulties after the death of the father.

2. Daniel Colladon (1802–1893), who studied law before turning to physics, played an important role in Sturm's life. A skillful experimenter and brilliant inventor, he conceived the idea of illuminated fountains, which were immediate successes in Paris, London, and Chicago. His research on the action of compressed air led him to construct drilling machines for boring tunnels, and he participated in the cutting of the Mont Cenis and St. Gotthard tunnels. He also was an expert in the building of gasworks.

3. See Countess Jean de Pange, *Le dernier amour de Madame de Staël* (Geneva, 1944), The author, who died at Paris in 1972, at the aga of eighty-four, was the sister of Louis de Broglie.

4. This six-page letter of 26 Apr. 1824 is reproduced in D. Colladon, *Souvenirs et mémoires* (Geneva, 1893). The original is at the Bibliothèque Publique et Universitaire. Geneva. MS 3255, fols. 219–22.

5. See R. Taton, "Les mathématiques dans le *Bulletin de Férussac,"* in *Archives internationales d'histoire des sciences*, **1** (1947), 100–125.

6. There is a passage concerning Sturm in *Souvenirs du duc de Broglie* (Paris, 1886), II, 454. published by his son, C. J. V. A. Albert de Broglie.

7. The complete text of Liouville's speech is in E. Prouhet. "Notice sur la vie et les travaux de Ch. Sturm." On the same day Colladon, who hurriedly left Geneva to attend his friend's funeral, sent Auguste de La Rive a long letter containing much information on Sturm; this unpublished letter is MS fr. 3748, fols. 206–207, at the Bibliothèque Publique et Universitaire, Geneva.

8.*Journal de l'École polytechnique*, **10** (1815), 457–548; see also *Oeuvres de Cauchy*, 2nd ser., **I** , 170–257; **II** , 187–193; **XV** , 11–16.

9. The statement of the theorem and the notation we follow is Mayer and Charles Choquet, *Traité élémentaire d'algèbre* (Paris, 1832), Sturm had given them permission to publish the results of his research.

10. A full study, already outdated, of Sturm's theorem is in Charles de Comberousse, *Cours de Comberousse*, *Cours de mathématiques*, 2nd ed., IV (Paris, 1890). pt. 2, 442–460.

11. "Memoir on Rational Derivation From Equations of Coexistence. That Is to Say, a New and Extended Theory of Elimination," in *Philosophical Magazine*, **15** (July–Dec. 1839), 428–435; and "On a Theory of the Conjugate Relations of Two Rational Integral Functions, Comprising an Application to the Theory of Sturm's Functions, and That of the Greatest Algebraic Common Measure," in *Abstracts of Papers Communicated to the Royal Society of London*, **6** (1850–1854), 324–327.

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I. Original Works. Sturm's books, both published posthumously, are *Cours d'analyse de l'École polytechnique*, 2 vols. (Paris, 1857–1859), prepared by E. Prouhet, 8th and subsequent eds. prepared by A. de Saint Germain–the 14th ed. appeared in 1909–translated into German by Theodor Fischer as *Lehrbuch der Analysis* (Berlin, 1897–1898); and *Cours de mécanique de l'École polytechnique*, 2 vols. (Paris, 1861), prepared by E. Prouhet, 5th ed. rev. and annotated by A. de Saint-Gerrmain (Paris, 1905).

Sturm's articles, notes, memoirs, and reports are listed below according to the journal in which they appeared.

In *Annales de mathématiques pures et appliquées*, edited by J. D. Gergonne: "Extension du problème des courbes de poursuite," **13** (1822–1923), 289–303. In *Mémoires présentés par divers savants á l'Académie royale de France*: "Mémoire sur la compression des liquides," 2nd ser., **5** (1838), 267–347, written with D. Colladon, who republished it thirty-two years after Sturm's death with his own paper of 1841, "Sur la transmission du son dans l'eau," as *Mémoire sur la compression des liquides et la vitesse du son dans l'eau* (Geneva, 1887); and "Mémoire sur la résolution des équations numériques," **6** (1835), 271–318, the complete text of the work containing the statement and demonstration of Sturm's theorem.

In Nouvelles annales de mathématiques or Journal des candidats aux écoles polytechnique et normale, edited by Orly Terquem and Camille Christophe Gerono: "Sur le mouvement d'un corps solide autour d'un point fixe," **10** (1851), 419–432.

The Bibliothéque Publique et Universitaire, Geneva, has nine original letters (plus one copy) sent by Sturm to Colladon, La Rive, and other Genevans. Colladon's correspondence contains sixteen letters directly concerning Sturm; among recipients are J. Liouville, Baron J.-F.-T. Maurice, Louis-Albert Necker, and Sturm's sister. All these documents are unpublished, except for two letters from Sturm to Colladon.

II. Secondary Literature. The first work on Sturm, appearing a year after his death, was E. Prouhet, "Notice sur la vie et les travaux de Ch. Sturm," in *Bulletin de bibliographie, d'histoire et de biographie mathématiques*, **2** (May–June 1856), 72–89; repr. in *Cours d'analyse*, 5th ed. (1877), I, xv–xxix. This article leaves much to be desired: the biographical data are incomplete and the analysis of Sturm's work is superficial; and although the list of writings is complete, it contains many errors. A fuller source is the autobiography of Daniel Colladon, *Souvenirs et mémoires* (Geneva, 1893), which contains long passages on Sturm's life and on their joint work, as well as the complete text of two long letters from Sturm (Coppet, 1823; Paris, 1824).

See also M. B. Porter, "On the Roots of Functions Connected by a Linerar Recurrent Relation of the Second Order," in *Annals of Mathematics*, 2nd ser., **3** (1901), 55 –70, in which the author discusses Sturm's first memoir on second-order homogeneous differential equations (which appeared in *Journal de mathématiques pures et appliquées*, **1** [1836], 106–186); J. E. Wright, "Note on the Practical Application of Sturm's Theorem," in *Bulletin of the American Mathematical Society*, **12** (1906), 246–347; and F. H. Safford, "Sturm's Method of Integrating ," *ibid.*, **17** (1910–1911), 9–15. With respect to the last article, it may be noted that one of the simplest methods for obtaining the addition theorem for the elliptic integrals of the first type is based on a procedure that appears in Sturm's *Cours d'analyse*, 5th ed., **11** (1877), 340–343.

Maxime Bôcher, "The Published and Unpublished Work of Charles Sturm on Algebraic and Differential Equations," in *Bulletin of the American Mathematical Society*, **18** (1911–1912), 1–18, is the best study on this subject. See also Bôcher's Charles Sturm et les mathématiques modernes," in *Revue du mois*, **17** (Jan.–June 1914), 88–104; and *Leçons sur les méthodes de Sturm dans la théorie des équations différentielles linéaires et leurs développements modernes*, Gaston Julia, ed. (Paris, 1917). Gaspare Mignosi, "Theorema di Sturm e sue estensioni," in *Rendiconti del Circulo matematico di Palermo*, **49** (1925), 1–164, is the most complete study of Sturm's theorem from both the theoretical and the historical points of view. It includes a long historical and critical introduction on works concerning the theorem and a chronological list of 65 notes and memoirs (pp. 152–158).

Gino Loria, "Charles Sturm et son oeuvre mathématique ," in *Enseignement mathématique*, **37** (1938), 249–274, with portrait, is very good and, despite its title, also deals with Sturm's work on mechanics, optics, and the theory of vision. Loria's chronological list of Sturm's works is partly based on that of Prouhet; although superior, it still contains several errors. Giorgio Vivanti, "Sur quelques théorémes géométriques de Charles Sturm," *ibid*., 275–291, was inspired by Sturm's article on regular polygons in *Annales mathématiques*, **15** (1825), 250–256. The first of the three theorems treated was developed by L'Huillier.

See also Henri Fehr, "Charles Sturm 1803–1855," in *Pionniers suisses de la science* (Zurich, 1939), 210–211, with portrait; and Pierre Speziali, *Charles-François Sturm (1803–1855). Documents inédits*, Conférences du Palais de la Découverte, ser. D, no. 96 (Paris, 1964). The latter is fully documented, especially with regard to Sturm's biography; it includes a reproduction of a profile of Sturm at age nineteen, based on a pencil drawing by Colladon. The portrait in the articles by Loria and Fehr was based on this drawing. There are no other likenesses of Sturm.

One may also consult the chapter on the Sturm-Liouville theory of differential equations in Garrett Birkhoff, *A Source Book in Classical Analysis* (Cambridge, Mass., 1973), 258–281.

Pierre Speziali