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(*b*. Nordhausen im Harz, Germany, 18 June 1913; *d*. Dnieper region, U.S.S.R., September 1943 [?])

mathematics.

Oswald Teichmüller was the only child of Julius Adolf Paul Teichmüller, an independent weaver by trade, and his wife, Gertrud Dinse. He grew up in the provincial Harz region around St. Andreasberg and Nordhausen. In the spring of 1931 he enrolled to study mathematics and physics at Gottingen University. Only a few months later, he joined the Nazi Party and the SA (Storm Troopers). Although he was a brilliant student of mathematics, he supported the expulsion of most of Göttingen’s mathematicians by the Nazi regime in 1933.

After Helmut Hasse’s call to a vacant chair at Gottingen in the early summer of 1934, Teichmiiller engaged in algebraic investigations (nos. 2, 3, 4, and 11 of his collected works) while also preparing a doctoral dissertation on spectral theory in quaternionic Hilbert space (no. 1), finished in 1935. After a short period of postdoctoral work at Göttingen, during which E. Ullrich and R. Nevanlinna introduced him to function theory (nos. 8 and 9), he transferred to the University of Berlin in April 1937, where a group of Nazi mathematicians had gathered around Ludwig Bieberbach and the journal *Deutsche Mathematik*. Teichmüller qualified as university lecturer in March 1938 with a good, though not spectacular, thesis on function theory (no. 13). One of the technical devices used there, quasi-conformal mappings, provided a clue to his main contribution to the theory of Riemann surfaces, the program of which he sketched in 1938 and 1939 while continuing work at Berlin, supported by a modest fellowship (no. 20).

Teichmüller was drafted into the army in the early summer of 1939, just before World War II, but continued his research, first as a soldier in Norway (no. 24), then in Berlin from 1941 to early 1943, working on decoding for the army high command (nos. 29 and 32). In early 1943, however, after the first successes of the Soviet army against the Germans, Teichmüller was sent to the eastern front. He disappeared in September 1943 at the Dnieper and very likely died in the same month, sharing the fate of a great majority of young men in his unit.

Teichmüller’s early algebraic investigations dealt with the valuation theory of fields and the structure of algebras. In valuation theory he introduced multiplicative systems of representatives of the residue field of valuation rings (no. 2), which, in a joint effort with E. Witt, led to a characterization of the structure of the whole field in terms of the residue field (no. 11). In the theory of algebras he started to generalize Emmy Noether’s concept of crossed products from fields to certain kind of algebras (*Normalringe*, no. 3), gaining new insights, for example, into the structure of p-algebras (algebra of rank p^n over a field of characteristic p; no. 4). Although from 1937 on, his main interests shifted to function theory, Teichmüller did not give up algebra, in a paper published in 1940, he explored further steps toward a Galois theory of algebras, resulting in the introduction of a group that was later recognized as a third Galois cohomology group (no. 22).

After his *Habilitation*, Teichmüller turned energetically to questions in the variation of conformal structures on surfaces, raised earlier by G. F. B. Riemann, H. Poincaré C. F. Klein, and R. Fricke. His most important innovation was the introduction of quasi-conformal mappings to this field, using ideas first developed by H. Grötzsch and L. Ahlfors in different contexts. That is, considering marked surfaces S of type (g, n), for example (that is, S orientable, closed of genus g with n distinct distinguished points, each S endowed with a homotopy class of sufficiently regular maps ø: $S_o \rightarrow S$, where S_o, is fixed of the same type), he concentrated on sufficiently regular homeomorphisms ø such that for z varying in S_o the dilatation dil ø(z) (ratio of maximal and minimal diameters of the image of a circle in the tangent plane T_zS_o with respect to conformal metrics on S_o and S) is bounded. Moreover, he analyzed the close relationship between such quasi-conformal ø and reciprocal Beltrami differentials q on is the local parameter, H is the complex-valued function on S_0) as invariants of the conformal metrics pulled back by ø.

Teichmüller’s main conjecture (I) may be stated as follows: In any homotopy class there is exactly one extremal quasi-conformal mapping $ø_o$—that is, a mapping with dilatation bounded from above by inf sup dil (z). That means variation of conformal structure can be realized uniquely by extremal quasi-conformal mappings (no. 20, secs. 46, 52, 122).

Teichmüller established a connection between extremal quasi-conformal mappings and regular quadratic differentials on S_o using a class of related reciprocal Beltrami differentials. That led him to another conjecture (II) proclaiming the existence of a bicontinuous bijective correspondence Φ between a space T_1, of real parts of certain reciprocal Beltrami differentials and $M_{g,n}$

the moduli space of all conformal structures considered. (T_1 consists of all expressions cRe{"/"}, where" is a regular quadratic differential on S_o and $0 < c < 1$.) In fact, he proved existence and injectivity of Φ (theorem A; no. 20, secs. 132–140).

Teichmüller attacked surjectivity along different lines. In his 1939 paper (no. 20) he analyzed infin-itesimal deformations of conformal structures on *S* heuristically, looking upon them as forming the tangent spaces of $M_{g.n.}$ After the introduction of an appropriate norm, $M_{g.n}$ was endowed with a Finsler space structure. On that basis he speculated about a possible path to a kind of continuity proof of surjectivity in another central conjecture (III): After appropriate change of norms, T_1 coincides with $T_S M_{g.n}$ and the exponential map of the Finsler metric coincides with Φ (20, secs. 115–123).

Because his heuristic arguments met with severe criticism. Teichmüller next showed existence of extremal quasi-conformal mappings in the special case of certain simply connected plane regions (pentagons; no. 24). Back in Berlin and working under slightly better conditons, he then gave an existence proof (theorem B) for surface of type (g, 0) by a clasical continuity argument from uniformizations and Finsler metrics (no. 29). But theorem B was also intended as a first step toward a deeper investigation of moduli spaces. In one of his last papers, Teichmüller sketched an idea of how to endow the moduli space $M_{g.o}$ with an analytic structure and how to construct an analytic structure and how to construct an analytic fiber space of Riemann surfaces parametrized by the points of $M_{g.o}$ (no. 32).

Owing to his being sent to the front and his early death, Teichmüller could not work out most of his ideas. They became seminal, however, for later work.

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II. Secondary Literature. Reference to some of Teichmüller's algebraic work is made in Saunders MacLane, "Topology and Logic as a Source of Algebra," in *Bulletin of the American Mathematical Society*, **82** (1976), 1–40. On his function theoretical contributions, see L. Bers, "Quasiconformal Mappings with Applications to Differential Equations, Function Theory and Topology," in *Bulletin of the American Mathematical Society*, **83** (1977), 1083–1100. For the broader context, see L. Furtmüller and M. Pini, "Mathematicians Under Hitler," in *Leo Baeck Yearbook*, XVIII (London, Jerusalem, and New York, 1973), 129–182; H. Mehrtens, "Ludwig Bieberbach and 'Deutsche Mathematik,'" in E. Phillips, ed., *History of Mathematics* (1987); and Norbert Schappacher, "Das Mathematische Institut der Universität Göttingen 1929–1950," in Heinrich Becker, Hans-Joachim Dahms, and Cornelia Wegeler, eds., *Die Universität Göttingen in Nazionalsozialismus* (Munich, 1987), 345–373. See also William Abikoff, "Oswald Teichmüller," in *Mathematical Intelligencer*, **8** no. 3 (1986), 8–16, 33.

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