Turán, Paul | Encyclopedia.com

(*b*. Budapest, Hungary, 28 August 1910; *d*. Budapest, 26 September 1976)

mathematics.

Turán was the eldest son of Aranha Beck and Béla Turán. He had two brothers and a sister, none of whom survived World War II. While in high school he showed considerable mathematical ability. Turán received his teaching diploma in 1933 and his Ph.D. (under Lipöt Féjer) at Pázmány Páter University, Budapest, in 1935. Because of the semi-fascist conditions in Hungary, Turán, who was Jewish, could not obtain a post even as high school teacher, and had to support himself by private tutoring. In 1938, when he was an internationally known mathematician, he finally became a teacher in the Budapest rabbinical high school.

After thirty-two months in a Nazi labor camp in Hungary in the years 1941–1944, Turán was liberated. He became a *Privatdozent* at the University of Budapest. In 1947 he went to Denmark for about six months and then spent six months at the Institute for Advanced Study at Princeton (during this period he completed two papers on polynomials and number theory). In 1948 he was elected corresponding member of the Hungarian Academy of Sciences, and became a full member in 1953. In 1948 and 1952 he received the Kossuth Prize, the highest scientific award in Hungary at that time. He became a full professor at the University of Budapest in 1949 and was the head of the department of algebra and number theory at the university and the head of the department of the theory of functions at the Mathematical Institute of the Hungarian Academy of Sciences.

Turán’s first major result, produced when he was twenty-four, was his simple proof of the Hardy-Ramanujan result that the number of prime factors of almost all integers is (1 + 0(l))*loglog n*. Further developments led to the Turán-Kubilius inequality, one of the starting points of probabilistic number theory.

By 1938 Turán had developed the basic ideas of his most important work, the power-sum method, on which he published some fifty papers, both alone and with collaborators (Stanislav Knapowski, Vera T. Sós [his wife], János Ptntz. Gabor Halász, and Istvàn Danes, among others). Turán worked on the power-sum method until his death (his last paper was a survey of the application of the method in explicit formulas for prime numbers). The method has its most significant applications in analytic number theory, but it also led to many important applications in the theory of differential equations, complex function theory, numerical algebra (approximative solution of algebraic equations), and theory of trigonometric series. Tur´n devoted three books (with the same title but different—increasingly rich—contents) to this subject. The last and most comprehensive one, *On a New Method in Analysis and Its Applications*, was published in 1984.

The essence of the method is to show that the power sum of n arbitrary complex numbers $z_1, \ldots, z_n$—that is, the sum —cannot be small for all *v* (compared with the maximal or minimal term, say). In fact, to cite just one (perhaps the most important) result of this theory, choosing *v* suitably from any interval of length *n*, we have

In order to understand this estimate, it should be noted that g(m + 1)= g(m + 2) = . . . g(m + n - 1) = 0 is certainly possible—for instance, the z_j’s are *n*th roots of unity. Similar results can be proved if we consider generalized power sums of the type , supposing very general conditions on the coefficients b_j.

To give an idea of the connection between the theory and its applications, it should be noted that these oscillatory results concerning power sums of complex numbers lead directly to oscillatory results on the solutions of some differential equations; but through a rather sophisticated technique (developed by Turán and Knapowski), via the connection of zeros of Riemann’s zeta function and primes, it is also possible to detect irregularities in the distribution of primes using these results. The variety of known applications is so rich that it is difficult to mention any area of classical analysis where the method would have no possible applications.

In 1952 Turán wrote a book in Hungarian and German on the power-sum method. A Chinese translation with some new results of this work appeared in 1954. An English version. *On a New Method of Analysis and Application*, completed by Halász and Pintz, appeared in 1984.

Besides his power-sum method, Turán did work in comparative prime number theory, analytic and quasi-analytic functions, differential equations, and other areas of analysis. In comparative prime number theory inequalities about the distribution of primes in different arithmetic progressions are studied. The subject goes back to Pafhuty Chebyshev and Edmund Landau, but Tur´n and Knapowski (his student and collaborator, who died young) developed it into a systematic theory.

An elementary power-sum problem posed by Turán in 1938 is the following:

Let $|z_1|= 1$, $|z_i| \leq 1$ $(1 < i < n)$ be n complex numbers.

Let where max $|s_\lambda| = f(z_1, \ldots z_n)$.

Turán conjectured first of all that $f(n) > c$ for all n. This was proved by F. V, Atkinson in 1960.

Turán further conjectured that This problem is still open.

Extremal graph theory was begun by Turán while he was in a labor camp. He wrote the first paper on this subject, and several more followed. Finally he gave birth to statistical group theory, in which he wrote seven fairly substantial papers with the author of this essay.

Paul ErdÖs