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(*b*. Philadelphia, Pennsylvania, 21 October 1882; *d*. Austin, Texas, 9 January 1973)

mathematics.

Harry Schultz Vandiver was the son of John Lyon and Ida Everett Vandiver. At an early age he developed an antagonism to public education that was to last the rest of his life. He dropped out of Central High School and went to work for twelve years as a customshouse broker for his father's firm. He was very much interested in the theory of numbers, and at the age of eighteen began his publishing career in the problem section of the *American Mathematical Monthly*. From 1904 to 1905 he attended some graduate courses in mathematics and made extensive use of the library at the University of Pennsylvania. In 1904 he collaborated with George David Birkhoff on a first-rate paper on the prime factors of $a^n - b^n$ that appeared in the *Annals of Mathematics*.

Vandiver continued to publish three or four papers a year until the end of World War I, during which he served in the U.S. Naval Reserve as a yeoman first class. Then, at the urging of Birkhoff, he turned professional by accepting an instructorship in mathematics at Cornell University in 1919, a position he held until 1924. Vandiver spent his summers at the University of Chicago, working with L. E. Dickson on the latter's monumental *History of the Theory of Numbers* (3 vols., Washington, D.C., 1919–1923) and on the *Report of the Committee on Algebraic Numbers of the National Research Council* (2 vols., Washington, D.C., 1923–1928).

In 1924 Vandiver accepted an associate professorship at the University of Texas at Austin, where he remained until his retirement in 1966. He also continually accepted visiting professorships and lectureships. He was in Princeton in 1934 when he was elected a member of the National Academy of Sciences. More than half of the one hundred papers he wrote thereafter appeared in the *Proceedings* of the academy.

In the summer of 1952 Vandiver visited the National Bureau of Standards Institute at Los Angeles to see what the high-speed computer SWAC could do with Fermat's last theorem. The problem is to show that the equation $x^p + y^p + z^p = 0$ is not solvable in nonzero integers x, y, z, p when p is a prime greater than 2. Vandiver and others had determined various criteria that p has to meet in order for the above equation to hold. He and his students over the years had shown $p > 600$. When the criteria were presented to the SWAC, it eliminated every $p < 2000$ in a couple of hours. (Later the limit of p was set at $p < 4002$ and more recently at $p < 125000$.)

In 1923 Vandiver married Maude Folmsbee. They had one son, Frank Vandiver, who became president of Texas A&M. In accordance with his father's views on public education, Frank was privately tutored for his secondary school and undergraduate college education.

Vandiver was awarded the F. N. Cole Prize in Number Theory by the American Mathematical Society in 1931, and in 1946 the University of Pennsylvania bestowed on him the honorary degree of doctor of science. He was a Guggenheim fellow in the academic year 1927–1928, and his work until 1961 was supported by the National Science Foundation (grants in 1955 and 1957–1961) and the American Philosophical Society (grants in 1934 and 1939).

The Vandivers never owned a home in Austin. For many years they occupied a permanent suite at the Alamo Hotel, where they had a large collection of classical recordings. When he was not doing research or refereeing or reviewing the work of others, Vandiver would relax by listening to Mozart or Beethoven or, on occasion, by attending a campus baseball game. At the age of eighty-four Vandiver gave up research and went into a rest home.

Vandiver's bibliography extends from 1900 to 1963 and contains 173 titles, of which 50 are directly concerned with Fermat's last theorem; the others are mainly on properties of Bernoulli numbers and on cyclotomy and commutative algebra. There are also half a dozen expository papers on Fermat's last theorem and one entitled "On the Desirability of Publishing Classified Bibliographies of the Mathematics Literature" (1960), no doubt inspired by his early work with Dickson's *History*. He also prepared a twenty-two-page bibliography of articles on Bernoulli and Euler numbers for the years 1869–1940, which he never published.

The Fermat problem of showing the nonexistence of integers x, y, z, none zero, such that $x^p + y^p = z^p$ holds for p a prime greater than 2, has long been separated into two cases. Case I is subject to the condition that p fails to divide xyz and is much simpler than case II, in which p may divide xyz. In case I we have Weiferich's criterion that 2^{p-1} is divisible by p^2. This condition is met by $p = 1093$ and $p = 3511$, but for no other prime less than six billion. However, these two primes fail to satisfy the Mirimanoff criterion that p^2 divides 3^{p-1} 1, so that case I is proved for $p < 6$ 10^9. Many times Vandiver expressed the conviction that case I would be disposed of in the near future.

His approach to case II was along the lines of the early German algebraic number theorist E. E. Rummer, who founded the arithmetic of numbers of the form $a_0+a_1w+a_2w^2+...+a_{p-1}w^{p-1}$,

where $w^p = 1$, $w \neq 1$, and the a's are ordinary integers.

The Ferraat equation can be written

$x^p=z^p-y^p=(z-y)(z-wy)\ ...\ (z-w_{p-1}y).$

Vandiver sought conditions on p alone to make this product a perfect of p. The criteria he derived had to do with the divisibility by p of the Bernoulli numbers B_2, B_4, B_6, . . ., B_{p-3} and the class number h of the cyclotomic field generated by w. Then there is the problem of combining the criteria to form a condition that should be impossible. Throughout this process one has to try to discover properties of the integers x, y, z, p, which one hopes do not exist after all. This sort of situation was commonplace in the scientific life of H. S. Vandiver.

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D. H. Lehmer