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(*b*. Fontenay-le-Comte, Poitou [now Vendèe], France, 1540; *d*. Paris, France, on February 1603), *mathematics*.

Viète's father, Étienne, was an attorney in Fontenay and notary at Le Busseau. His mother was Marguerite Dupont, daughter of Françoise Brison and thus a first cousin of Barnabé Brisson. Viète was married twice: to Barbe Cothereau and, after her death, to Juliette Leclerc. After an education in Fontenay, Viète entered the University of Poitiers to study law. He received a bachelor's degree in law in 1560 but four years later abandoned the profession to enter the service of Antoinette d'Aubeterre, mother of Catherine of Parthenay, supervising the latter's education and remaining her loyal friend and adviser throughout his life. After Antoinette d'Aubeterre was widowed in 1566, Viète followed her to La Rochelle. From 1570 to 1573 he was at Paris, and on 24 October of that year Charles IX appointed him counselor to the *parlement* of Brittany at Rennes. He remained at Rennes for six years, and on 25 March 1580 he became *maître de requêtes* at Paris (an office attached to the *parlement*) and royal privy counselor. From the end of 1584 until April 1589 Viète was banished from the royal court by political enemies and spent some time at Beauvoir-sur-Mer. He was recalled to court by Henry III when the latter was obliged to leave Paris and to move the government to Tours, where Viète became counselor of the *parlement*. During the war against Spain, Viète served Henry IV by decoding intercepted letters written in cipher. A letter from the liaison officer Juan de Moreo to Philip II of Spain, dated 28 October 1589, fell into Henry's hands. The message, in a new cipher that Philip had given Moreo when he departed for France, consisted of the usual alphabet with homophonous substitutions, plus a code list of 413 terms represented by groups of two or three letters or of two numbers. either underlined or dotted. A line above a two-digit group indicated that it could be ignored. It was not until 15 March 1590 that Viète was able to send Henry the completed solution, although he had previously submitted parts of it. He returned to Paris in 1594 and to Fontenay in 1597. He was in Paris in 1599 but was dismissed by Henry IV on 14 December 1602.

Viète had only two periods of leisure (1564-1568 and 1584-1589). His first scientific works were his lectures to Catherine of Parthenay, only one of which has survived in a French translation: *Principes de cosmographie, tirés d'un manuscrit de Viette, et traduits en françois* (Paris, 1637). This tract, containing essays on the sphere, on the elements of geography, and on the element of astronomy, has little in common with his "Harmonicon coeleste," which was never published but is available in manuscript (an autograph in Florence, Biblioteca Nazionale Centrale, MSS della Biblioteca Magliabechiana, cl, XI, cod. XXXVI, and a copy in cod. XXXVII; a copy by G. Borelli in Rome, Biblioteca Nazionale Centrale Vittorio Emanuele **II** , fondo San Pantaleone; and the LibriCarucci copy in Paris, Bibliothèque Nationale, fonds lat. 7274. Part of the treatise is in Paris, Bibliothèque Nationale fonds Nouv. acqu. lat. 1644, fols. 67^r-79^v; and a French index of the part of Bibliothèque Nationale, fonds lat. 7274, is in Bibliothèque Nationale, Nouv. acqu. franç. 3282, fols. 119^r-123^r. The "Harmonicon coeleste," in five books, is Ptolemaic because Viète did not believe that Copernicus' hypothesis was geometrically valid).

All of viète's mathematical investigations are closely connected with his cosmological and astronomical work. The *Canon mathematicus, seu ad triangula cum appendicibus*, publication of which began in 1571, was intended to form the preparatory, trigonometric part of the "Harmonicon coeleste." The *Canon* is composed of four parts, only the first two of which were published in 1579; "Canon mathematicus," which contains a table of trigonometric lines with some additional tables, and "Universalium inspectionum ad Canonem mathematicum liber singularis," which gives the computational methods used in the construtíon of the canon and explains the computation of plane and spherical triangles with the aid of the general trigonometric relations existing among the determinant components of such triangles. These relations were brought together in tables that allow the relevant proportion obtaining among three known and one unknown component of the triangle to be read off directly. The two other parts, devoted to astronomy, were not published. Viète certainly knew the work of Rheticus, for he adopted the triangles of three series that the latter had developed.

The *Canon* has six tables, the first of which gives, minute by minute, the values of the six trigonometric lines. For the construction of this table Viète applied the method given by Ptolemy in his *Almagest*, which was improved by the Arabs and introduced into the West through the translation of al-Zarqālī's *Canones sive regulae super tabulas astronomiae* by Gerard of Cremona; John de Lignères's *Canones tabularum primi mobilis* in the fourteenth century: and John of Gmunden's *Tractatus de sinibus, chordis et arcubus*, which inspired Peurbach and Regiomontanus. All these took as their point of departure an arc of 15° called a *Kardaga*. The second table, "Canon triangulorum laterum rationalium," was based on the following proposition: "If there is a right-angled triangle having h for the hypotenuse, b for the base, and p for the perpendicular, and the semi-difference $(h-p)/2 = 1$; then $h = (b^2) + 1$ and $p = (b^2/4) - 1$. If b is given successive values of an arithmetical progression, the difference will be constant in the table of values of h and p thus formed. The third table, "Ad logisticem per Εξεχοντaδας tabella," is a multiplication table in the form of a right triangle that immediately gives, in degrees and minutes, the product $n.n'/60$ for all the numbers n and n' included between 0 and 60. "Fractionum apud mathematicos usitarum, alterius in alterum reductionibus tabella adcommodata," the fourth table, gives the quotients obtaining by dividing the Egyptian year, the day, and the hour, and their principal subdivisions by each other and also by the most commonly used integers. The fifth table, “Mathematici canonis epitome,” gives the values of the trigonometric lines from degree to degree and the length of the arc expressed in parts of the radius. The sixth table, “Canon triangulorum ad singulas partes quadranti circuli secundum Εξεχονταδων logisticem,” gives the value of the six trigonometric lines from degree to degree, the radius 1 being divided into sixty parts, each part into sixty primes, and each prime into sixty seconds.

After the canon of triangles with rational sides, in the second part of the *Canon*, Viète gave as functions of the radius the values of the sides of inscribed polygons with three, four, six, ten, and fifteen sides and the relations that exist among these trigonometric lines, which permit easy calculation of the tables. In his solution of oblique triangles, Viète solved all the cases (except where three sides are given) by proportionality of sides to the sines of the angles opposite the sides; for the case of three sides, he follows the ancients in subdividing the triangle into right triangles. For spherical triangles he employed the same notation as for plane triangles and established that a spherical right triangle is determined by the total sine and two other elements. In spherical oblique triangles, Viète followed the ancients and Regiomontanus in subdividing the triangle into two right triangles by an arc of a great circle perpendicular to one of the sides and passing through the vertex of the angle opposite. Also in the second part of the *Canon*, Viète wrote decimal fractions with the fractional part printed in smaller type than the integral and separated from the latter by a vertical line.

The most important of Viète’s many works on algebra was *In artem analyticem isagoge*, the earliest work on symbolic algebra (Tours, 1591). It also introduced the use of letters both for known quantities, which were denoted by the consonants *B, C, D,* and so on, and for unknown quantities, which were denoted by the vowels. Furthermore, in using *A* to denote the unknown quantity *x*, Viète sometimes employed *A* quadratus, *A* cubus…to represents x^2, x^3… This innovation, considered one of the most significant advances in the history of mathematics, prepared the way for the development of algebra.

The two main Greek sources on which Viète drew appear in the opening chapter: book **VII** of Pappus’ *Collection* and Diophantus’ *Arithmetica*. The point of departure for Viète’s “renovation” was his joining of facts, presented by Pappus only in reference to geometric theorems and problems, to the procedure of Diophantus’ *Arithmetica*. On the basis of Pappus’ exposition, Viète called this procedure *ars analytice*. In chapter 1 he undertook a new organization of the “analytic” art. To the two kinds of analysis mentioned by Pappus, the “theoretical” and the “problematical” (which he called “zetetic,” or “seeking [the truth],” and “poristic,” i.e.; “productive [of the proposed theorem]”), he added a third, which he called “rhetic” (“telling” in respect to the numbers), or “exegetic” (” exhibiting” in respect to the geometric magnitudes). He defined the new kind of analysis as the procedure through which the magnitude sought is produced from the equation or proportion set up in canonical form.

In chapter 2 Viète amalgamates some of “common notion” enumerated in book I of Euclid’s *Elements* with some definitions and theorems of book V, of the geometric books II and VI, and of the “arithmetical” books VII and VIII to form his stipulations for equations and proportions. In chapter 3 he gives the fundamental “law of homogeity,” according to which only magnitudes of “like genus” can be compared with each other, and in the fourth chapter he lays down “the canonical rules of species calculation.” These correspond to the rules for addition, subtraction, multiplication, and division used for instruction in ordinary calculation. In this chapter he presents a mode of calculation carried out completely in terms of “species” of numbers and calls it *logistice speciosa*-in contrast with calculation using determinate numbers, which is *logistice numerosa*. Of significance for formation of the concepts of modern mathematics, viète devotes the *logistice speciosa* to pure algebra, understood as the most comprehensive possible analytic art, applicable indifferently to numbers and to geometric magnitudes. By this process the concept of *eidos*, or species, undergoes a universalizing extension while preserving its link to the realm of numbers. In this general magnitudes. Viète’s *logistice speciosa*, on the other hand, is understood as the procedure analogous to geometric analysis and is directly related to Diophantus’ *Arithmetica*.

In chapter 5 viète presents the *leges zeteticae*, which refer to elementary operations with equations: to antithesis (proposition I), the transfer of one of the parts of one side of the equation to the other; to hypobibasm (proposition II), the reduction of the degree of an equation by the division of all members by the species common to all of them; and to parabolism (proposition III), the removal of the coefficient of the *potestas* (conversion of the equation into the form of a proportion). The sixth chapter, “De theorematum per poristicen examinatione,” deals more with synthesis and its relation to analysis than with poristics. It states that the poristic way is to be taken when a problem does not fit immediately into the systematic context.

In chapter 7, on the function of the rhetic art, viète treats the third kind of analysis (rhetic or exegetic), which is applied to numbers if the search is for a magnitude expressible in a number, as well as to lengths, planes, or solids if the thing itself must be shown, starting from canonically ordered equations.

In chapter 8, the final one, viète gives some definitions-such as of “equation” ; An equation is a comparison of an unknown magnitude with a determinate one-some rules, and some outlines of his works *De numerosa potestatum purarum, atque adfectarum ad exegesin resolutione tractatus, Effectionum geometricarum canonica recensio*, and *Supplementum geometriae*. In 1630 the work was translated into French by A. Vasset (very probably Claude Hardy) as *L’algèbre nouvelle de M. Viette* and J. L. de Valuezard as *Introducation en l’art analytique, ou nouvelle algèbre de François viète*. Both also contain a translation of viète’s *Zeteticorum libri quinque*. A modern French translation of the work was published by F. Ritter in *Bullettino di bibliografia*… **1** (1868), 223–244. An English version by J. W. Smith appeared as an appendix to Jacob Klein’s *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, Mass., 1968).

In 1593 viète published *Zeteticorum libri quinque*, which he very probably had completed in 1591. In it he offered a sample of *logistice speciosa* and contrasted it directly with Diophantus' *Arithmetica*, which, in his opinion, remained too much within the limits of the *logistice numerosa*. In order to stress the parallelism of the two works,viète ended the fifth book of his *Zetetics* with the same problem that concludes the fifth book of Diophantus' *Arithmetica*. In other parts of the book he also takes series of problems from the Diophantus work. References by Peletier and Peter Ramus, as well as Guilielmus Xylander's translation (1575), must certainly have introduced viète to the *Arithmetica*, which he undoubtedly also came to know in the original.

Moreover, as K. Reich has proved in her paper "Diophant, Cardano, Bombelli. viète, ein Vergleich ihrer Aufgaben," he was acquainted with Cardno's *De numerorum proprietatibus, Ars magna*, and *Ars magna arithmeticae* and mentioned his name in problems **II** , 21 and **II** , 22. According to Reich, however, it is not known whether viète, in preparing his *Zetetics*, considered Bombelli's *Algebra*. The *Zetetics* is composed of five books, the first of which contains ten problems that seek to determine quantities of which the sum, difference, or ratio is known. The problems of the second book give the sum or difference of the squares or cubes of the unknown quantities, their product, and the ratio of this product to the sum or the difference of their squares. In the third book the unknown quantities are proportional, and one is required to find them if the sum or the difference of the extremes or means is given. This book contains the application of these problems to right triangles. The fourth book gives the solutions of second-and third-degree indeterminate problems, such as **IV** , 2,3, to divide a number, which is the sum of two squares, into two other squares. The fifth book contains problems of the same kind, but generally concerning three numbers: for instance (V ,9), to find a right triangle in such a way that the area augmented with a given number, which is the sum of two squares, is a square.

Viète's notation in his early publications is somewhat different from that in his collected works, edited by F. van School in 1646. For example, the modern $(3BD^2 - 3BA^2)/4$ is printed in the *Zetetics* as (*B* in *D* quadratum 3 - *B* in *A* quadratum 3)/4, while in 1646 it is reprinted in the form (*B* in *Dq* 3– *B* in *Aq* 3)/4. Moreover, the radical sign found in the 1646 edition is a modification introduced by van Schooten. Viète rejected the radical, using instead the letter *l* in the *Zetetics* - for example, *l.* 121 for The same holds for Viète's *Effectionum geometricarum canonica receniso*, the outline of which he had given in his *Isagoge*: "With a view to exegetic in geometry, the analytical art selects and enumerates more regular procedures by which equations of 'sides' and 'squares' may be completely interpreted" -that is, it concerns a convenient method for solving geometrical problems by using the coefficients of the equation in question, without solving, the corresponding equation. All the solutions he gives in this tract have been carried out by geometric construction with the ruler an compass: for instance, the proof of proposition X, which leads to the equation $x^2 - px = q^2$, and that of proposition XVII, which leads to the equation $x^4 + p^2x^2 = p^2q^2$.

In 1593 at Tours, Jamet Mettayer edited *Francisci Vietae Supplementum geometriae, ex opere restitutae mathematicae analyseos seu algebra nova*. The following statement from proposition XXV - "Enimvero ostensum est in tractatu de aequationum recognitione, aequationes quarato-quadratorum ad aequationes cuborum reduci" -is important because it shows that by 1593 his tract *De aeqiationum recognitione* had already been completed, long before its publication by Alexander Anderson (1615). The tract begins with the following postulate: A straight line can be drawn from any point across any two lines (or a circle and a straight line) in such a way that the intercept between these two lines (or the line and the circle) will be equal to a given distance, any possible intercept having been predefined. the twenty-five propositions that follow can be divided into four groups:

1. Propositions 1-7 contain the solution of the problem of the mesographicum-to find two mean proportionals between two given straight line segments-and its solution immediately yields the solution of the problem of doubling the cube.

2. Propositions 8-18 contain the solution of the problem of the trisection of an angle and the corresponding cubic equation. The trigonometric solution of the cubic equation occurs twice: in proposition 16 and 17.

3. Proposition 19-24 contain the solution of the problem of finding the side of the regular heptagon that is to be inscribed in a given circle.

4. Proposition 25 explains the importance of the applied method: the construction of two mean proportionals, the trisection of an angle, and all problems that cannot be solved only by means of the ruler and compass but that lead to cubic and biquadratic equations, can be solved with the aid of the ancient *neusis* procedure.

In 1592 Viète began a lively dispute with J. J. Scaliger when the latter published a purported solution of the quadrature of the circle, the trisection of an angle, and the constructiom of two mean proportionals between two given line segments by means of the ruler and compass only. In that year Viète gave public lectures at Tours and proved that Scaliger's assertions were incorrect, without mentioning the name of the author. For this reason he decided in 1593 to publish book VIII of his *Variorum de rebus mathematicis responsorum Liber VIII, cuius praecipua capita sunt: De duplicatione cubi et quadratione circuli, quae claudit πϱχόειϱον seu ad usum mathematici canonis methodica*. In chapters 1, 2, and 5 Viète treats the traditional problem of the doubling of the cube, that is, of the constuction of two mean proportionals. In the first chapter, on the basis of Plutarch's *Life of Marcellus* (ch. 14), he calls this an irrational problem. In the fifth chapter he treats it synthetically, referring to the "ex Poristicis methodous" that he had presented in the *Supplementum geometria*. In chapter 3 he is concerned with the trisection of the angle and, in chapter 7, with the construction of the regular heptagon to be inscribed in a given circle, proposed by François de Foix, count of Candale, the most important contemporary editor and reviser of Euclid. Chapters 6 and 14 are related to Archimedes' *On Spirals*, already known in the Latin West through the Moerbeke translation of 1269.

In chapter 8 Viète discusses the quardratix and, in chapter 11, the lunes that can be squared. He investigates the problem of the corniculate angle in chapter 13 and sides with Peletier, maintaining that the angle of contact is no angle. Viète's proof is new: the circle may be regarded as a plane figure with an infinite number of sides and angles; but a straight line touching a straight line, however short it may be, will coincide with that straight line and will not form an angle. Never before had the meaning of "contact" been stated so plainly. In chapter 16 Viète gives a very interesting construction of the tangent to the Archimedean spiral and, in chapter 18, the earliest explicit expression for π by an infinite number of operations. Considering regular polygons of 4, 8, 16,… sides, inscribed in a circle of unit radius, he found that the area of the circle is

2.

from which he obtained

The trigonometric portion of this treatise begins with chapter 19 and concerns right and oblique plane and spherical triangles. In regard to the polar triangle and Viète's use of it, Braunmühl in his *Vorlesungen* assures the reader that Viète's reciprocal figure is the same as the polar triangle. He arrives at this conclusion because Viète's theorems are arranged in such a manner that each theorem is the dual of the one immediately preceding it.

Since Scaliger could not defend himself against Viète's criticism, he left France for the Netherlands, where soon after his arrival in 1594 he published his *Cyclometrica elementa*, followed some months later by his *Mesolabium*. Viète responded with *Munimen adversus cyclometrica nova* (1594) and *Pseudomesolabium* (1595). In the first, through a nice consideration based on the use of the Archimedean spiral, he gives two interesting approximations of a segment of a circle. In the second he seeks those chords cutting the diameter in such a way that the four parts increase in geometric series. In the appendix Viète refutes Scaliger's assertion that in the inscribed quadrilateral the diameter and both diagonals are in arithmetical proportion.

Viète's mathematical reputation was already consdierable when the ambassador from the Netherlands remarked to Henry IV that France did not possess any geometricians capable of solving a problem propounded in 1593 by Adrian Romanus to all mathematicians and that required the solution of a forty-fifth-degree equation. The king there-upon summoned Viète and informed him of the challenge. Viète saw that the equation was satisfied by the chord of a circle (of unit radius) that subtends an angle 2π/45 at the center. In a few minutes he gave the king one solution of the problem written in pencil and, the next day, twenty-two more. He did not find forty-five solutions because the remaining ones involve negative sines, which were unintelligible to him.

Viète published his answer, *Ad problema, quod omnibus mathematicis totius orbis construendum proposuit Adrianus Romanus, responsum*, in 1595. In the introduction he says: "I, who do not profess to be a mathematician, but who, whenever there is leisure, delight in mathematical studies.…" Regarding Romanus' equation, Viète had seen at once that since 45 = 3 . 3 . 5, it was necessary only to divide an angle once into five equal parts, and then twice into three, a division that could be effected by corresponding fifth- and third-degree equations. In the above problem he solved the equation $3x - x^3 = a$; using the roots x he determined y by $3y - y^3 = x$, and by the equation $5z - 5z^3 + z^5 = y$ he found the required roots z.

At the end of his work Viète proposed to Romanus, referring to Apollonius' *Tangencies*, the problem to draw a circle that touches three given circles. Romanus was acquainted with Regiomontanus' statement that he doubted the possibility of a soultion by means of the ruler and compass only. He therefore solved the problem by determining the center of the required circle by means of the intersection of two hyperbolas; this solution did not, however, possess the rigor of the ancient geometry. In 1600 Viète presented a solution that had all the rigor desirable in his *Apollonius Gallus, seu exsuscitata Apollonii Pergaei Περί ἐπα □ ωῦ geometria ad V.C.A. Romanum*, in which he gave a Euclidean solution using the center of similitude of two circles. Romanus was so impressed that he traveled to Fontenay to meet Viète, beginning an acquaintanceship that soon became warm friendship. Viète himself did not publish the book; very probably it was done by Marino Ghetaldi. A Greek letter dedicated to Viète precedes the text in the original edition. In appendix I, confronted with certain problems that Regiomontanus could solve algebraically but not geometrically, Viète provides their geometric construction and notes, by way of introduction, that these geometric construction are important. In appendix II he vehemently attacks Copernicus, and there is also a reference to a work intended to correct the errors in the work of Copernicus and the defects in that of Ptolemy. It was to have been entitled *Francelinis* and to have contained a composition, "Epilogistice montuum coelestium Pruteniana," based on hypotheses termed Apollonian, such as the hypothesis of the movable eccentric.

In the 1591 edition of the *Isagoge*, Viète had already given the outline of the *De numerosa potestatum purarum, atque adfectarum ad exegesin resolutione tractatus*. The "numerical resolution of powers" referred to in the title means solving equations that have numerical solutions, such as $x^2 = 2916$ or $x^2 + 7x = 60750$. The work was published in 1600 at Paris, edited by Marino Ghetaldi, with Viète's consent. (All information concerning the edition is taken from a letter written by Ghetaldi to Michel Coignet, dated 15 February 1600, which is printed at the end of the work.) Viète gave some of his manuscripts to Ghetaldi when the latter was in Paris. Ghetaldi took them to Rome and allowed his friends there to make a copy. After Viète's death his heirs gave other manuscripts to his friend Pierre Alleaume, who left them to his son jacques, a pupil of Viète's. Jacques entrusted Anderson with the treatises *De aequationum recognitione, Notae ad logisticem posteriores*, and *Analytica angularium sectionum*.

In *De numerosa potestatum*, Viète gives a method of approximation to the roots of numerical equations that resembles the one for ordinary root extraction. Taking $f(x) = k$, where k is positive, Viète separates the required root from the rest, then substitutes an approxiamte value for it and shows that another digit of the root can be obtained by division. A repetition of this process gives the next digit, and so on. Thus, in $x^5 - 5x^3 + 500x = 7{,}905{,}504$, he takes $r = 20$, then computes $7{,}905{,}504 - r^5 + 5r^3 - 500r$ and divides the result by a value that in modern notation would be $|(f[r + s_1] - f[r]| - s_1^n$, where n is the degree of the equation and s_1 is a unit of the denomination of the next digit to be found. Thus ,if the required root is 243 and r has been taken to be 200, then s_1 is 1. In the example above, where $r = 20$, the divisor is 878,295 and the quotient yields the next digit of the root, 4. One obtains $x = 20 + 4 = 24$, the required root.

Viète also had a role in the improvements of the Julian calendar. The yearly determination of the movable feasts had long resulted in great confusion. The rapid progress of astronomy led to the consideration of this subject, and many new calendars were proposed. Pope Gregory XIII convoked a large number of mathematicians, astronomers, and prelates, who decided upon the adoption of the calendar proposed by Clavisu. To rectify the errors of the Julian calendar, it was agreed to write 15 October into the new calendar immediately after 4 October 1582. The Gregorian calendar met with great opposition among scientists, including Viète and Tobias Müller. Viète valued the studies involved in a reform of the calendar; and toward the end of his life he allowed himself to be carried away by them and to engage in unjustified polemics against Clavius, the result of which was the publication with Mettayer of *Libellorum supplicum in regia magistri relatio kalendarii vere Gregoriani ad eccelesiasticos doctores exhibita pontifici maximo Clementi VIII anno Christo 1600 iubilaeo* (1600). He gave the work to cardinal Cinzio Aldobrandini, who transmitted it to Clavius. Since Clavius rejected the proposed corrections. Viète and Pierre Mettayer, the son of Jean, published a libel against Clavius that was as vehement as it was unjust: *Francisci Vietae adversus Christophorum Clavium expostulatio* (1602).

Francisci vietae fontenaensis de aequationum recognitione et emendatione tractatus duo was published in 1615, under the editorship of Viète's Scottish friend Alexander Anderson. The treatise "De emendatione" contains the subject matter of the work as announced in the *Isagoge* under the title "Ad logisticen speciosam notae posteriores" and sets forth a series of formulas (*notae*) concerning transformations of equations. In particular it presents general methods for solving third- and fourth-degree equations. This work reveals Viète's partial knowledge of the relations between the co-efficients and the roots of an equation. Viète demonstrates that if the coefficient of the second term in a second-degree equation is minus the sum of two numbers the product of which is the third term, then the two numbers are roots of the equation. Viète rejected all but positive roots, however, so it was impossible for him to perceive fully the relations in question.

Viète's solution of a cubic equation is as follows: Given $x^3 + 3B^2x = 2Z^3$. To solve this let $y^2 + yx = B^2$. Since from the constitution of such an equation B^2 is understood to be a rectangle of which the lesser of the two sides is y, and the difference between it and the larger side is x, $(B^2 - y^2)/y = x$. Therefore $(B^6 - 3B^4y^2 + 3B^2y^4 - y^6)/y^3 + (3B^4 - 3B^2y^2)/\mathrm{y} = 2Z^3$. When all terms have been multiplied by y^3 and properly ordered, one obtains $y^6 + 2X^3y^3 = B^6$. Since this equation is quadratic with a positive root it also has a cube root. Thus the required reduction is effected. Conclusion: If, therefore, $x^3 +3B^2x = 2Z^3$, and $–Z^3 = D^3$, then $(B^2 - D^2)/D$ is x as required.

In the soultion of biquadratics, Viète remains true to his principle of reduction. He first removes the term involving x^3 to obtain the form $x^4 + a^2x^2 + b^3x = c^4$. He then moves the terms involving x^2 and x to the right-hand side of the equation and adds $x^2y^2 + y^4/4$ to each side, so that the equation becomes $(x^2 + y^2/2)^2 = x^2 (y^2 - a^2) - b^3x + y^4/4 + c^4$. He then chooses y so that the right-hand side of this equation is a perfect square. Substituting this value of y, he can take the square root of both sides and thus obtain two quadratic equations for x, each of which can be solved.

In theorem 3 of chapter VI, Viète gives a trigonometrical solution of Cardano's irreducible case in cubics. He applies the equation $(2 \cos \alpha)^3 - 3 (2 \cos \alpha) = 2 \cos 3\alpha$ to the solution of $x^3 - 3a^2x = a^2b$, when $a> b/2$, by setting $\mathrm{x} = 2a \cos \alpha$ and determining 3α from $b =2\mathrm{a} \cos3\alpha$. In the last chapter Viète resolves into linear factors $x - x_k$ the first member of an algebraic equation $\phi(x) = 0$ from the second up to the fifth degree. Anderson's edition is the only one besides the *Opera* of 1646. There is still a manuscripts that contains the text (Paris, Bibliothèque Nationale, Nouv. acqu. lat. 1644, fols. 1r -31^r. "De recognitione aequationum tractatus," and fols. 32^r -60^r. "De aqeuationum emendatione tractatus secundus").

In 1615 Anderson published Viète's treatiseon angle sections. *Ad angularium sectionum analyticem theoremata καvολίκωτεϱα a Francisco Vieta fontenaensis primum excogitata at absque ulla demonstratione ad nos transmissa, jam tandem demonstratioibus confirmata*. This treatise deals, in part, with general formulas of chords, sines, cosines, and tangents of multiple arcs in terms of the trigonometric lines of the simple arcs. Viète first applies algebraic transformation to trigonometry, particularly to the multisection of angles, but without proofs and calculations, which were added by Anderson. In theorem 6 Viète considers the equations for multiple angles: letting $2 \cos a = x$, he expresses $\cos na$ as a function of x for all integers $n < 11$; and at the end he presents a table for determining the coefficients. In theorem 7 he expresses $2x^{n-2} \sin na$ in terms of x and y using $2 \sin a = x$ and $2 \sin 2a = y$. After theorem 10 Viète states: "Thus the analysis of angular sections involves geometric and arithmetic secrets which hitherto have been penetrated by no one." To the treatises of the *Isagoge* belong "Ad logiscticen speciosam, notae priores" and "Ad logisticen speciosam notae priores," the latter now lost. The first was not published during his life, because Viète believed that the manuscript was not yet suitable for publication. (It was published by Jean de Beaugrand in 1631.) It represents a collection of elementary general algebraic formulas that correspond to the arithmetical propositions of the second and ninth books of Euclid's *Elements*, as well as some interesting propositions that combine algebra with geometry. In propositions 48-51 Viète derives the formulas for $\sin 2x$; $\cos 2x$; $\sin 3x$; $\cos 3x$; $\sin 4x$; $\cos 4x$; $\sin 5x$ and $\cos 5x$ expressed in $\sin x$ and $\cos x$ by applying proposition 46, "From two rightangled triangles construct a third right-angled triangle," to two congruent right triangles: to right triangles with simple and double angles; with simple and triple angles, and with simple and quadruple angles respectively. He remarks, that the coefficients are equal to those in the expansion $(B + D)^n$ (*B* being the perpendicular and *D* the base of the original right triangle), that the various terms must be "homogeneosus" and that the signs are alternately + and -. (A French translation of this work was published by F. Ritter in *Bullettino di bibliografia* …**1** [1868], 245–276.) Besides Viète's published works there are manuscripts containing works of him or attributed to him. In addition to Nouv, acqu. lat. 1644, Bibliothèque Nationale, fonds lat., nouv, acqu. 1643, contains few new elements. The author was very well acquainted with Viète's work, particularly with his *De numerosa potestatum… ad exegesin resolutione*…; he betrays the influence of simon Stevin's *Arithmétique* because his manner of denoting on the method used by stevin and he uses the signs for equality and square root, London, British Museum, Sloane 652, fols. 1–9, contains the *Isagoge,* and fols. 10–40 the *Zetetics*.

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