# Abraham Wald | Encyclopedia.com

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Abraham Wald (1902-1950) was a mathematical statistician and a geometer. Given the fashions of this century, his fame as a statistician is by far the greater.

**Mathematical statistics**

Wald's interest in mathematical statistics became primary around 1938 and continued without interruption until his death. At ease in mathematical analysis, Wald contributed to the solutions of many of the specialized statistical problems of that period (see Wolfowitz 1952). However, it is with two broad lines of statistical research that his name is always linked: statistical decision theory and sequential analysis.

**Statistical decision theory.** By 1938 there was available a considerable body of theory dealing with the relationship between observable data and decision making, resulting from work along two closely related lines. One line dealt with the estimation problem—the problem of forming, from observable sample data, estimates, which are in some sense "best," of characteristics of populations described by probability distributions. The other line began with hypotheses concerning these probability distributions and sought "best" tests, based on observable sample data, of these hypotheses. In these two lines of research, R. A. Fisher, J. Ney-man, and E. S. Pearson, all working in England, were particularly prominent.

Both of these developments can, of course, be viewed as branches of the more general problem of making decisions in the face of uncertainty, and others must have thought of them as such. But it was Wald who first formally dealt with them in this way. As early as 1939, in one of his first papers (and possibly his finest) in mathematical statistics, Wald introduced a general mathematical structure for (single-sample) decision making, sufficiently general to include both estimation (point and interval) and hypothesis testing. He introduced such fundamental concepts as the multiple decision space and weight and risk functions, and for one of the solutions of the decision problem he introduced the principle of minimization of maximum risk. (There is currently some difference of opinion as to the dependence, in this last area, of Wald's work on von Neumann's great paper of 1928.) Such concepts as the least favorable a priori distribution and admissible regions are also found in this first paper. Wald continued his broad analysis of the decision problem, with a long interruption during the war, and it slowly but steadily gained acceptance. His work culminated in his formal and very general *Statistical Decision Functions* (1950), which incorporates his earlier researches into Bayes' and minimax solutions, as well as his later researches on complete classes of decision functions. The important connection between the decision problem and the zero-sum two-person game is also described at length in this book. Wald continued his work on decision theory in the short time he lived after the publication of his book, his research centering on the role of randomization in the decision process.

His total work in decision theory is probably his most important contribution to mathematical statistics.

**Sequential analysis.** Wald's second major achievement in mathematical statistics is sequential analysis. The notion that in some sense it is economical to observe and analyze data sequentially, rather than to observe and analyze a single sample of predetermined fixed size, was not a new one. Intuitive support for this notion is immediate; if the evidence shown in sequentially unfolding data is sharply one-sided, it seems reasonable to believe that the inquiry can be terminated early, with lengthier inquiries reserved for those situations in which the issue at hand appears, via the sequentially unfolding data, to be in greater doubt. This notion and the partial mathematical formulation of it were to be found in the statistical literature; among those who dealt with it before Wald was Walter Bartky of Chicago, and among Wald's contemporaries, George Barnard, working in England. But again it was Wald, in 1943, who first formulated mathematically and solved quite generally the problem of sequential tests of statistical hypotheses. He introduced the particular method of the sequential probability ratio test and, with Wolfowitz (1948), showed its optimal properties. He found operating characteristic and average sample number functions; he introduced, if he did not completely solve, the problem of sequential tests of composite hypotheses (utilizing weight functions); and he began vital discussions of such basic topics as multivalued decisions and optimal sequential estimation. All this, plus many special problems, were gathered together in *Sequential Analysis* (1947), a book surprisingly easy to read, less formal and more elementary in structure than his work on decision functions.

**Influence on statistical research.** Wald's strictly mathematical approach to problems had heavy impact on American research in statistical theory. Up to 1939, one finds excellent researches in statistical theory that nevertheless sometimes lack a firm mathematical basis. Wald's approach was different: his formulations of decision theory and sequential tests of hypotheses were strictly mathematical. Wald can be associated with the beginning of a separation, continuing through the present, of American statistical research from (the parent) British statistical research. With notable exceptions, of course, current issues of *Biometrika* (a leading British statistical journal) and of the *Annals of Mathematical Statistics* (a journal of predominantly American authorship) will show at a glance the difference between the more formal, more mathematical American school—largely inspired and to some extent trained by Wald—and the more intuitive, more applied, less mathematical British school—influenced by such statistical innovators as Fisher, who were less impressed by the value of formal mathematical structure.

A second consequence of Wald's *modus operandi* is notable. Up to 1939, theoretical statisticians were primarily interested in rather limited problems. Wald's formulation of problems was often so broad that his work was difficult to read, but in setting out problems in broad terms, he greatly facilitated later research by others. For example, research in such difficult areas as sequential tests of composite hypotheses was much facilitated by Wald's extensive outline, however incomplete, of this area in his general formulation of sequential theory.

Wald was at heart a mathematician. Although he was not openly opposed to intuitive justification or to popularization, he had no serious interest in either and he asserted that such activities, in the absence of or as substitutes for logical structure, are not permanently useful. Nevertheless, Part 1 of Wald's first full-scale report of his researches in sequential analysis (see Columbia … 1943) includes numerous heuristic and intuitive arguments and justifications of the sequential idea and of approximate formulas for risks of error, many of them originated by Wald himself. These surely help the reader understand, in a nonmathematical way, the nature of this new development; but they do not seem quite in the character of Wald. All this changes in Part 2, where Wald introduced cylindric random variables and abruptly tackled the difficult mathematical problem at hand (see Columbia … 1945).

Wald's attitude toward specialized application was similar. He was always willing to help practical statisticians; and although his improvisations, approximations, guesses, and *ad hoc* solutions did not generally match the quality of his formal work, he nevertheless offered them freely. Yet his interest in such areas was casual.

With respect to the originality of his contributions to mathematical statistics, Wald is in a class with Fisher and Neyman. But of all workers in this field Wald combined best a profound understanding of the value of the precise formulation of broad and significant areas of statistical inference with the mathematical equipment to handle them. His ability to recognize a major statistical area when he saw one and to do something about it was impressive. Were he alive today, he might well be able to formulate the Bayesian inference problem in such a way that its mathematical structure and its consequences would be clearly set apart from the philosophical and intuitive controversy which no amount of mathematics can ever settle.

### Work in geometry and other fields

Wald's other major contribution was in geometry. Far closer than mathematical statistics to the core of mathematics itself, Wald's work here may someday be regarded as his major achievement. At present it is hardly known. Wald went to Vienna briefly in 1927, permanently in 1930; during the period 1931-1936 he worked in geometry with Karl Menger. His major work centered on the problem of the curvature of surfaces. He wrote on many topics in topology and metric spaces, measure and set theory, and lattice theory; and he was the first to prove the existence of a collective in probability theory. His activity in this area had ended by 1943—in fact, there was little after 1936.

Wald also did important work in econometrics and mathematical economics. From 1932 to 1937 and, sporadically, later, he made valuable contributions to such diverse subjects as seasonal corrections to time series, approximate formulas for economic index numbers, indifference surfaces, the existence and uniqueness of solutions of extended forms of the Walrasian system of equations of production, the Cournot duopoly problem, and finally, in his much-used work written with Mann (1943), stochastic difference equations. By all odds, the most important of these were the papers on the existence of a solution to the competitive economic model, written in 1935 and 1936 for Menger's colloquium; an expository version, published in 1936, was translated in the October 1951 issue of *Econometrica*. These papers, along with von Neumann's slightly earlier oral discussion using Brouwer's fixed-point theorem, are the first in which a competitive existence theorem is rigorously proved. Some of Wald's conditions would be deemed overly strong today, but it was a pioneering accomplishment to have provided such a rigorous proof—some 26 years before Uzawa's demonstration of the equivalence of Wald's existence theorem and the fixed-point theorem. This paper alone guarantees Wald's permanent fame in economics.

### Intellectual career

Wald was a superb teacher. There were no gimmicks or jokes—only precision and clarity. Sometimes, as Wolfowitz has noted (1952), the precision was labored, for Wald was generally content with any solid proof and seldom went to the trouble of searching for briefer and more elegant proofs. But his lectures were effective. The present author was the only student in a course of Wald's in the early days of sequential analysis, and with care and skill Wald taught him the content of his "green book" (see Columbia …1943; the contents of this book were classified by the U.S. government until after the war). Wald was not often electrifying, but his admirable teaching during the 1940s still helps to sustain statistical research and teaching. The notebooks (194?; 1941; 1946) created by his students from his lectures are testimony to the quality of Wald's instruction; they are rigorous at the level Wald had in mind, and they remain, some 25 years after their appearance, useful and even provocative for the modern teacher and student.

In Wald's case, more than in the case of most, the work and the man were the same; he lived his work, and his happiest hours were devoted to it. It could have been Wald who said, "Let's go down to the beach and prove some theorems."

Wald was born in 1902, in Cluj, Rumania. After private schooling and self-schooling (the consequence of complications arising from his family's Jewish orthodoxy), Wald, well-trained in mathematics, finally settled in Vienna in 1930. Soon after, he worked for five years in geometry under Menger. In 1932 he began five years of work in econometrics and mathematical economics at the Austrian Institute for Business Cycle Research. In 1938, the year of the *Anschluss,* Wald accepted an invitation—one which probably saved his life— from the Cowles Commission to do econometric research in the United States. Later in 1938 he was brought by Harold Hotelling to Columbia to work in mathematical statistics, and he remained there for the rest of his life. While on a lecture tour of India in 1950, he died in an airplane crash.

Wald was a quiet and gentle man, deeply immersed in his work. He was fairly aloof from small talk, and he had few hobbies. But he was not indifferent to recognition; in the controversies that occasionally developed in the hyperactive and hypersensitive wartime atmosphere of Columbia's Statistical Research Group (of which Hotelling was official investigator and of which W. Allen Wallis was director of research), Wald displayed an entirely normal combination of passive distaste for dispute and active interest in the handling of his work.

Apart from the pleasure he took in his work, Wald had a reasonable share of joy during his life. His marriage to Lucille Lang, who perished with him in India, and his two children, Betty and Robert, were sources of happiness to him. He also had his full share of sorrows, chief among them the death of eight of the nine European members of his immediate family in the gas chambers of Auschwitz.

The scholars whose professional lives were most closely related to Wald's include Harold Hotelling at the University of North Carolina and J. Wolfo-witz at Cornell. Hotelling, himself one of the most distinguished figures in American statistical research, brought Wald to Columbia in 1938, securing for him a Carnegie fellowship and an assistant professorship, and helped him through a difficult period of adjustment. However, although Wald's early interest in certain areas of mathematical statistics was initiated by problems brought to his attention by Hotelling, they did not work together; their approach to problems, as well as the kind of problems that interested them, was somewhat different. In particular, HoteUing's interdisciplinary interests contrasted with Wald's strictly statistical interests. But they had great respect for each other, and Hotelling played a major role in Wald's career.

Wolfowitz was Wald's leading student. Oriented mathematically almost exactly as Wald was, Wolfowitz wrote no fewer than 15 papers with Wald and was his closest friend. It is nearer to the truth to say that it was the team of Wald and Wolfowitz — rather than Wald alone—that gave much of American statistical inference the rather severe mathematical character it has today, though this is not to imply that either would be in sympathy with mathematically difficult work divorced from statistical reality.

Harold Freeman

*[For the context of Wald's work, see*Estimation; GameTheory; HypothesisTesting. For *discussion of the subsequent development of Wald's ideas, see*DecisionTheory; SequentialAnalysis.]

# BIBLIOGRAPHY

For *an account of Wald's many specialized contributions to mathematical statistics, see* Wolfowitz 1952. For *comment on his early contribution to probability, see* Menger 1952; von Mises 1946. *Discussions of his contributions to mathematical economics and econometrics are found in* Morgenstern 1951; Tintner 1952. *His major work in statistical decision theory is in* Wald 1950. For *his major work in sequential analysis, see* Wald 1947. For *a commentary on his work in geometry, see* Menger 1952. *A bibliography of Wald's published work is contained in* Selected Papers …1955. For *lecture notes compiled by his students, see* Wald 194?; 1941; 1946.

# WORKS BY WALD

(1936) 1951 On Some Systems of Equations of Mathematical Economics. *Econometrica* 19:368-403. → First published in German in Volume 7 of *Zeitschrift fur Nationalbkonomie*.

(1939) 1955 Contributions to the Theory of Statistical Estimation and Testing Hypotheses. Pages 87-114 in Abraham Wald, *Selected Papers in Statistics and Probability*. New York: McGraw-Hill. → First published in Volume 10 of the *Annals of Mathematical Statistics*.

194? Notes on the Theory of Statistical Estimation and of Testing Hypotheses. Notes prepared by Ralph J. Brookner. Unpublished manuscript. → Lectures given

in a one-semester course at Columbia University, and available in the Columbia University Libraries.

1941 Lectures on the Analysis of Variance and Covari-ance. Notes prepared by Ralph J. Brookner. Unpublished manuscript. → Lectures given in 1941 at Columbia University, and available in the Columbia University Libraries.

1943 Wald, Abraham; and Mann, H. B. On the Statistical Treatment of Linear Stochastic Difference Equations. *Econometrica* 11:173-220.

1943 Columbia University, Statistical Research Group*Sequential Analysis of Statistical Data: Theory*. New York: Columbia Univ. Press.

1945 Columbia University, Statistical Research Group*Sequential Analysis of Statistical Data: Applications*. New York: Columbia Univ. Press.

1946 Notes on the Efficient Design of Experimental Investigation. Unpublished manuscript. -→ Lecture notes of a one-semester course given in 1943 at Columbia University, and available in the Columbia University Libraries.

1947 *Sequential Analysis*. New York: Wiley.

(1948) 1955 Wald, Abraham; and Wolfowitz, J. Optimum Character of the Sequential Probability Ratio Test. Pages 521-534 in Abraham Wald, *Selected Papers in Statistics and Probability*. New York: McGraw-Hill. → First published in Volume 19 of the *Annals of Mathematical Statistics*.

(1950) 1964 *Statistical Decision Functions*. New York: Wiley.

*Selected Papers in Statistics and Probability*. New York: McGraw-Hill, 1955. → Published posthumously. Contains writings first published between 1938 and 1952, and a bibliography of Wald's works on pages 20-24.

# SUPPLEMENTARY BIBLIOGRAPHY

Hotelling, H. 1951 Abraham Wald. *American Statistician* 5:18-19.

Menger, K. 1952 The Formative Years of Abraham Wald and His Work in Geometry. *Annals of Mathematical Statistics* 23:14-20.

Morgenstern, Oskar 1951 Abraham Wald: 1902-1950. *Econometrica* 19:361-367.

Tintner, G. 1952 Abraham Wald's Contributions to Econometrics. *Annals of Mathematical Statistics* 23: 21-28.

Von Mises, Richard (1946) 1964 *Mathematical Theory of Probability and Statistics*. Edited and augmented by Hilda Geiringer. New York: Academic Press.

Wolfowitz, J. 1952 Abraham Wald: 1902-1950. *Annals of Mathematical Statistics* 23:1-13.