John Wallis | Encyclopedia.com

(*b*. Ashford, Kent, England, 3 December 1616; *d*. Oxford, England, 8 November 1703), *mathematics*

Wallis was the third child of John Wallis and his second wife, Joanna Chapman. His father studied at Trinity College, Cambridge, and after having taken holy orders became minister at Ashford, about 1603. Standing in great esteem and reputation in his town and parish, he died when John was barely six.

Young John grew up, together with his two older sisters and two younger brothers, in the care of his mother. After he had received his first education, he was sent in 1625 to a grammar school at Tenterden, Kent, where, according to his autobiography.[1] he enjoyed a thorough training in Latin. In 1631 – 1632 Wallis attended the famous school of Martin Holbeach at Felsted, Essex. Besides more Latin and Greek he also learned some Hebrew and was introduced to the elements of logic. As mathematics was not part of the grammar school curriculum, he obtained his first insight into this field during a vacation; he studied what a brother of his had learned in approximately three months as preparation for a trade.

Wallis entered Emanuel College, Cambridge, the "Puritan College," about Christmas 1632 as a pensioner. He not only took the traditional undergraduate courses (obtaining his bachelor of arts degree early in 1637), followed by studies in theology, but he also studied physic, anatomy, astronomy, geography, and other parts of natural philosophy and what was then called mathematics—although the latter "were scarce looked upon, with us, as Academical Studies then in fashion." He was the first student of Francis Glisson to defend the doctrine of the circulation of the blood in a public disputation.

In 1640 Wallis received the degree of master of arts and was ordained by the bishop of Winchester. For some years he earned his living as private chaplain and as minister in London. From 1644, after the outbreak of the Civil War, he also acted as secretary to the Assembly of Divines at West-minister, which was charged with proposing a new form of church government. For about a year he also held a fellowship at Queens' College, Cambridge, in consequence of a Parliamentary ordinance. He gave up this position when he married Susanna Glyde of Northiam, Sussex, on 14 March 1645.

Wallis' appointment as Savilian professor of geometry at Oxford on 14 June 1649 must have come as a surprise to many; his accomplishments thus far, with one exception, had had little to do with mathematics. His predecessor, Peter Turner, was a Royalist who had been dismissed by an order of Parliament; Wallis had rendered valuable services not only as a secretary to the Assembly of Divines but also by his skill in deciphering captured coded letters for the Parliamentarians. Few people in 1649 could have for seen that within a few years the thirty-two-year-old theologian would become one of the leading mathematicians of his time.

This appointment determined Wallis' career; he held the chair until his death more than half a century later. In addition, in 1657 – 1658 he was elected—by a somewhat doubtful procedure—custos archivarum (keeper of the archives) to the university, an office he also held for life. In 1654 he had been admitted doctor of divinity. At the Restoration Wallis was confirmed in his offices for having possessed the courage to sign the remonstrance against the execution of King Charles I; he also received the title of royal chaplain to Charles II. When in 1692 Queen Mary II offered Wallis the deanery of Hereford, he declined, hinting that favors for his son and his son-in-law Blencowe would be more welcome signs of recognition of his services to his country.

These achievements include the mathematical works, helping found the Royal Society; his work in the decipherment of code letters for the government; logic; teaching deaf mutes to speak and the related grammatical and phonetical writings; archival studies and his assistance to the university in legal affairs; theological activities as a preacher and author of treatises and books; and the editions (many of them first editions) of mathematical and musical manuscripts of ancient Greek authors.

The first two decades of the Savilian professorship were the most creative period in Wallis's life. He later increasingly turned to editing works of other scientists (J. Horrox, W. Oughtred, and Greek authors) and his own earlier works, and to the preparation of historical and theological discourses. His *Opera mathematica* appeared between 1693 and 1699, financed by and printed at the university.

Wallis enjoyed vigorous health throughout his life. His powers of intellect were remarkable, and he was renowned for his skill in public disputations. But he also possessed a highly contentious disposition and became involved in many violent controversies—the more so since modesty does not seem to have been one of his virtues. Nevertheless he had many devoted friends. It was for Thomas Smith, vice-president of Magdalen College, Oxford, and librarian at the Cottonian Library, London, that Wallis wrote his autobiography in 1697; and Samuel Pepys commissioned Sir Godfrey Kneller to paint a full-length portrait of "that great man and my most honoured friend, Dr. Wallis, to be lodged as an humble present of mine (though a

Cambridge-man) to my dear Aunt the University of Oxford."[2] Wallis was interred in St. Mary's, the university church, and an epitaph by his son was placed in the wall near his burial place: "Joannes Wallis, S.T.P., Geometriae Professor Savilianus, et Custos Archivarum Oxon. Hic dormit. Opera reliquit immortalia …" ("Here sleeps John Wallis, Doctor of Theology, Savilian Professor of Geometry, and Keeper of the Oxford Archives. He left immortal works …"[3])

Mathematics Wallis reports in his *Algebra*[4] that his interest in mathematics (beyond the little that he may have learned at Cambridge) was first aroused in 1647 or 1648, when he chanced upon a copy of William Oughtred's *Clavis mathematicae*. After having mastered it in a few weeks, he rediscovered Cardano's solution of the cubic equation (not given by Oughtred) and, continuing where Oughtred had left off, composed in 1648 a *Treatise of Angular Sections*, which remained unpublished until 1685. In the same year, at the request of Cambridge professor of mathematics John Smith, the Platonist (1618 – 1652), he gave an explanation of Descartes's treatment of the fourth-degree equation. The basic idea, to write the equation as a product of two quadratic factors, could be derived from Harriot's *Artis analyticae praxis* (published posthumously in 1631); yet Wallis repeatedly claimed not to have known this book in 1648. Such was the total evidence of his mathematical talents that Wallis presented when he was made Savilian professor of geometry in 1649.

With a rare energy and perseverance, he now took up the systematic study of all the major mathematical literature available to him in the Savilian and the Bodleian libraries in Oxford. According to the statutes of his chair, Wallis had to give public lectures on the thirteen books of Euclid, on the *Conics* of Apollonius, and on all of Archimedes' work. He was also to offer introductory courses in practical and theoretical arithmetic—with a free choice of textbooks therein. Lectures on other subjects such as cosmography, plane and spherical trigonometry, applied geometry, mechanics, and the theory of music were suggested but not obligatory according to the statutes.

An outcome of his elementary lectures was the *Mathesis universalis, seu opus arithmeticum* (1657). Its treatment of notation, including a historical survey, stressed the great advantages of a suggestive and unified symbolism; yet the influence of Oughtred (who had developed a rather special notation) sometimes makes itself felt—to no great advantage. On the whole, this work reflects the rather weak state of mathematical learning in the universities at the time.

In the treatise *De sectionibus conicis* (1655) Wallis dealt with a classical subject in a new way.[5] He considered the conic sections merely as plane curves, once he had obtained them by sections of a cone, and subjected them to the analytical treatment introduced by Descartes rather than to the traditional synthetic approach. In addition, he employed infinitesimals in the sense of Cavalieri and Torricelli. Here he also first introduced the sign for infinity and used $1/\infty$ to represent, for example, the height of an infinitely small triangle. Although Mydorge in adherence to the ancient methods had obtained a certain simplification of the treatment in 1631, Wallis was rather proud of his achievement; he may not have known Mydorge's *De sectionibus conicis* at the time of writing. Shortly afterward, in 1659, Jan de Witt's valuable treatise *Elementa curvarum linearum*, also employing the analytic symbolism, appeared in Amsterdam. Yet, on the whole, the new viewpoint was accepted only slowly by mathematicians.

Together with his conic sections Wallis published the book on which his fame as a mathematician is grounded, *Arithmetica infinitorum*; the title page is dated 1656, but printing had been completed in the summer of 1655. It resulted mainly from his study of Torricelli's *Opera geometrica* (1644), for Cavalieri's basic work on the methods of indivisibles was unavailable. At first Wallis' attempts to apply these methods to the quadrature of the circle met with failure; and not even a study of the voluminous *Opus geometricum* (1647) of Gregory of St. Vincent, which was devoted to this subject, would help. But then, by an ingenious and daring sequence of interpolations, he produced his famous result[6]

Although the method was mistrusted by such eminent mathematicians as Fermat and Huygens, the result was ascertained by numerical computation. Wallis' main interest lay not with the demonstration, but with the investigation. Actually searching for the value of

he considered the generalized integral

Its reciprocal $1:l\ (k,n)$ he tabulated first for integral values of k and n (receiving the symmetric array of the binomical coefficients or figurated numbers), then for the fractions for , with $k = n = 1/2$, this should yield for which he wrote the symbol □. Then each second value of the row and column which met at □ was a certain (fractional) multiple of □ Assuming that all rows and columns in his table would continually increase, Wallis was able to derive two sequences of upper and lower bounds for □, respectively. When these sequences are continued indefinitely, they yield his famous infinite product. William Brouncker soon transformed it into a regular continued fraction, which Wallis included in his book.

Wallis' method of interpolation—he himself gave it this name, which has become a *terminus technicus*—is based on the assumption of continuity, and, incidentally, seems closely related to the procedure he had to apply when he deciphered coded letters. To preserve this continuity and thereby the underlying mathematical law in his table, Wallis went to the utmost limit. He admitted fractional multiples of the type , claiming that A here should be infinite so that the value of the product was a finite number. One must emphasize the kind of "functional thinking" revealed here—not on the basis of geometric curves but of sequences of numerical expressions, that is, tabulated functions.

There are many more remarkable results of a related nature in the *Arithmetica infinitorum*, in the tracts on the cycloid and the cissoid, and in the *Mechanica*.[7] The integral $l\ (k, n)$ may in fact, by the substitution $x \rightarrow y^k$, be transformed into the normal form of the beta integral. He soon derived analytically the integral for the arc length of an ellipse and reduced other integrals to the elliptic one. But more important than the individual problems that Wallis mastered was the novelty of his approach—his analytic viewpoint, in contrast to the traditional geometric one—at a time when the symbolism of analysis had not yet been properly developed. The best documentation of his new "functional thinking" is provided in the *Arithmetia Infinitorum*; he finally plots the graphs of the family of functions the values of which he had so far evaluated only for a sequence of distinct points. There he considers not so much the single curves as the sequence of them, since the parameter changes from one integral value to the next. The answer to his question of what the equations of these curves would be for fractional values of the parameter—another type of interpolation and example of "continuous thinking" —was given by Euler by means of the gamma function, the generalized factorial.

The *Arithmetica infinitorum* exerted a singularly important effect on Newton when he studied it in the winter of 1664 – 1665.[8] Newton generalized even more than Wallis by keeping the upper limit of the integrals $I\ (k, n)$ variable. He thus arrived at the binomial theorem by way of Wallisian interpolation procedures. In a few cases the binomial expansion could be checked by algebraic division and root extraction; but, just as in the case of Wallis' product, a rigorous justification had to wait until mathematical techniques had been much refined.

The publication of the *Arithmetica infinitorum* immediately provoked a mathematical challenge from Fermat. He directed "to Wallis and the other English mathematicians," some numerical questions: To find a cube, which added to all its aliquot parts will make a square (such as $7^3 + 7^2 + 7 + 1 = 20^2$), and to find a square number, which added to all its aliquot parts, will make a cube.[9] Fermat, lawyer and councillor of the *parlement* in Toulouse, had added: "We await these solutions, which, if England or Belgic or Celtic Gaul do not produce, Narbonese Gaul will." Besides Wallis, Brouncker, later the first president of the Royal Society, participated in the contest on the side of the English. On the Continent, Frenicle de Bessy applied his great skill in handling large numbers. Wallis at first highly underestimated the difficulty as well as the theoretical foundation of Fermat's questions; and Fermat added further problems in 1657 – 1658. Wallis maintained the number 1 to be a valid solution, and in return drew up some superficially similar questions. His method of solution was more or less that of trial and error, based on intelligent guessing, and in some ways was not unrelated to the procedures employed in his *Arithmetica infinitorum*. Until the end of his life Wallis had no idea of the number-theoretical insights that Fermat had obtained—nor could he, since his challenger did not reveal them. Afraid that the French mathematicians might reap all the glory from this contest, Wallis obtained permission to publish the letters: the *Commercium epistolicum* appeared in 1658. The last chapter of his *Discourse of Combinations, Alternations, and Aliquot Parts* (1685) deals with "Monsieur Fermat's Problems Concerning Divisors and Aliquopt Parts." Finally, among his manuscripts there are also a number of attempts to solve some of Fermat's problems, including the "Theorema Fermatianum Negativum" that $a^3 + b^3 = c^3$ is not possible in integral or rational numbers and another negative theorem that there does not exist a right triangle with square area.[10]

But number theory had no special appeal to Wallis—nor to any other mathematician of the time, Frenicle excepted. This was so partly because it was hardly applicable, as Wallis himself emphasized and partly because it did not suit the taste of seventeenth- and eighteenth-century mathematicians, Euler being a notable exception. Fermat, who had glimpsed the treasures of number theory and had recognized its intrinsic mathematical value, did little to introduce his fellow mathematicians to the subject. Thus the general judgment about the contest had to be based on Wallis' *Commercium epistolicum*, and the editor did not hesitate to underline the achievements he and Brouncker had made. No wonder that his fame was now firmly established throughout Europe.

Wallis also participated in the competition in which Pascal in the summer of 1658 asked for quadratures, cubatures, and centers of gravity of certain figures limited by cycloidal arcs.[11] Neither Wallis nor Lalouvère, who also competed for the prize, satisfied Pascal, and no prize was awarded. This was not quite fair, and in 1659 Wallis replied with *Tractatus duo …de cycloide et …de cissoide*. Here, as well as in the second part of his voluminous *Mechanica, sive de motu tractatus geometricus* (1669 – 1671), he again relied on his analytic methods. This second part, on the calculation of centers of gravity, is the major part of the *Mechanica*, and in it Wallis carried on the analytical investigations of the 1650's

The first part deals with various forms of motion in a strictly "geometrical," that is, Euclidean, manner, starting with definitions followed by propositions. The motion of bodies under the action of gravity is covered in particular. The final chapter of the first part is devoted to a treatment of the balance and introduces the idea of moment, which is essential for the inquiries into the centers of gravity. In the third part, Wallis returns not only to the elementary machines, according to ancient tradition, but above all to a thorough treatment of the problems on percussion. In 1668 percussion and impact were a major topic of discussion at the Royal Society, and Wallis, Wren, and Huygens submitted papers.[12] In the *Mechanica*, Wallis extended his investigations, studying the behavior of both elastic and inelastic bodies. Although in style and subject matter it is not a uniform book, at the time it certainly was one of the most important and comprehensive in its field. It represents a major advance in the mathematization of mechanics, but it was superseded in 1687 by a much greater one—Newton's *Principia*

Wallis' last great mathematical book was *Treatise of Algebra, Both Historical and Practical* (1685), the fruit of many years' labor.[13] As its title suggests, it was to combine a full exposition of algebra with its history, a feat never previously attempted by any author. The book was Wallis' only major mathematical work to be published in the vernacular. (In 1693 an augmented Latin translation was issued as vol. II of his *Opera mathematica*.)

Of the 100 chapters, the first fourteen trace the history of the subject up to the time of Viète, with emphasis on the development of mathematical notation. The subsequent practical introduction to algebra (chapters 15 – 63) was based almost entirely on Oughtred's *Clavis mathematicae*, Harriot's *Artis analyticae praxis*, and *An Introduction to Algebra* (1668). Thomas Brancker's translation J. H. Rahn's *Teutsche Algebra* (1659), with numerous additions by John Pell, Rahn's former teacher. This fact alone signals the great bias Wallis had developed in favor of his countrymen. It becomes even more obvious in the passages where the author claimed that Descartes had obtained his algebraical knowledge from Harriot. Criticisms of Wallis' one-sided account were raised immediately and have continued since. After an insertion concerning the application of algebra to geometry and geometrical interpretations of algebraic facts (chapters 64 – 72, including an attempt to give a representation of imaginary numbers),[14] Wallis devoted the final twenty-eight chapters to a subject that one would hardly look for in a book on algebra today: a discussion of the methods of exhaustion and of indivisibles, again with reference to the *Arithmetica infinitorum*. Thus the new methods were still considered as an extension of an old subject rather than as a wholly new field of mathematics.

The *Algebra* also includes an exposition of the method of infinite series and the first printed account, much augmented in the second edition, of some of Newton's pioneering results. Wallis had long been afraid that foreigners might claim the glory of Newton's achievements by publishing some of his ideas as their own before Newton himself had done so. He therefore repeatedly warned his younger colleague at Cambridge not to delay but to leave perfection of his methods to later editions.[15] (Volume III of the *Opera*[1699] contains an *Epistolarum collectio*, of which the most important part is the correspondence between Newton and Leibniz, in particular Newton's famous "Epistola prior" and "Epistola posterior" of 1676.)

A part from some editions of Greek mathematical classics, the *Algebra* with its several supplementary treatises— *Cono-Cuneus* (a study in analytic three-dimensional geometry), *Angular Sections,*[16] *Angle of Contact, and Combinations, Alternations, and Aliquot Parts*—marked the end of the stream of mathematical works. Even without the polemics against Hobbes and some minor pieces, they fill three large volumes.

Wallis helped shape over half a century of mathematics in England. He bore the greatest share of all the efforts made during this time to raise mathematics to the eminence it enjoyed on the Continent. The center of mathematical research and of the "new science" in Galileo's time lay in Italy. It then shifted northward, especially to France and the Netherlands. Because of Wallis' preparative work and Newton's genius, it rested in Britain for a while, until through the influence of Leibniz, the Bernoullis, and Euler it moved back to the Continent.

Nonmathematical Work. Wallis first exhibited his mental powers early in the Civil War (1642 or 1643), when by chance he was shown a letter written in cipher and succeeded in decoding it within a few hours.[17] Because more letters were given to him by the Parliamentarians, rumors were later spread that he had deciphered important royal letters that had fallen into their hands. Wallis strenuously denied the accusation, and it is very unlikely that he revealed anything harmful to the royal family or the public safety—if indeed he came across such information. On the contrary, the confirmation of his offices at the Restoration may well have been a sign of gratitude to him by Charles II. For many years Wallis continued to decipher intercepted letters for the government for the government, especially after the Revolution. In old age he taught the art to his grandson William Blencowe but refused to disclose it when Leibniz on behalf of his government requested information on it.

In his autobiography, written in January 1697, when he was over eighty, Wallis referred to one of his first successes more than half a century earlier:

Being encouraged by this success, beyond expectation: I afterwards ventured on many others (some of more, some of less difficulty) and scarce missed of any that I undertook, for many years, during our civil Wars, and afterwards. But of late years, *the French Methods of Cipher* are grown so intricate beyond what it was wont to be, that I have failed of many: tho' I did have master'd divers of them.[18]

Of great importance for much of his later scientific work was his introduction, while living in London, to a group interested in the "new" natural and experimental sciences—the circle from which the Royal Society emerged soon after the Restoration.[19] To Wallis we owe one of the few reports on those early meeting that give direct evidence.

About the year 1645, while I lived in *London* (at a time, when, by our Civil Wars, Academical Studies were much interrupted in both our Universities:) beside the Conversation of divers eminent Divines, as to matters Theological; I had the opportunity of being acquainted with divers worthy Persons, inquisitive into Natural Philosophy, and other parts of Humane Learning; and Particularly of what hath been called the *New philosophy or Experimental Philosophy*.

We did by agreement, divers of us, meet weekly in *London* on a certain day, to treat and discours of such affairs…

These meetings we held sometimes at *Dr. Goddards* lodgings in *Woodstreet* (or some convenient place near) on occasion of his keeping an Operator in his house, for grinding Glasses for Telescopes and Microscopes; and sometimes at a convenient place in *Cheap-side;* sometime at *Gresham College* or some place near adjoyning.

Our business was (precluding matters of Theology and State Affairs) to discours and consider of *Philosophical Enquiries*, and such as related thereunto; as *Philosophical, Anatomy, Geometry, Astronomy, Navigation, Staticks, Magneticks, Chymicks*

Mechanicks, and *Natural Experiments*; with the State of these Studies, as then cultivated, at home and abroad. We there discoursed of the *Circulation of the Blood, the Valves in the Veins, the Venae Lacteae, the Lymphatick vessels, the Copernican Hypothesis, the Nature of Comets, and New Stars, the Satellites of Jupiter, the Oval Shape* (as it then appeared) *of Saturn, the spots in the sun, and its Turing on its own Axis, the Inequalities and Selelnography of the Moon. the several Phases of Venus and Mercury, the Improvement of Telescopes, and grinding of Glasses for that purpose, the Weight of Air, the possibility or Impossibility of Vacuities, and Natures Abhorrence thereof: the Torricellian Experiment in Quicksilver, the Descent of heavy Bodies, and the degrees of Acceleration therein;* and divers other things of like nature. Some of which were then but New Discoveries, and others not so generally Known and imbraced,as now they are; With other things appertining to what hath been called *The New Philosphy;* which from the time of *Galileo at Florence, and S^r^ Francis Bacon* (*Lord Verulam* in *England* hath been much cultivated in *Italy France, Germany*, and other parts abroad, as well as with us in *England*

About the year 1648, 1649, some of our company being removed to *Oxford* (first D^r^*Wilkins*, then I, and soon after *D^r^ Goddard*) our company divided. Those in *London* continued to meet there as before (and we with them, when we had occasion to be there;) and those of us *Oxford*… continued such meetings in *Oxford*; and brought those Studies into fashion there …

Those meetings in *London* continued, and (after the Kings Return in 1660) were increased with the accession of divers worthy and Honorable persons; and were afterwards incorporated by the name of *the Royal Society, etc.* and so continue to this day.

While the Royal Society of London did indeed grow and continue , the Oxford offspring suffered a less happy fate. After a period of decline and interruption it seems to have flourished again in the 1680's when Wallis was elected its president and tried to establish closer contacts with the mother society and similar groups in Scotland. But Oldenburg, secretary of the London society, initiated publication of the *philosophical Transactions* and thereby provided a more permanent means of scientific exchange than personal intercourse and weekly discussions[20] Wallis made ample use of the *Transactions*; and between 1666 and 1702 he published more than sixty papers and book reviews. The reviews concerned mathematical books, but the papers were more wide-ranging[21] One of the leading scientists among the early fellows of the Royal Society, he was also one of the most energetic in promoting it and helping it to achieve its goals, at a time when not a few of these virtuosi were men without a real understanding of the scientific experiments conducted and of the complex theories behind them.

Wallis' most successful work was his *Grammatica Linguage anglicanae* with a *Praxis grammatica*and a treatise, *De loquela* on the production of the sounds of speech. First published in 1652, the sixth, and last, edition in England appeared in 1765; it was also published on the Continent.

In his *History of Modern Colloquial English*, H. C. Wyld emphasized that Wallis "has considerable merits as an observer of sounds, he has good powers of discrimination, nor is he led astray by the spelling like all the sixteenth–century grammarians, and Bullokar, Gill, and Butler in the seventeenth."[22] He then continued to discuss some of Wallis' more noteworthy observations. A much more detailed account is given in M. Lehnert's monograph.[23]

Wallis' *Treatise of Speech* formed a useful theoretical foundation for h is pioneering attempts to teach deaf-mutes how to speak. In 1661 and 1662 Wallis instructed two young men, Daniel Whaley and Alexander Popham; the latter had previously been taught by Dr. William Holder. Wallis presented Whaley to the Royal Society on 21 May 1662 and in 1670 reported on his instruction of Popham in the *Philosophical Transactions*—failing to mention Holder's teaching.[24] This unfair act eventually (1678) led to a bitter attack by Holder, to which Wallis replied in no less hostile words.[25]

This was one of the many violent quarrels in which Wallis became involved. Although readily inclined to boast of his achievements and to appropriate the ideas of others for further development, he did not always acknowledge his debt to his predecessors. Furthermore he was often carried away by his temper and would reply without restraint to criticism. He thus quarreled with Holder, Henry Stubbe, Lewis Maydwell, and Fermat; and his longest and most bittered dispute, with Thomas Hobbes, dragged on for over a quarter of a century.[26] Despite, or rather because of, his limited mathematical knowledge, Hobbes claimed in 1655 to possess an absolute quadrature of the circle. Somewhat later he also purported to have solved another of the great mathematical problems—the duplication of the cube. Hobbes's chief transgression, however, was in having dared to criticize Wallis' *Arithmetica infinitorum*. The controversy soon degenerated into the most virulent hostility, which gave rise to wild accusations and abusive language. The quarrel ended only with Hobbes's death in 1679. J. F. Scott has suggested that Wallis' relentless attacks may have been partly motivated by Hobbes's increasing influence, especially as author of the *Leviathan* and by Wallis' fear that Hobbes's teachings would undermine respect for the Christian religion.

As keeper of the archives, Wallis rendered considerable services to his university. In his brief account of Wallis' life, David Gregory said, "He put the records, and other papers belonging to the University that were under his care into such exact order, and managed its lawsuits with such dexterity and success that he quickly convinced all, even those who made the greatest noise against this election, how fitt he was for the post."[27] A successor as keeper, Reginald L. Poole, also praised Wallis' work: "He left his mark on the Archives in numerous transcripts, but above all by the Repertory of the entire collection which he made on the basis of Mr. Twyne's list in 1664 and which continues to this day the standard catalogue."[28] Wallis' catalogue was not replaced until even later in the twentieth century. Although not a practicing musician, Wallis composed some papers on musical theory that were published in the *Philosophical Transactions*,[29] and he edited works on harmony by Ptolemy, Porphyrius, and Bryennius. One of his papers reports his observation of the "trembling" of consonant strings, while others contain a mathematical discussion of the intervals of the musical scale and the resulting need for temperament in tuning an organ or other keyboard instrument. In an appendix to Ptolemy's *Harmonics*, Wallis attempted to explain the surprising effects attributed to ancient music (which he rendered in modern notation); and he also dealt with these effects in a separate paper. Finally he contributed extended remarks on Thomas Salmon's *Proposal to Perform Musick, in Perfect and Mathematical Proportions* (London, 1688), the forerunner of which, *An Essay to the Advancement of Musick* (London, 1672), had aroused great interest as well as conflicting views.

Theology . From 1690 to 1692 Wallis published a series of eight letters and three sermons on the doctrine of the Holy Trinity, directed against the Unitarians. In order to explain this doctrine he introduced an analogous example from mathematics: a cubical body with three dimensions, length, breadth, and height; and compared the mystery of the Trinity with the cube:

This longum, latum, profundum, (Long Broad, and Tall), is but *One* Cube; of *Three Dimensions*, and yet but *One Body*, And this *Father, Son* and *HolyGhost*: Three Persons, and yet but One God.[30]

Wallis' discourses on the Trinity met with marked approval from various theological quarters. It was even used in Pierre Bayle's famous *Dictionnairehistorique et critique* in a note to the article on Abailard. Bayle wished to vindicate Abailard of the charge of Tritheism,[31] which had been raised against him for having used an analogy between the Trinity and the syllogism that consists of proposition, assumption, and conclusion. Just as nobody doubts the orthodoxy of Wallis on the basis of his geometrical example, Bayle argued, there was no reason to attack Abailard for his analogy of the syllogism.

Wallis' sermons and other theological works, often praised for their simple and straightforward language, testify that his religious principles were Calvinist, according to the literal sense of the Church of England. He never denied the Puritanism in which he had grown up, although he remained a loyal member of the official church.

From his student days, Wallis sided with the Parliamentarians, and Cromwell is said to have had a great respect for him. As secretary to the Assembly of Divines at Westminster during the Civil War, Wallis became thoroughly familiar with the controversial issues within the Episcopal Church and between the Church and Parliament. Included in his autobiography is a rather long intercalation about this assembly, which was convened to suggest a new form of church government in place of the episcopacy.[32] His interpretation of proceedings carried on half a century earlier might have been somewhat colored by the actual events that followed. The episcopacy was, after all, not abolished; and Wallis had tried to stay on good terms with the bishops and archbishops. Toward the end of the century he strongly opposed the introduction of the Gregorian calendar in England, considering it a kind of submission to Rome. The new calendar was not in fact adopted in Britain until 1752. Some of Wallis' friends and colleagues in the Royal Society exchanged their university posts for careers in the church, but Wallis himself was never given the opportunity. Obviously his trimming politics had made him not totally acceptable to the monarchy, although he did enjoy signs of royal favor. As he himself expressed it, he was "willing whatever side was upmost, to promote (as I was able) any good design for the true Interest of Religion, of Learning, and the publick good."[33]

NOTES

1. C.J. Scriba, "The Autobiography …," 24.

2. J. R. Tanner, ed., *Private Correspondence and Miscellaneous Papers of Samuel Pepys, 1679-1703*, **II** (London, 1926), 257.

3. "S.T.P." is the usual abbreviation for Doctors of Divinity in inscriptions: Wallis was never created professor of theology.

4. J. Wallis. *Algebra, ch. 46.*

5. 5. See H. Wieleitner, "Die Verdienste."

6. For a more detailed description, see Sir T. P. Nunn, "The Arithmetic"; J. F. Scott, *The Mathematical Work*, ch. 4; and D. T. Whiteside, "Patterns of Mathematical Thought in the Later Seventeenth Century," in *Archives for History of Exact Sciences*. **1** (1961), 179 – 388, esp. 236 – 243.

7. See W. Kutta, "Elliptische," and A. Prag. "John Wallis," esp. 391 – 395.

8. D. T. Whiteside, "Newton's Discovery of the General Binomial Theorem," in *Mathematical Gazette*, **45** (1961), 175 – 180; and *The Mathematical Papers of Isaac Newton*. D.T. Whiteside,ed., 1 (Cambridge, 1967), 96 – 111.

9. See G. Wertheim, "P. Fermats Streit…" and J.E. Hofmann, "Neues über Fermats…"

10. See C.J. Scriba, *Studien* …,chs. 2 – 3.

11. See K. Hara, "Pascal et Wallis …," and J. Hofmann and J. E. Hofmann, "Erste Quadratur der Kissoide," in *Deutsche Mathematik*, **5** (1941), 571 – 584.

12. See A. R. Hall, “Mechanics and the Royal Society, 1668 – 1670,” in *British Journal for the History of science*, **3** (1966 – 1967), 24 – 38.

13. See J. F. Scott, “John Wallis.”

14. See G. Eneström, “Die geometrische Darstellung.”

15. See C. J. Scriba, “Neue Dokumente zur Entstehungsgeschichte des Prioritätsstreites zwischen Leibniz und Newton um die Erfindung der Infinitesimalrechnung,” in *Akten des Internationalen Leibniz—Kongresses Hannover, 14.–19. November 1966, II. Mathematik—Naturwissen schaften* (Wiesbaden, 1969), 69–78.

16. See C. J. Scriba, *Studien*, ch. 1.

17. See D.E. Smith, “John Wallis,” and D. Kahn, *The Code breakers* (New York, 1967), 166 – 169.

18. See C. J. Scriba, “The Autobiography.” 38.

19. Different opinions have been expressed as to whether the Royal Society emerged from the London group described by Wallis or from an independent Oxford group in existence before Wallis came to Oxford in 1649. For a champion of the latter view, see M. Purver, *The Royal Society: Concept and Creation* (London, 1967). A brief review of this is C.J. Scriba, “Zur Entstehung der Royal Society,” in *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften*, **52** (1968), 269 – 271. There is an extended debate, in three articles by P.M. Rattansi, C. Hill, and A. R. Hall and M. B. Hall, in *Notes and Records. Royal Society of London*, **23** (1968), 129 – 168, where further references are given. It seems to be without doubt that the London group cannot be ignored. Wallis’ report is taken from “The Autobiography,” 39–40.

20. Wallis’ correspondence with Oldenburg is printed in *The Correspondence of Henry Oldenburg*, A. R. Hall and M.Boas Hall, eds. (Madison, Wis., 1965–).

21. For a not quite complete list of Wallis’ publications in the *Philosophical Transactions of the Royal Society*, see J.F. Scott. *The Mathematical Work*, 231 – 233; paper no. 62 is not by Wallis.

22. H.C. Wyld, *A History of Modern Colloquial English*, 3rd ed. (Oxford. 1936: repr. 1953), 170.

23. M. Lehnert, in *Die Grammatik*, criticizes the older work by L. Morel, *De Johannis Wallisii*, as insufficient. See also A. B. Melchior, “Sir Thomas Smith and John Wallis,” in *English Studies*, **53** (1972), and his review of John Wallis, “Grammar of the English Language,” in *English Studies*. **55** (1974). 83 – 85.

24.*Philosophical Transactions of the Royal Society*, **5** no. 61 (18 July 1670), 1087 – 1097 (pagination repeated).

25. W. Holder. *A Supplement;* J. Wallis, *A Defense*.

26. See J.F. Scott, *The Mathematical Work*, ch. 10.

27. Bodleian Library Oxford, MS Smith 31, p. 58; J. Collier, *A Supplement*.

28. R. L. Poole, *A Lecture on the History of the University Archives* (Oxford, 1912), 25.

29.*Philosophical Transactions of the Royal Society*, **12** , no. 134 (23 Apr. 1677), 839 – 842; **20** , no. 238 (Mar. 1698), 80 – 84; **20** , no. 242 (July 1698), 249 – 256; **20** , no. 243 (Aug. 1698), 297 – 303. See L. S. Lloyd, “Musical Theory in the Early *Philosophical Transactions*,” in *Notes and Records. Royal Society of London*, **3** (1940 – 1941), 149 – 157.

30. Quoted from R. C. Archibald, “Wallis on the Trinity,” 36.

31. See the query by E. H. Neville, “Wallis on the Trinity,” 197, who quotes the 5th ed., I (Amsterdam, 1734), 30. In the new ed. (Paris, 1820), it is I, 59 – 60, note M.

32. See Scriba, “The Autobiography,” 31 – 37.

33.*Ibid*., 43.

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The *Opera mathematica* should not be confused with the *Operum mathematicorum pars prima* and *pars secunda*, published in 1657 and 1656 [sic], respectively. Vol. I contains *Oratio inauguralis: Mathesis universalis, sive arithmeticum opus integrum: Adversus Meibomii De proportionibus dialogum, tractatus elenctibus;* and *M. Mersenni locus notatur*. Vol. II contains *De angulo contactus et semicirculo disquisitio geometrica: De sectionibus conicis, nova methodo expositis, tractatus; Arithmetica infinitorum* (already printed and in some copies distributed in 1655), and the brief *Eclipsis solaris observatio*.

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