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(*b*. Vestby, near Dröbak, Norway, 8 June 1745; *d*. Copenhagen, Denmark, 25 March 1818), *surveying, mathematics.*

Although he was born when Norway was part of Denmark and spent most of his life in Denmark, Wessel is regarded as a Norwegian. (Niels Nielsen, in his *Matematiken i Danmark, 1528-1800* [Copenhagen-Christiania, 1912], gives his birthplace as Jonsrud in Akershus.) His father, Jonas Wessel, was a vicar in the parish of which his grandfather was the pastor; his mother was Maria Schumacher.

After attending the Christiania Cathedral School in Oslo from 1757 to 1763, Wessel spent a year at the University of Copenhagen. In 1764 he began work on the cartography of Denmark, as an assistant with the Danish Survey Commission operating under the Royal Danish Academy of Sciences. He passed th University examination in Roman law in 1778 and became survey superintendent in 1798. He continued as a surveyor and cortographer even after his retirement in 1805, working on special projects until rheumatism forced him to stop in 1812. He was frequently in financial difficulty; but since he would accept no remuneration for special maps of Schleswig and Holstein requested by the French government, the Royal Danish Academy awarded him a silver medal and a set of its *Mémoires* and maps. He was made a Knight of Danebrog in 1815.

Wessel's fame as a mathematician is based entirely on one paper, written in Danish and published in the *Mémoires* of the Royal Danish Academy, that established his priority in publication of the geometric representation of complex numbers. John Wallis had given a geometric representation of the complex roots of quadratic equations in 1685; Gauss had had the idea as early as 1799 but did not explicitly publish it until 1831. Robert Argand's independent publication in 1806 must be credited as the source of this concept in modern mathematics because Wessel's work remained essentially unknown until 1895, when its significance was pointed out by Christian Juel. Despite its lack of influence upon the development of mathematics, Wessel's publication was remarkable in many ways: he was not a professional mathematician; Norway and Denmark were not mathematically productive or stimulating at that time; he was not a member of the Royal Danish Academy (he had been helped and encouraged by J. N. Tetens, councillor of state and president of the science section of the Academy); and yet his exposition was, in some respects, superior to and more modern in spirit than Argand's.

The title of Wessel's treatise calls it an "attempt" to give an analytic representation of both distance and direction that could be used to solve plane and spherical polygons. The connection of this goal with Wessel's work as a surveyor and cartographer is obvious. The statement of the problem also suggests that Wessel should be credited with an early formulation of vector addition. In fact, Michael J. Crowe, in *A History of Vector Analysis* (University of Notre Dame Press, 1967), defines the first period in that history as that of a search for hypercomplex numbers to be used in space analysis and dates it from the time of Wessel, whom he calls the first to add vectors in three-dimensional space. Wessel's first step was to note that two segments of the same line, whether of the same or opposite sense, are added by placing the initial point of one at the terminus of the other and defining the sum to be the segment extending from the initial point of the first to the terminal point of the second. He immediately defined the sum of two nonparallel segments in the same way and extended this definition to apply to any number of segments.

For multiplication of line segments, Wessel drew his motivation from the fact that, in arithmetic, a product of two factors has the same ratio to one factor as the other factor has to 1. Assuming that the product and the two factors are in the same plane and have the same initial point as the unit segment, he reasoned that the product vector should differ in direction from one factor by the same amount by which the other factor differed from unity. Wessel then designated two oppositely directed unit segments having the same origin by +1 and -1, and assigned to them the direction angles 0° and 180°. To unit segments perpendicular to these he assigned the symbols $+\epsilon$ and $-\epsilon$ and the angles 90° and either 270° or -90°. Wessel immediately pointed out that multiplication of these numbers corresponded to addition of their angles and gave a table in which $(+\epsilon) \cdot (+\epsilon) = -1$. He then noted that this means that ε = , and that these operations do not contradict the ordinary rules of algebra. From this and his definition of addition of vectors, Wessel next wrote $\cos v + \epsilon \sin v$ as the algebraic formula for a unit segment and then derived the algebraic formula $(a + \epsilon b) \cdot (c + \epsilon d) = ac - bd + \epsilon(ad + bc)$ for the product of any two segments from the formula for the product of two unit segments derived from the equation $(\cos v + \epsilon \sin v)(\cos u + \epsilon \sin u) = \cos(u + v) + \epsilon \sin(u + v)$, using trigonometric identities.

Thus Wessel's development proceeded rather directly from a geometric problem, through geometric-intuitive reasoning, to an algebraic formula. Argand began with algebraic quantities and sought a geometric representation for them.

Wessel was more modern than Argand in his recognition of the arbitrary nature of the definitions of operations that Argand initially attempted to justify by intuitive arguments. Wessel also sought to extend his definition of multiplication to lines in space. T. N. Thiele's view that Wessel should be credited with anticipating Hamilton's formulation of quaternion multiplication, however, seems to exaggerate the extent of his work, which Wessel himself recognized as incomplete.

Wessel used in his development trigonometric identities that Argand derived by means of his definitions of operations on complex numbers. Argand presented a greater variety of applications of his work, including a proof of the fundamental theorem of algebra and of Ptolemy's theorem. Wessel worked at his original problem of applying algebra to the solution of plane and spherical polygons, after expanding his discussion to include formulas for division, powers, and roots to complex numbers.

Wessel's initial formulation was remarkably clear, direct, concise, and modern. It is regrettable that it was not appreciated for nearly a century and hence did not have the influence it merited.

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In the meantime, Sophus Lie published a reproduction in *Archiv for mathematik og naturavidenskab*, **18** (1896). The English trans. by Martin A. Nordgaard of portions of the original paper first appeared in D. E. Smith, ed., *A Source Book in Mathematics* (New York, 1929), 55–66, repr. in H. O. Midonick, ed., *The Treasury of Mathematics* (New York, 1965), 805–814.

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Much information on Wessel's cartographic and surveying activity is in Otto Harms, "Die amtliche Topographie in Oldenburg und ihre kartographischen Ergebnisse," in *Oldenburger Jaharbuch*, **60** , pt. **1** (1961), 1–38.

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