Weyl, Hermann Claus Hugo | Encyclopedia.com

(*b*. Elmshorn, Germany, 9 November 1885;

d. Zürich, Switzerland, 8 December 1955), *mathematics, mathematical physics, philosophy*. For the original article on Weyl, see *DSB* vol. 14.

The original *DSB* article focuses on Weyl's contributions to analysis, analytical number theory, and the representation theory of Lie groups. Other fields of study are mentioned only in passing. This appendix presents some aspects of Weyl's ideas in the foundations of mathematics, differential geometry, and mathematical physics.

It was already mentioned in the main article that Weyl changed his research orientation under the impact of the experience of World War I and the crisis in its aftermath. As Weyl later said, he looked for safeness in his main fields of research. He turned toward the foundations of analysis and to the newly founded theory of general relativity with its strong appeal to philosophically intriguing interrelations of mathematics and physics. In 1918 Weyl published two books on these two rather distinct topics: *The Continuum* and *Space, Time, Matter*.

Constructivist Sympathies and Transitional Turn to Intuitionism. In *The Continuum* Weyl proposed a constructivist arithmetical foundation of analysis or, more properly, for a part of analysis. This began his public intervention into the foundations of mathematics, in clear opposition to David Hilbert's program of a purely formalistic axiomatic foundation. He sketched how a reduced part of real analysis could be secured by constructions in a semiformalized arithmetical language, respecting the restrictions of predicativity, although the constructivist (reduced) continuum stood in stark contrast to the intuitive continuum and physical ideas of space-time. That seems to have contributed to his change of mind already a year after the publication of the book. He started to support Luitzen Egbertus Jan Brouwer's more radical intuitionistic program and attacked Hilbert's foundational views even more strongly in a programmatic article on the recent foundational crisis of mathematics (1921). But his initial hopes of a unified intuitionist foundation of mathematics, which would also serve the purpose of a deeper understanding of the physical space-time were not fulfilled.

Weyl soon realized that the intuitionist foundation of mathematics led to undesirable technical complications and relied too much on a kind of evidence that was difficult to reconcile with his interests in mathematical physics. Hilbert's foundational program seemed to be better adapted to providing the symbolical tools needed in most advanced contemporary mathematical physics, in particular the rising quantum mechanics. In the late 1920s Weyl showed more sympathy with Hilbert's foundational view, but could not avoid being irritated by Kurt Gödel's negative result of 1930 for Hilbert's strategy of founding modern analysis on a finitistic consistency proof. He even returned after World War II to a weak preference of his arithmetical constructive approach of 1918.

Linking Geometry and Physics. Parallel to the work on the foundations of analysis and the concept of the continuum, Weyl started to analyze and to deepen the links between differential geometry and the newly established general theory of relativity. *Space, Time, Matter* was one of the first text books on the subject and among the most influential ones over decades to come. It was revised in five successive editions until 1923. In the first part of this book, Weyl gave an up-to-date introduction to Riemannian and Lorentzian geometry, which made it an important introductory monograph to modern differential geometry. At the time when the book first went to print, Weyl developed a generalization of Riemannian geometry, which was more firmly built on what he called a "purely infinitesimal'' point of view. He avoided a direct comparison of lengths and other quantities at different points of the manifold and introduced the seminal concept of point-dependent gauges and a *gauge field*(a scale connection) to allow the transfer of metrical concepts from one point to another. The scale connection had formal properties that made it appear as an appropriate mathematical expression for the potential of the electromagnetic field. It became the embryonic core of a tradition of physical field theories which—in a more general form—became important in high energy physics of the last third of the twentieth century.

The gauge geometry of 1918 served Weyl as a starting point for an attempt at unifying electromagnetism and gravity. In this respect Weyl took up and tried to improve Hilbert's program to derive the basic matter structures from purely field theoretical considerations (extending an approach of Gustav Mie; see Weyl, "Gravitation und Elektrizität"). For about two years he believed he had found a clue to the problem of understanding matter by such a classical field theoretical approach. In the third edition of *Space, Time, Matter* he added a passage on a gauge geometric generalization of semi-Riemannian geometry and his unified field theory. But already a year later he realized that the derivation of matter structures from pure field theory had no chance for success.

Space, Time, Matter was only a small part of the contributions to the interchange between differential geometry and general relativity. Weyl pursued it in his own peculiar way based on a broad conceptual and philosophical view. He studied the interrelation of conformal and projective differential geometric structure and realized that both together specify a (Weylian) metric uniquely. An even deeper conceptual approach was contained in his *Mathematische Analyse des Raumproblems* (1923; Analysis of the Problem of Space), in which the older space problem of the nineteenth century was transplanted into the context of purely infinitesimal geometry. Here Weyl (sketchily) introduced concepts of infinitesimal group operations and connections. Independently developed by Élie Cartan in 1922, they were later turned into the language of fiber bundles and led to the study of geometries characterized by gauge structures.

Group Theory and Physics. The study of differential geometry and the problem of space led Weyl to studying the representation theory of Lie groups. In the mid-1920s he delved into what became his most influential contributions to pure mathematics, the study of the representations of semisimple Lie groups (1925–1926). Extended and refined, this work formed the core of his later book *The Classical Groups* (1939), written as a harvest of his work and his lecturing activities on this topic during his Princeton years.

During his work on the representation theory of Lie groups, Weyl actively followed the turn toward the new

quantum mechanics of Werner Heisenberg, Max Born, and Pascual Jordan and started to explore the possibilities opened up by the interplay of infinitesimal and finite group operations in quantum mechanics. In 1927–1928 he gave a lecture at Zürich Eidgenössische Technische Hochschule (ETH) on the topic, which gave rise to his second, again very influential, book on mathematical physics, *The Theory of Groups and Quantum Mechanics*(1928; Engl. trans. 1931). Here he emphasized the conceptual role of group methods in the symbolic representation of quantum structures, in particular the intriguing interplay of representations of the special linear group and permutations groups. In this interplay he also saw the mathematical clue to understanding the phenomenon of spin coupling, studied at the time by some of the young physicists turning toward quantum chemistry (Fritz London, Walter Heitler, and others). Moreover, Weyl explored the central role of the discrete symmetries (parity, charge conjugation, and time inversion) in the first steps toward a quantized version of electrodynamics, thus anticipating structural elements that turned into important questions for physics three decades later.

Not covered in the book, but published separately, was a second step for his gauge theory of the electromagnetic field. Quantum theorists had proposed reconsidering the gauge idea for the phase of wave functions rather than for scale gauge as in Weyl's original idea of 1918. Weyl (1929) took up the proposal and explored it after the rise of Paul Dirac's spinor fields for the electron (as was done similarly and independently by Vladimir Fock). He came to the conclusion that the new context demanded considering symmetry extension of the Lorentz group by unitary transformations in $U(1)$. That gave rise to a modified gauge theory of electromagnetism, which was endorsed by leading theoretical physicists (among them Wolfgang Pauli, Erwin Schrödinger, and Fock) and served as a starting point for the next generation of physicists, who molded the symbolic frame of gauge field theories in the 1950s and 1960s.

Weyl's Philosophical Writings. Hermann Weyl was not only a philosophically interested researcher in mathematics and physics. He had close, but changing relations to philosophers such as Edmund Husserl, Fritz Medicus, and later to the existential philosophies of Karl Jaspers and Martin Heidegger. He gave active literary expression to his philosophical reflections of scientific activity in many of his publications. Most influential was his philosophical handbook *Philosophy of Mathematics and Natural Science*, originally published in German in 1927, and translated into English in 1949. It became a classic in the philosophy of science. A central topic of his thought was the mode and the role of symbolic construction in scientific knowledge of the world. It reappeared in his later reflections on twentieth-century science (1953). Like others of his generation, Weyl was shocked not only by the atrocities of the Nazi regime but also by the destructiveness of the nuclear weapons developed during the war. He considered it as a kind of hubris of modern techno-scientific culture, which had started to step beyond the circle of activities that can be accounted for. He hesitatingly hoped that it might perhaps be contained, if at all, by a slow rise of moral awareness of users and producers of scientific knowledge.

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