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(*b*. Czestochowa, Poland, 2 May 1902; *d*. London, England, 5 September 1959)

mathematics.

Zarankiewicz's contributions to mathematics were in topology, the theory of graphs, the theory of complex functions, number theory, and mathematical education. In addition, he founded and for several years headed the Polish Astronautical Society.

Zarankiewicz was born and raised in a moderately well-to-do family. After obtaining his baccalaureate degree in 1919 in Bedzin, near Czesto-chowa, he studied mathematics at the University of Warsaw. He was awarded a Ph.D. there in 1923 for a dissertation (published in 1927) on the cut points in connected sets, and in 1924 he was made assistant to the professor of mathematics at the Warsaw Polytechnic. In 1929, following publication of his habilitation essay on a topological property of the plane (concerning the mutual cuttings of three regions and three continua), Zarankiewicz became *Dozent* in mathematics. He spent the academic year 1930-1931 working with Karl Menger at Vienna and with Richard von Mises at Berlin, where he collaborated with Stefan Bergman and other mathematicians. Upon returning to Warsaw, Zarankiewicz was assigned to teach a course in rational mechanics at the Polytechnic and courses in mathematics and statistics at the High Agricultural College. In 1936 he was invited to the University of Tomsk to lecture for a semester on conformal mappings, particularly on several problems that he had solved. He was named substitute for the professor at the Warsaw Polytechnic in 1937; but his nomination to the professorship (1939) was not confirmed until after the war.

During the German occupation of Poland, Zarankiewicz taught mathematics, clandestinely, to underground groups of high school and college students. In 1944 he was deported to a forced labor camp in Germany. Returning to the ruins of Warsaw in 1945, he resumed his courses at the Polytechnic and continued them for the rest of his life.

Zarankiewicz was appointed full professor in 1948 and spent several months of that year working at Harvard and at several other American universities. From 1949 to 1957 he supervised in Poland the Mathematical Olympics for high school students, remaining a member of its central board thereafter. At the same time he was a member of the editorial committees of *Applied Mechanics Reviews* and *Matenmhvka,* a Polish journal written primarily for secondary-school teachers. From 1948 to 1951 Zarankiewicz headed the Warsaw section of the Polish Mathematical Society. He maintained a long-standing active interest in as tronautics and in the organization in Poland of a society founded in 1956 devoted to this field. Zarankiewicz died in London while presiding over a plenary session of the Tenth Congress of the International Astronautical Federation, of which he was vice–president. His funeral was held in Warsaw, and one of the city's streets is named for him. Zarankiewicz's topological writings deal mainly with cut points, that is, those points which disconnect (and locally disconnect) the continua, and with the continua that disconnect the spaces. In 1926 he showed, among other things, that if C is a locally connected continuum (a continuous image of an interval), then the set $_T$(C) of all the cut points of C possesses a special structure, which he characterized. In particular, the closures of the constituents of $_T$(C) are dendrites, that is, one-dimensional, acyclic, locally connected continua. In his doctoral dissertation Zarankiewicz introduced and investigated the important notion of the continua of convergence. He characterized the locally connected continua by the equivalence, for their closed subsets F, of the connectedness and of the semicontinuity of the set between all the pairs of its points. He also characterized the dendrites C by the structure of the set and-independently of Pavel Urysohn-the hereditarily locally connected continua by the absence, among their subsets, of continua of convergence (1927). In 1932 and in 1951 Zarankiewicz resumed and extended the study of the set $_T$(C) by a series of remarkable theorems. In particular, he gave a new definition of the cyclic element in G. T. Whyburn's sense, for the locally connected continua.

Zarankiewicz's studies of the cutting of spaces by continua were concerned especially with local cuttings of the plane or, which is topologically equivalent, of the sphere. In publications of 1927, 1929, and 1932, he established interesting topological characterizations of the circumference, of the straight line, and of several other lines with the aid of the number of their connectedness points, another notion that he originated. These theorems, which are more quantitative than qualitative, reflect Zarankiewicz's tendency to seek numerical solutions in every field of mathematics in which he worked. For example, generalizing R. L. Moore's theorem concerning triods in the plane, he showed that in Euclidean spaces of more than two dimensions, every family of disjoint continua each of which locally cuts the space at a point that cuts locally itself *(doppelt zerlegender Punkt)* is, at most, countable (1934).

Zarankiewicz's last publication in topology (1952), written with C. Kuratowski, deals with a problem that is still unsolved; Given n disjoint regions in the plane (or on the sphere) with connected boundaries $R_1, R_2, \ldots, R_n$ and *k* continua $C_1, C_2, \ldots, C_k$, each of which meets each of these regions, what is the minimum number $s_{k,n}$ of couples i,j such that C_i cuts R_j (where $i = 1,2,$

…, *k* and j = 1, 2, …, n)? Zarakiewicz's conjecture is that $s_{k,n}$ = (k–2)(n–2) for all integers k≥2 and n≥2. In 1928 he showed that $s_{3,3}$ –1 and, in the joint work of 1952, that the presumed formula holds for all k≥4 and n≥4.

In theory of graphs Zarankiewicz developed, among other topics, a criterion for the existence of a complete subgraph of highest possible order in every graph of a given order in which the minimum number of edges arising from a vertex is sufficiently high (1947). Later, Pál Turán improved this criterion and devoted an interesting study to it. Zarankiewicz, in publications of 1953 and 1954, solved, independently of K. Urbanik, a problem posed by Turán by showing that if A and B are finite sets of the plane, composed of a and b points, respectively, and if each point of A is joined by a simple arc to all the points of B in such a way that outside of A and B every point of intersection belongs to two arcs, then the number of these points of intersection is at least equal to

and that this minimum is attained. In 1951 Zarankiewicz posed the problem of finding the least number k_j (n) such that every set of k_j(n)points of a plane net of n^2 points (where nɰ3) contains j^2 points on *j* lines and on *j* columns. Several other authors have subsequently treated this problem.

Zarankiewicz's works on complex functions (1934, 1938, 1956) deal principally with the kernel *(Kernfunktion)* and its applications. Given a complete system where *v=1, 2,…, of orthonormal analytic functions in a domain **D** of the complex plane of z=x+iy*, the function is called the kernel of the domain **D** . It is known that it exists for all z and of **D** and depends only on **D** ; that if **D** is simply connected–that is, if the boundary of **D** is connected–the–function transforms **D** onto the interior of the circle , where c is a constant; that the function $\mathbf{K}_D$ is a relative invariant–that is,

for every analytic function z*z mapping the domain **D** of the complex z–plane onto the domain **D** * of the complex z*–plane–and, consequently, that the formula

represents the square of the length of the line or element of an invariant metric; that the curvature of the metric

is an absolute invariant of the conformal mappings; and that if the boundary of **D** is connected, $J_D(z, z)$ is a constant.

Zarankiewicz showed that when the boundary of D is doubly connected–that is, has exactly two components–the function $J_D(z, z)$ is no longer constant; in this case he represented it by doubly periodic functions. He established a criterion for recognizing, with the aid of this representation, when it is that a boundary domain with two components can be transformed conformally into another domain of this type. (**P. P** . Kufarev determined the minimum domain, that is, into which every domain of which the boundary consists of two components is transformable by the function W[z].) This Zarankiewicz result played an important role in the development of the theory of the kernel and its generalizations to several variables, notably to pseudo–conformal transformations in space of more than three dimensions.

In number theory Zarankiewicz devoted particular attention to what are called triangular numbers–that is, triplets of integers equal to the lengths of the sides of right triangles (1949). His ideas inspired a work by Sierpiński (1954), at the end of which the author reproduces an ingenious example, inspired by Zarankiewicz, of a decomposition of the set of natural numbers into two disjoint classes, neither of which contains a triplet of consecutive numbers or an infinite arithmetic progression.

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