Do I not actually believe that Hilbert's list had a great role in the mathematics of this century. It certainly was psychologically important for many mathematicians. For example Arnold told that while being a young graduate student he had copied the list of Hilbert's problems and always kept it with him. Later I heard a story that Goldsmith learns about that, actually he mock Arnold on this. Arnold saw problem solving as an essential part of mathematical creativity. He was the only way we know the truth of our thoughts; that is actually the only way of mathematics can die as an accepted part of the culture of humanity. I think, in our century, it was played by set theory. Initially conceived by Cantor as a new chapter of mathematics, "the theory of infinity", set theory, gradually changed its status and developed into the universal mathematical language. It was understood that starting with a rather short list of basic terms and operations, one could generate recursively the linguistic constructions which apparently conveyed equally well the intuition of the founding fathers of calculus, probability, number theory, topology, differential geometry and what not. Thus the whole mathematical community acquired a common idiom. Moreover, allowing the clear distinction between the logical and the analytic-theoretic content of the mathematical constructions on the one hand, and their flexible linguistic expression (notation, formulas, theorems) on the other, set theory greatly simplified the interaction between the right and left brain of the mathematical mind as the whole new function of the set-theoretic language became the basis for the development of new technical tools, for the formulation of new mathematical notions (the notion of the zero, the notion of the genericity), the development of the foundations of mathematics was connected first of all with external social phenomena: the rapid growth of the scientific community in general and the ground for a new generation of extremely difficult problems fell into the span of thirty years. And right brain, the geometric intuition to new domains. I have in mind topology, algebraic geometry, quantum field theory, quantum cohomology, quantum computing.


YURI I. MANIN

‘Good Proofs are Proofs that Make us Wiser’

This year’s International Congress is the last ICM in this century. Do you think a Hilbert is still possible? Or are we witnessing an era of more localized problems?

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Are there certain specific topics that come to your mind, in which our century was really a top level?

In the 18th and 19th century mathematical language was much vaguer than we are accustomed to. I think the 20th century started with rethinking the basis. When the classics were clear enough there was a great search of technical methods of incredibly strong which led to the creation of powerful tools allowing us to develop and expand our geometric intuitions. The number of mathematicians as topologists and algebraic geometry. As soon as the technical development was accomplished, the solution of several of very difficult problems and the development of new mathematical tools: Weil conjectures, falling off the Mordell conjecture, Wiles’ proof of Fermat. All of them could have been done in the last century just because mathematics was not developed enough.

Some people — some of them mathematicians — proclaim the end of proof, partly in view of the universal availability of computers. How would you comment on this?

If you are speaking of mathematics without proofs you are speaking of something intrinsically contradictory. The proof cannot die.

When one considers the question of how many mathematical notions changed in the last twenty years in a way that the new notions are quantum versions of the old ones — it is amazing. Look at quantum gravity! We have quantum cosmology, quantum computers, quantum cryptography and quantum computing in physics. In my opinion, the mathematics of the last hundred years did not produce anything comparable to quantum theory or general relativity in terms of the resulting change of our total world perception. But I do believe that without the mathematical tools that physicists invented with fantastic intuition and which they used in a very stimulating but somewhat careless way from the point of view of a pure mathematician.

How do you think the 20th century will be looked at from an historical point of view? Was it an important century?

I think so. Mathematics of this century succeeded in harmonizing and unifying diverse fields on a scale probably never seen before. The most prominent role in this unification was played by set theory. Initially conceived by Cantor as a new chapter of mathematics, "the theory of infinity", set theory, gradually changed its status and developed into the universal mathematical language. It was understood that starting with a rather short list of basic terms and operations, one could generate recursively the linguistic constructions which apparently conveyed equally well the intuition of the founding fathers of calculus, probability, number theory, topology, differential geometry and what not. Thus the whole mathematical community acquired a common idiom. Moreover, allowing the clear distinction between the logical and the analytic-theoretic content of the mathematical constructions on the one hand, and their flexible linguistic expression (notation, formulas, theorems) on the other, set theory greatly simplified the interaction between the right and left brain of the mathematical mind as the whole new function of the set-theoretic language became the basis for the development of new technical tools, for the formulation of new mathematical notions (the notion of the zero, the notion of the genericity), the development of the foundations of mathematics was connected first of all with external social phenomena: the rapid growth of the scientific community in general and the ground for a new generation of extremely difficult problems fell into the span of thirty years. And right brain, the geometric intuition to new domains. I have in mind topology, algebraic geometry, quantum field theory, quantum cohomology, quantum computing.

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Why are you so pessimistic?

Contemporary public debate on science and human values.

For this reason, it is essential for the content of the scientific discourse. Everything that is essential, is carried out either by long lists of more or less well structured data, or by mathematics. For this reason, it is also important to understand the limits. With the advent of Galileo, Kepler and Newton, the natural language in sciences was relegated to the role of a high level mediator between the actual scientific knowledge and the essential of cultural traditions. Using the natural language in studying and teaching sciences, we bring with it our values and meanings.

Natural language is an extremely flexible tool of communicating human knowledge. It is the language of poetry and religion, of seduction and of love. With all its power, natural language is an essential tool of expressing one's emotions and enforcing one's will, of communicating human culture. We are not machines. Natural language is what separates us from the machines.

In my opinion, the basis of all human culture is language, and mathematics is only a special kind of language. It is a kind of language that is extremely flexible, and is extremely powerful. It is a kind of language that is essential for the content of the scientific discourse. It is a kind of language that is essential for the understanding of the natural world.

Mathematics is not only understanding of formulas on a piece of paper. There is nothing useful about it. It becomes useful if it is implemented in something else. For example, in quantum mechanics (or chips or whatever), it is only understanding of formulas on a piece of paper. The point is that there is an inherent weakness in trying to justify one's concerns by saying that mathematics is not useful if it is not implemented in something else. It is the problem of money in terms of allocating limited resources.

Applications ask for and get much more money than pure mathematics. But I think that pure mathematics is essential for the content of the scientific discourse.

Let us go back to the question of applied mathematics. Isn't it true that mathematics is not as isolated as it is so often portrayed, and the content of the scientific discourse is not as isolated as it is so often portrayed?

Let me ask you a question about mathematics internally. In recent years, the mathematical community seems to emphasize applications. Do you think that pure mathematics will have problems, as compared to applied mathematics? Do you have the impression that the money will go in the future only to those fields?

Should the mathematicians go on the offensive? Should they step out into the world and sell culture? Probably must try to prove that they are important, but I think it is difficult. How could one possibly try to prove that mathematics is important?

Perhaps we have problems, as compared to applied mathematics? Do you have the impression that the money will go in the future only to those fields?

Let me ask you a question about mathematics internally. In recent years, the mathematical community seems to emphasize applications. Do you think that pure mathematics will have problems, as compared to applied mathematics? Do you have the impression that the money will go in the future only to those fields?

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Mathematicians don't need and don't use mathematics. It is a problem of the public attention and the public scale of values. I see the growing estrangement of our society from the scientific discourses. It is not our fault to not spend on mathematics, probably on universities in general. Mathematics — if it will be a victim — will be a victim of the decrease of the funding of the foundations of pure mathematics. But, surely, I do think that there will be a continuous shift to applications in terms of the quantitative resources allocated to applications, and the attractiveness of this kind of occupation is much more to the applied mathematicians. The old system of mathematical simulation — computers at large, database programs and things like that. I have once translated a famous Russian book on mathematics from Russian into English. It was a book dedicated to Al'khorezmi. Knuth started his talk with a funny statement. In his opinion, the primary importance of computers for the mathematical community is that those people finally took to mathematics who were interested in mathematics but had an algorithmic sort of mind. Now they were able to do what they wanted. Before that, this was a problem of the human mind. Now it is a problem of the computers. And the community of future potential mathematicians there is a subcommunity whose minds are better for knowing computers than for proving theorems. In the last century many mathematicians probably would have proved theorems but nowadays they do not have a great success. Just recently, there was a notice in the newspaper that a computer has proved a conjecture. And the proof is huge. And if it is not a proof by hand I remain as stupid as I was before. How could Rembrandt have argued against the fact that he was dying in total misery as a poor man? How could he argue? I don't really know what mathematics is all about. But this is so with culture, because in the same way, we don't really know what Rembrandt's picture is all about, why he portrayed persons — as he did — an old man and background. Why is it important? Isn't that the problem of money in terms of allocating limited resources?

What do you think is the cultural role of mathematics?

In my opinion, the basis of all human culture is language, and mathematics is a special kind of language. It is a kind of language that is extremely flexible, and is extremely powerful. It is a kind of language that is essential for the content of the scientific discourse. It is a kind of language that is essential for the understanding of the natural world. Natural language is not very well fit for acquiring, organizing, and keeping our growing understanding of the natural world. It carries in a kind of primitive form of metaphorical thinking. Aristotle was arguably the last great mind that stretched this capability of language to its limits. With the advent of Galileo, Kepler and Newton, the natural language in sciences was relegated to the role of a high level mediator between the actual scientific knowledge and the essential of cultural traditions. Using the natural language in studying and teaching sciences, we bring with it our values and meanings, prejudices, poetical imagery, passion for power and trickster's skills, but nothing really essential for acquiring, organizing, and understanding of the natural world. Everything, mathematical, is carried out either by long lists of more or less well structured data, or by mathematics. For this reason, I believe that these two communities are influenced by different merits and my life-long preoccupation with mathematics in the capacity of researcher and teacher still remains an advertisement by a pure mathematician.

However, I do not believe that I can convincingly defend this conviction in the context of contemporary public debate on science and human values.

Why are you so pessimistic?
I will start explaining my pessimism by reminding that in the current usage “culture” became a profoundly self-referential word. Namely, it is taken for granted that any definition of culture is determined by the pre-existing cultural background, even if the latter is not made explicit. This means that no objective account and evaluation of culture is possible. Furthermore, any statement about culture that becomes authoritative changes the public image of culture and thus changes the culture itself. Most importantly, the modern discourse on culture is largely subordinate to the political discourse. We were less aware of all this when four decades ago, C. P. Snow launched the discussion of the “two cultures”. Basically, Snow was worried by the fact that in his milieu the scientific knowledge was not considered as an organic part of the education of a cultured person, as opposed to the Greeks and Shakespeare. Moreover, one could openly and even boastfully acknowledge his or her ignorance of basic laws of physics without damaging his or her image as a cultured person. Snow saw this as a result of the distorted public perception of what constituted the actual content of culture and hoped that public debate and reformed education could help to restore the balance.

Is the thesis of the two cultures still relevant?

The relevance of his observation for us depends on our ability to identify ourselves with respect to his idealized Culture with capital C, embracing Homer and Bach, Galileo and Shakespeare, Tolstoy and Einstein. I am afraid that this ability is largely lost. In fact, the popular idea of multiculturalism creates the image of many equally valid cultures. Grand culture of European origin and/or cultivation is put on a par with other regional cultures and is diminished in stature by such pejorative connotations as cultural imperialism and eurocentrism. Environmentalists blame science and technology for the destructive uses we made of them, thus further diminishing their cultural appeal. Ironically, the same arguments that scientists employed in order to justify their occupation, are now turned against them. Deconstructionist and postmodern trends of discourse put in doubt the basic criteria of recognizing the scientific truth going back at least to Galileo and Bacon, and try to replace them by wildly arbitrary intellectual constructions. In this way many of the influential thinkers do not just ignore but aggressively dismiss the scientific counterpart of the contemporary culture. I may (as I do) find this situation deplorable, but I cannot realistically count on an improvement in the foreseeable future.

Coming back to the future of mathematics, do you personally have a theory for which you say: “If I live long enough, this is what I would like to see.”?

This I do not know for the following reason: During my scientific career I have changed my subjects several times and not so much because I found something more interesting than something else. Basically I find everything very interesting, but there is no possibility to do everything at the same time. The second best strategy is to try mastering several fields in turn. Two main things I was always interested in were number theory on the one hand and physics on the other. So I think in both domains I always tried to use the intuition developed in both domains. Understanding problems in number theory helped me to understand problems in physics and vice versa. On my private list of values a place of honor is held by the Renaissance term “varietà”—richness of life and world matched with variety of experience and thought, achieved by great minds which we try to emulate.

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