JACOB BERNOULLI (January 06, 1655 – August 16, 1705)

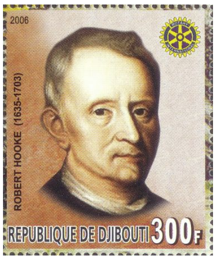
by HEINZ KLAUS STRICK, Germany

When in 1567, FERNANDO ÁLVAREZ DE TOLEDO, THIRD DUKE OF ALBA, governor of the Spanish Netherlands under KING PHILIPP II, began his bloody suppression of the Protestant uprising, many citizens fled their homeland, including the BERNOULLI family of Antwerp. The spice merchant NICHOLAS BERNOULLI (1623–1708) quickly built a new life in Basel, and as an influential citizen was elected to the municipal administration. His marriage to a banker's daughter produced a large number of children, including two sons, JACOB (1655–1705) and JOHANN (1667–1748), who became famous for their work in mathematics and physics.

Other important scientists of this family were JOHANN BERNOULLI's son DANIEL (1700–1782), who as mathematician, physicist, and physician made numerous discoveries (circulation of the blood, inoculation, medical statistics, fluid mechanics) and nephew NICHOLAS (1687–1759), who held successive professorships in mathematics, logic, and law.

JACOB BERNOULLI, to whose memory the Swiss post office dedicated the stamp pictured above in 1994 (though without mentioning his name), studied philosophy and theology, in accord with his parents' wishes. Secretly, however, he attended lectures on mathematics and astronomy.

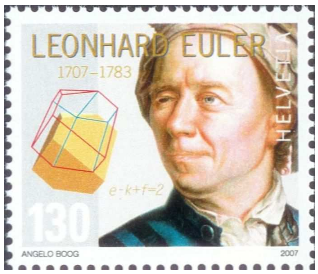
After completing his studies at the age of 21, he travelled through Europe as a private tutor, making the acquaintance of the most important mathematicians and natural scientists of his time, including ROBERT BOYLE (1627–1691) and ROBERT HOOKE (1635–1703). Seven years later, he returned to Basel and accepted a lectureship in experimental physics at the university.



At the age of 32, JACOB BERNOULLI, though qualified as a theologian, accepted a chair in mathematics, a subject to which he now devoted himself entirely. He also encouraged his brother JOHANN, his junior by thirteen years, who was studying medicine at his parents' wish, to take an interest in mathematics.

JACOB BERNOULLI applied the principle of induction as a method of proof, and in his investigation of series, used the inequality that today is known as Bernoulli's inequality: For $x > -1$ ($x \neq 0$), one has $(1+x)^n > 1 + n \cdot x$.

He also studied infinite series, proving that the harmonic series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...$ grows without bound and that the sum of the reciprocals of squares of integers is bounded, satisfying the inequality $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+... < 2$; that is, the series is convergent.



It was LEONHARD EULER (1707–1783), who became interested in mathematics through lectures given by JOHANN BERNOULLI, who was the first to prove that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \approx 1{,}645$.

Although at first he had some difficulties with the theory of GOTTFRIED WILHELM LEIBNIZ (1646–1716), he applied the differential calculus with great success and published papers on the calculation of tangents and surface areas.

In 1690, he succeeded in solving a problem posed by LEIBNIZ using the differential calculus: what is the curve along which a body falls with constant speed (the so-called isochrone)?

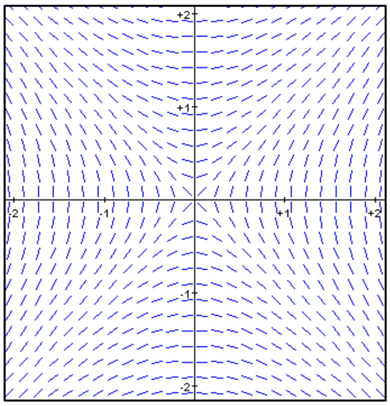
In his article he was the first to speak of an integral calculus, after which LEIBNIZ adopted the term "integral" in his writings.

Equations representing the relationship between one or more quantities and their rates of change, so-called differential equations, arise frequently in physics. Some of these can be solved using the method of *separation of variables* (an idea originating with JACOB BERNOULLI).

For example, the relationship $y' = \frac{x}{y}$ between the variables x and y und the derivative of the latter, becomes, after rearrangement and integration, $yy' = x$ and hence $\int y dy = \int x dx$, that is,

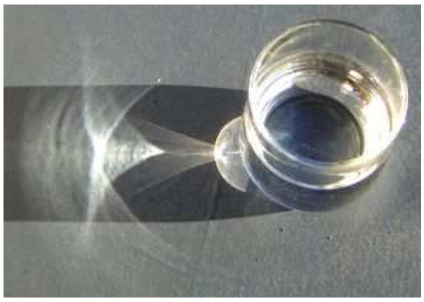
$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$, or $y^2 - x^2 = 2C$,

which is the equation of a hyperbola; in the figure can be seen the associated field of tangents of the differential equation (an idea originating with JOHANN BERNOULLI):



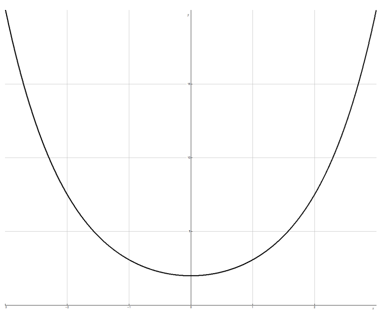
At the lattice points of the coordinate system are shown the tangents, whose slope can be calculated from the differential equation.

Together, the Bernoulli brothers studied caustics (envelopes of reflected rays) and in this connection, derived a formula for the osculating circle of a curve; for a differentiable function, its radius r can be calculated as follows: $r = \frac{(1+f'(a)^2)^{3/2}}{f''(a)}$

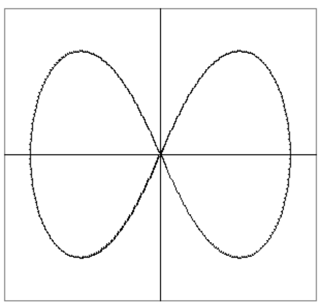


(picture of a caustic from Wikipedia).

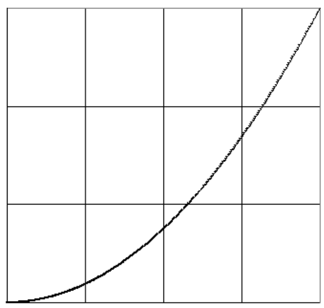
Other papers prove that JACOB BERNOULLI knew how to apply the new calculus:



Catenary



Lemniscate



Brachistochrone

- What is the curve that takes the form of a chain hanging from two points of equal height?
  The solution is the so-called *catenary*: $f(x) = \frac{a}{2} \cdot (e^{\frac{x}{a}} + e^{-\frac{x}{a}})$
- What is the geometric locus of all points such that the product of their distance from two fixed points is constant?
  The solution is the *lemniscate*: $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$
- Through what curve must two points at different heights be joined so that a body falling without friction travels from the upper point to the lower point in the least amount of time? The *brachistochrone* curve was found as a solution by NEWTON, LEIBNIZ and L'HOSPITAL.

Together, the brothers played a significant role in the dissemination and development of the calculus. However, beginning with little sensitivities and petty jealousies that made working together difficult, over the course of years there developed an implacable hatred that did not remain hidden from other scientists.

The ambitious JOHANN BERNOULLI left Basel to take up a professorship of mathematics in Groningen. Only after his brother's death did he return to Basel, succeeding him in his chair at the university.

The year 1713 saw the beginning of the priority dispute between LEIBNIZ and NEWTON over who had "invented" the differential calculus, and JOHANN BERNOULLI took LEIBNIZ's side.

From correspondence between JACOB BERNOULLI and CHRISTIAAN HUYGENS (1629–1695) on games of chance there arose a comprehensive theory of probability. JACOB's book *Ars conjectandi* (The art of guessing) was brought out posthumously by his nephew NICHOLAS in 1713. JACOB BERNOULLI's work generalized the results that HUYGENS had collected in his 1657 *De ratiociniis in ludo aleae* (On calculation in games of chance). In particular, he explored combinatorial problems systematically and showed how their solution could be applied to games of chance.

The last section contains the "golden theorem," which since SIMEON DENIS POISSON is also knows as BERNOULLI's *law of large numbers*:

A random trial is repeated n times under the same conditions, the results of each trial being assumed independent of the results of previous trials (so-called BERNOULLI trials). The probability that a particular event A ("success") will occur each time a trial is held is denoted by p.

Then if X is the number of successes, the probability that $X = k$ is given by the following:

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

As the number of trials increases, the relative frequency X/n stochastically approaches the probability p of the event; that is, for every $\varepsilon > 0$, one has $\lim_{n\to\infty} P(\left|\frac{X}{n} - p\right| < \varepsilon) = 1$

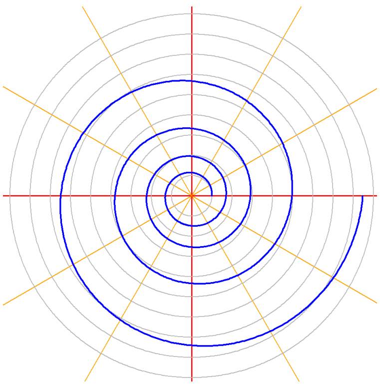
BERNOULLI's law of large numbers is shown on the depicted Swiss postage stamp in the more general form $\frac{1}{n} \cdot (x_1 + ... + x_n) \to E(X)$ and is represented graphically as well: the sequence of arithmetic means $x_1, ..., x_n$ of the results of the trials approaches the expectation $E(X)$ of the associated random variable.

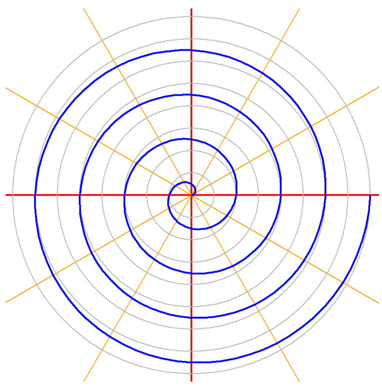
In his study of sums of powers, JACOB BERNOULLI encountered certain numbers that today are known as the BERNOULLI *numbers* B_n. These appear in the series expansion of $f(x) = \frac{x}{e^x - 1}$ around the point 0. The function and its derivatives are not thus defined at the point 0, but f(x) can be continuously extended to this point, and one has $f(x) = \sum_{n=0}^{\infty} B_n \cdot \frac{x^n}{n!}$ with

$$B_0 = 1; B_1 = -\tfrac{1}{2}; B_2 = \tfrac{1}{6}; B_3 = 0; B_4 = -\tfrac{1}{30}; B_5 = 0; B_6 = \tfrac{1}{42}; B_7 = 0; B_8 = -\tfrac{1}{30}; B_9 = 0; B_{10} = \tfrac{5}{66}; ...$$

For these BERNOULLI numbers, one has the following formula for n > 1: $\sum_{k=0}^{n-1} \binom{n}{k} \cdot B_k = 0$

These numbers also play a role in the series expansions of $\tan(x)$, $\ln(\frac{\sin(x)}{x})$ and $x \cdot \cot(x)$.





In solving the question as to what curve is intersected at the same angle by every ray from the origin, JACOB BERNOULLI discovered the *logarithmic spiral*.

He was so enthusiastic about the properties of this *spira mirabilis* – even after a central dilation, another spiral of the same type results – that he requested that the curve together with the motto *Resurgo eadem mutata* (changed, I return the same) be placed on his gravestone. However, the stonemason in his ignorance engraved an ARCHIMEDIAN spiral instead of a logarithmic one.

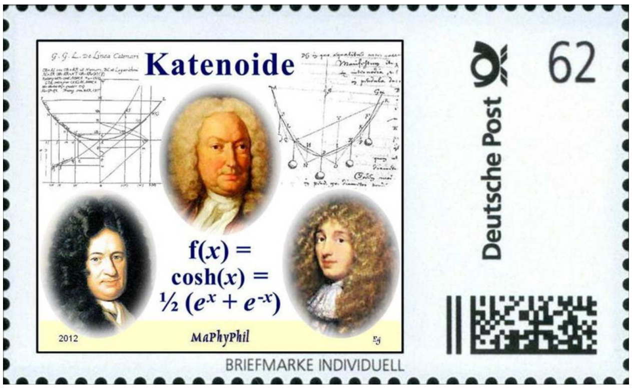
First published 2010 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

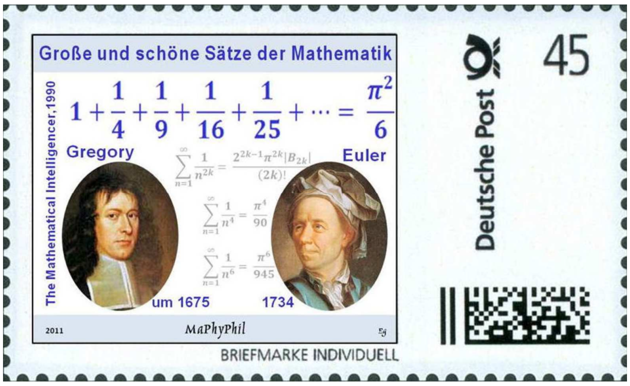
https://www.spektrum.de/wissen/jakob-bernoulli-1655-1705/1039591

Translated by David Kramer

English version first published by the *European Mathematical Society* 2013

Here an important hint for philatelists who also like individual (not officially issued) stamps:





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