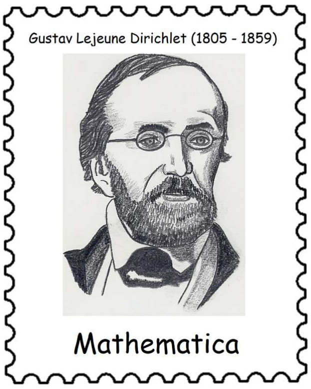
GUSTAV LEJEUNE DIRICHLET (February 13, 1805 – May 5, 1859)

by HEINZ KLAUS STRICK, Germany



When CARL FRIEDRICH GAUSS, the princeps mathematicorum, died on February 23, 1855, in Göttingen, a worthy successor to his academic chair in mathematics was quickly found in JOHANN PETER GUSTAV LEJEUNE DIRICHLET. Thus began a new golden age of mathematics in Germany.

The surname DIRICHLET hints at the family's origins: his ancestors came from the region of Richelette (DIRICHLET = de Richelette), near Liège. DIRICHLET's grandfather, JOHANN DIRICHLET, had settled in Düren (in North Rhine-Westphalia), where he married a daughter of a local family. It was JOHANN's father, with the same given name as his father, who had added LEJEUNE (= the younger) to make it clear just who was who, and it then became part of the family name.

(drawing: © Andreas Strick)

When in 1805, GUSTAV was born, the youngest of seven children, in Düren, the town belonged among the regions that had been annexed by France. GUSTAV's father was the local postmaster. In 1815, following the Congress of Vienna, Prussia obtained control over the Rhine province, though it was several years before the administrative structures were fully transferred. GUSTAV was to have followed the profession of a merchant, but his unusually strong interest in mathematics became apparent when he was still quite young. Thus at the age of 12, he was allowed to transfer to a gymnasium in Bonn, where a family acquaintance, who was a teacher at the gymnasium, was prepared to keep an eye on the boy. As it turned out, the lad did not need much looking after; he was universally praised for his diligence and pleasing manner.

When this family acquaintance took a position in Koblenz, GUSTAV transferred to a Jesuit gymnasium in Cologne. Here, the instruction in mathematics that he received from his teacher GEORG SIMON OHM (who is renowned for his discovery in 1826 of the law about electric currents that bears his name) was of decisive importance for the boy's future.

Unfortunately, GUSTAV's performance in Latin did not meet the school's standards, and thus he was compelled to leave the school in 1821 without obtaining a diploma. Fortunately for him, though, such certification for pursuing university studies in mathematics was unnecessary at that time.

At the beginning of the nineteenth century it was extremely difficult in Germany to obtain a serious education in higher mathematics. GAUSS, in Göttingen, was busy with his position as director of the observatory and professor of astronomy and showed little interest in lecturing on mathematics. Therefore, the seventeen-year-old youth decided to study in Paris. At the *Collège de France*, he flourished in his studies with FOURIER, LAPLACE, LEGENDRE, and POISSON. In his free time he devoted his attention to reading a single work: GAUSS's *Disquisitiones arithmeticae*.

When in the summer of 1823 he was offered – the result of a recommendation – a lucrative position as language instructor to the family of GENERAL FOY, a famous general of the Napoleonic Wars and leader of the Liberal Party in the Chamber of Deputies, he accepted, in order to ease the financial strain on his family.

In 1825, the twenty-year-old GUSTAV submitted a paper to the *Académie des Sciences* that dealt with FERMAT's famous conjecture from the year 1637, that is, the conjecture that there are no nontrivial integer solutions to the equation $x^n + y^n = z^n$ for n > 2.

Indeed, Fermat had indicated that he possessed a marvellous solution for an arbitrary value of *n*, though in his posthumous papers a proof was found only for the case $n = 4$. Euler had published a proof in 1770 for the case $n = 3$. And now, 55 years after Euler, the young Dirichlet, an unknown German student without even a high-school diploma, was attacking the case $n = 5$. He was able to show that if there were in fact an integer solution, then one of the variables x, y, z must be divisible by 5 and moreover, that this number could not be even. Remarkably, he was given the opportunity to present his ideas to the *Académie*, and he did so.

There was, to be sure, a gap to be filled in order for the proof of the impossibility of a solution to the Fermat equation for the case $n = 5$ to be complete, namely a proof that the variable divisible by 5 also could not be odd. This was shown by Legendre, a member of the *Académie*. Moreover, a short time later, Dirichlet came up with an independent proof of the result.

The sensation created by his talk at the *Académie* opened new opportunities for Dirichlet. He became acquainted with Alexander von Humboldt, who was working in Paris on the analysis of his expedition through Central and South America and in whose salon all the scientific luminaries gathered regularly.

A result of this acquaintanceship was that after the death of his “employer” General Foy, Dirichlet obtained employment in Prussia: Dirichlet who by birth was a citizen of Prussia, sent a letter of inquiry to the Prussian minister von Altenstein in Berlin, enclosing in it a copy of his work on Fermat’s conjecture. He also sent his article to Gauss and asked him to provide a recommendation for Berlin. Alexander von Humboldt, who together with his brother Wilhelm wished to turn Berlin into the scientific centre of Europe, wrote to his friend von Altenstein and to Gauss as well, seeking support for his plans. Obviously impressed by Dirichlet’s article, Gauss actually sent the sought-for letter to Berlin (contrary to his usual manner of dealing with such requests).

von Altenstein now offered Dirichlet a well-paid position as a lecturer at the University of Breslau, with the possibility of habilitation. To take the position, he had to obtain his doctorate, and to that end, he confidently turned to the University of Bonn (which is not far from Düren). But there was a problem, for the twenty-one-year-old Dirichlet had no high-school or university diploma and had never studied at a Prussian university, and to top things off, his article on the Fermat conjecture was not written in Latin. Above all, Dirichlet was unable to engage in a public defence of his thesis in the Latin language. The university faculty finally arrived at the solution of awarding Dirichlet an honorary doctorate.

In the spring of 1827, Dirichlet was able to take up his duties as lecturer in Breslau. On his way there, he visited Gauss in Göttingen. Gauss received him graciously and listened to news about the current state of mathematical research in Paris.

Dirichlet was able to satisfy only two of the three requirements for his habilitation. He presented a demonstration lecture (on the irrationality of π) and wrote an article (on the relationships between $b \in IN$, $\sqrt{b} \notin IN$, and the coefficients u, v in the development of $(a+\sqrt{b})^n = u + v \cdot \sqrt{b}$ for arbitrary $a, b, n \in IN$. von Altenstein was willing to forgo the defence being held in Latin.

Dirichlet was not very happy in the Silesian province. He missed the exchange of ideas with other scientists to which he had become accustomed in Paris. Nonetheless, he used his time there – somewhat over a year – to produce some noteworthy publications.

In 1825 and 1831, Gauss published articles on what is known as the *law of biquadratic reciprocity* (a characterization of equations of the fourth degree in terms of arithmetic *modulo p*). Dirichlet found shorter proofs to some of the theorems found by Gauss and thereby supplemented Gauss's work. Friedrich Wilhelm Bessel, an astronomer and mathematician in Königsberg, was so impressed with this work that he wrote a letter to Alexander von Humboldt, who took the letter again to the minister von Altenstein to encourage him finally to secure a position for Dirichlet in Berlin.

At first, this was nothing more than a position as teacher at a military academy (army college) in Berlin at which Prussian officers were educated. Within three years, he had taught courses on the theory of equations (solution of equations up to degree 4), series and sequences, descriptive geometry, trigonometry, conic sections, the geometry of space, mechanics, and geodesy. Over the years, Dirichlet saw to it that topics from calculus and its application to mechanics were added to the instructional program. Dirichlet was extremely effective as a teacher, with the result that over a period of twenty-eight years, the military administration was unwilling to relieve him of his teaching duties of up to eighteen hours per week.

From the very beginning, Dirichlet gave supplementary mathematical lectures at the University of Berlin, which in 1831 invited him to join the philosophical faculty, though only as a professor designatus, since the university insisted on compliance with the rules of habilitation. In 1832, he was inducted into the *Prussian Academy of Sciences* as its youngest member.

Dirichlet's reputation grew from year to year, even without a formal habilitation. His students were deeply impressed with the clarity of his lectures, which he presented with great enthusiasm and without notes, only rarely making use of the chalkboard. When the prescribed time for his lecture had elapsed, he made a note and then resumed the next lecture precisely at the last formulated idea. Without a habilitation, however, he did not have the right to serve as a doctoral advisor. It was not until 1851 that Dirichlet finally was able to bring himself to hold a public lecture in Latin so that he could obtain the privileges of a full professor.

In the year 1828, Alexander von Humboldt organized a congress of the *Gesellschaft Deutscher Naturforscher und Ärzte* (Society of German Naturalists and Physicians) in Berlin, in which over six hundred scientists from Germany and abroad participated. Gauss was a guest of honour at the home of von Humboldt. Felix Mendelsohn-Bartholdy, son of the wealthy banker Abraham Mendelsohn-Bartholdy and grandson of the philosopher Moses Mendelsohn, produced an original composition for the occasion. At a reception at the banker's house (today the seat of the *Federal Council of Germany*), Dirichlet met Mendelsohn-Bartholdy's daughters Fanny and Rebecca, and he fell in love with Rebecca. Rebecca had many admirers, but she chose the diffident Gustav Dirichlet. Two children resulted from their happy marriage.

The year 1843 saw the beginning of a close friendship between DIRICHLET and JACOBI, who occupied a chair in mathematics at Königsberg. The friendship was so close that DIRICHLET was willing to accompany JACOBI on a convalescence trip to Italy; ALEXANDER VON HUMBOLDT saw to it that neither suffered any financial hardship on that account. And when DIRICHLET became severely ill, JACOBI took over his duties at the military academy, since payment was given only when instruction actually took place.

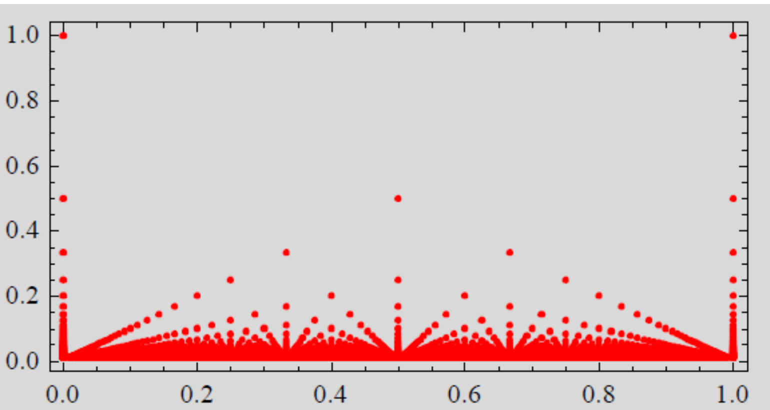
In 1846, the University of Heidelberg made DIRICHLET an enticing offer. JACOBI applied to ALEXANDER VON HUMBOLDT to use his influence to make sure that DIRICHLET would remain in Berlin. He wrote, "He alone – not I, not CAUCHY, not GAUSS, know what a complete mathematical proof is. When GAUSS says that he has proved something, I consider it quite probable that he has done so; when CAUCHY says so, one would offer equal odds as to whether he has in fact done so; when DIRICHLET says so, it is a certainty." DIRICHLET's annual salary was doubled, though he was still burdened with his teaching duties at the military academy.

When GAUSS died in 1855, the University of Göttingen sought DIRICHLET as his successor. DIRICHLET wished to remain in Berlin, but only provided that his teaching obligations at the military school would be removed and that his salary would be raised to a level commensurate with that of a university professor. Since the decision in Berlin appeared to be a long time coming, he accepted the call to Göttingen. There he and his family quickly settled in. He encouraged the further mathematical development of RIEMANN and DEDEKIND, the last two doctoral students of GAUSS. DEDEKIND attended every lecture that DIRICHLET gave in Göttingen; he was thereby able later to reconstruct those lectures from his notes and publish them.

In the summer of 1858, DIRICHLET travelled to Switzerland for rest and recuperation, there to compose in peace a memorial oration in memory of GAUSS. Following a heart attack, he returned to Göttingen deathly ill, and only gradually recovered. But when his wife suddenly and unexpectedly died of a stroke, he lost all will to live. He died on May 5, 1859, one day before his friend and patron ALEXANDER VON HUMBOLDT.

DIRICHLET's enormous importance in the history of mathematics can be seen even today in the number of concepts and theorems bearing his name:

- **DIRICHLET condition** (1829): DIRICHLET discovered an error in an article of CAUCHY's on the convergence of trigonometric series (FOURIER series). He corrected it and gave a condition for the convergence of such series. In this connection he introduced a special function, the **DIRICHLET function** D, defined by $D(x)=1$ for $x \notin Q$ and $D(x)=0$ for $x \in Q$. This function is discontinuous at every point of its domain of definition; it is not (RIEMANN) integrable, since the limit values from above and below do not agree. He also recognized that the summands in infinite series cannot always be reordered; depending on the arrangement, the limiting value can be finite or infinite (DIRICHLET paradox).



Modified DIRICHLET function

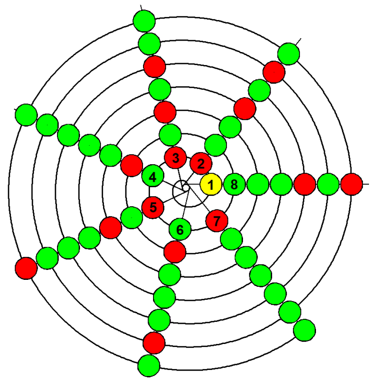
$$D(x) = \begin{cases} 0 & \text{if } x \notin \mathbf{Q} \text{ (irrational)} \\ 1/q & \text{if } x = p/q \in \mathbf{Q}, \text{GCD}[p,q] = 1 \end{cases}$$

(graphics: © Robert Kragler)

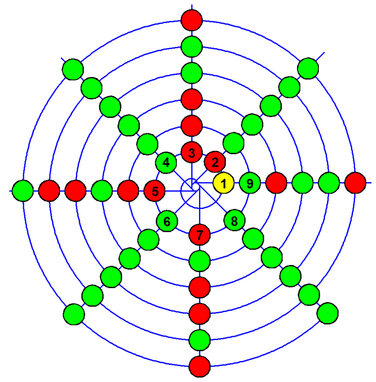
- **Dirichlet's definition of a function** (1837): If a variable *y* stands in relation to a variable *x* in such a way that to every numeric value of *x* there corresponds a unique value of *y* based on some rule, then *y* is said to be a function of the independent variable *x*.
- **Dirichlet approximation theorem**: For every real number *x* there exist infinitely many pairs *p*, q of integers such that $\left|x-\frac{p}{q}\right| \le \frac{1}{q^2}$. This estimate for the approximation of real numbers by rational numbers was later sharpened by Adolf Hurwitz (1859-1919) to $\left|x-\frac{p}{q}\right| \le \frac{1}{\sqrt{5}\cdot q^2}$, whereby the factor $\sqrt{5}$ cannot be improved. To prove this theorem, one uses the simple but highly effective *Dirichlet pigeonhole principle*, used for the first time by Dirichlet in 1834: if one distributes *n* objects among m pigeonholes, where $n > m$, then at least one of the pigeonholes must contain at least two objects.
- **Dirichlet**'s **theorem on primes in arithmetic progression**: In 1837, Dirichlet proved a fundamental theorem on prime numbers: if *a*, *m* are integers that are relatively prime, then every arithmetic progression $a, a+m, a+2m, a+3m, \ldots$ contains infinitely many prime numbers. In the base-ten system (that is, $m=10$), this means, for example, that there are infinitely many primes that end in $a=1$, infinitely many ending in $a=3$, and likewise for $a=7$ and $a=9$. The proof of the theorem is carried out with the help of so-called **Dirichlet series** $\sum_{n=1}^{\infty}\frac{a_n}{n^s}$.

In addition to works on number theory and complex analysis, Dirichlet also published several works on the theory of probability (**Dirichlet distribution**) and theoretical physics.

While the number of his publications may seem small, nevertheless, as Gauss wrote in 1845, "His individual works would scarcely fill a book of no great magnitude. But they are each of them gems, and gems are not weighed on a shopkeeper's scale."



m = 7



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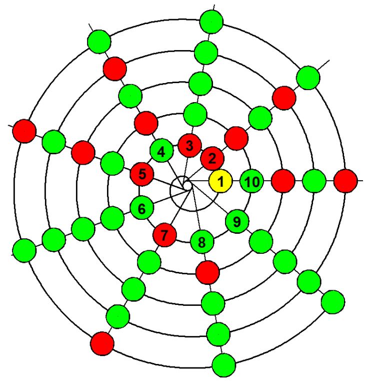

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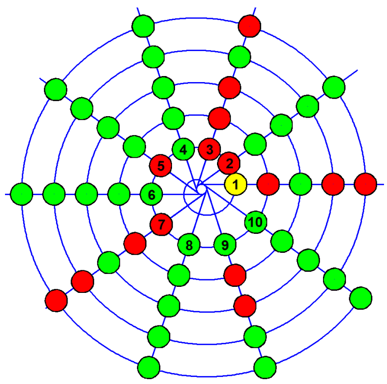

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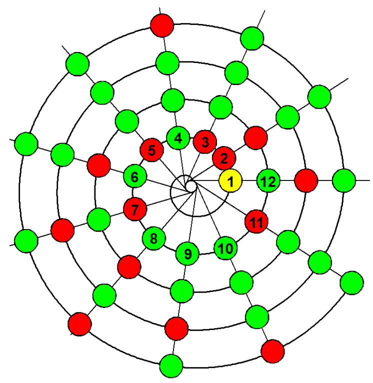

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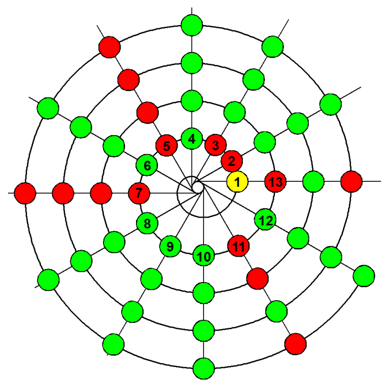

m = 12

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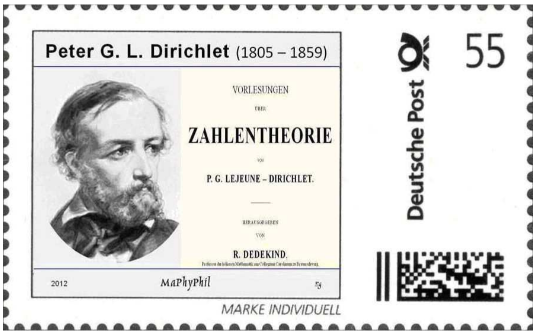

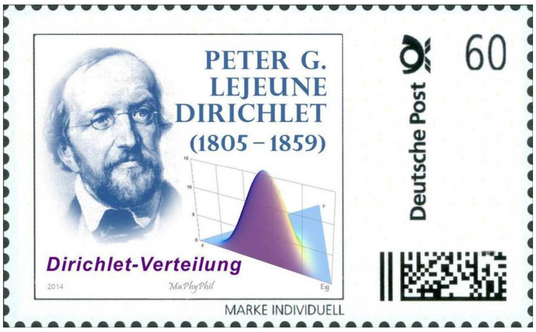

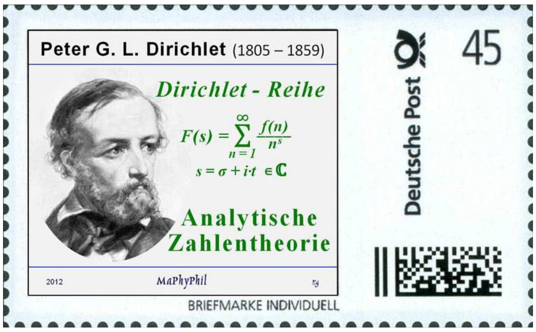

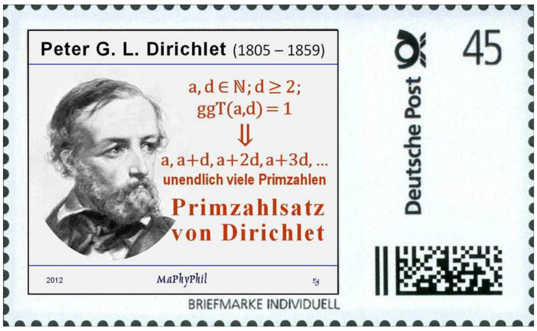

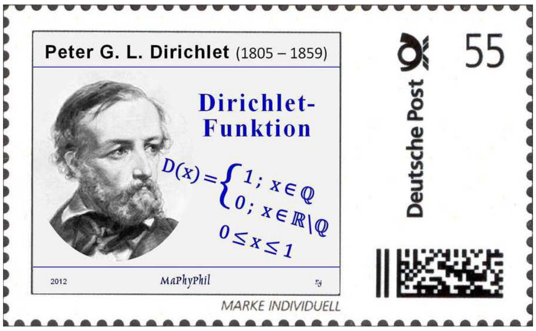

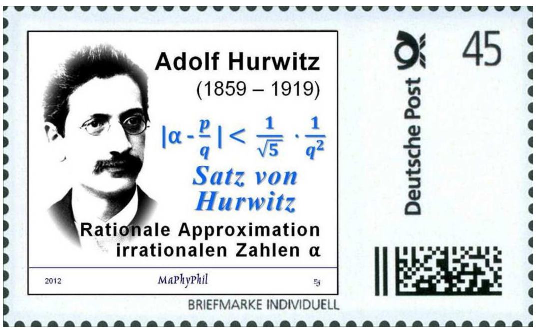

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