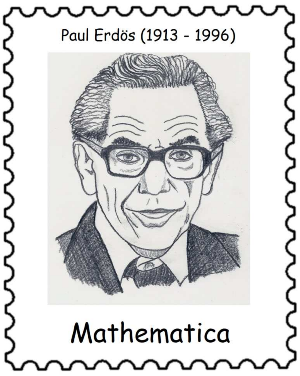
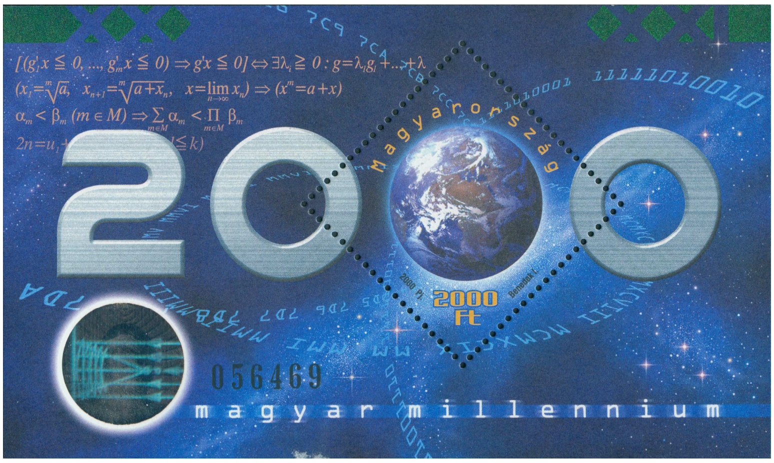
PAUL ERDŐS (March 26, 1913–September 20, 1996)

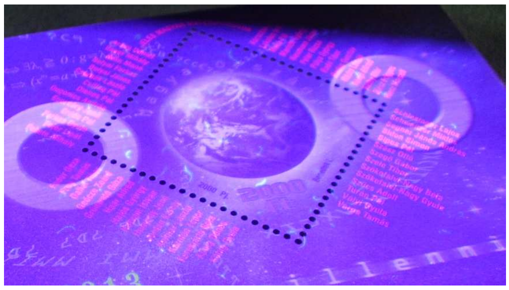
by HEINZ KLAUS STRICK, Germany



On the occasion of the new millennium, the Hungarian postal service released a block containing a postage stamp with a hologram portrait of DÉNES GÁBOR (1900-1979), the Hungarian-born inventor of holography. One can also see some numbers represented in various number systems as well as some mathematical theorems.

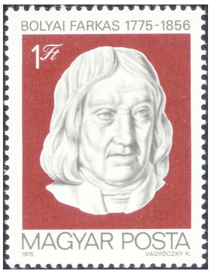
(drawing: Andreas Strick ©)

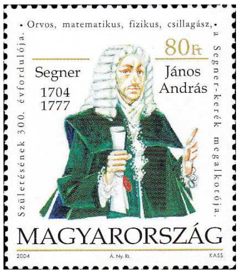




If one shines ultraviolet light on the block, then an inscription appears around the perimeter of the stamp: *Jeles magyar matematikusok* (outstanding Hungarian mathematicians), along with the names of 57 Hungarian mathematicians, including PAUL ERDŐS (by Hungarian writing convention, the given name appears after the family name, whence ERDŐS PÁL), as well as (with last name first)

BOLYAI FARKAS, BOLYAI JÁNOS, NEUMANN JÁNOS, SEGNER JÁNOS ANDRÁS.





On account of the vast number of his publications, ERDŐS PÁL has been called the EULER of the twentieth century: He wrote over 1500 scientific papers on a variety of topics, primarily on the theory of numbers and combinatorics, but also on classical analysis, graph theory, set theory, and the theory of probability. ERDOS stimulated over five hundred coauthors to work with him collaboratively.

It was such an honour to publish a joint paper with ERDOS that these coauthors are said to have "ERDOS number" 1. (Anyone who has coauthored a paper with someone with ERDOS number 1 is considered to have ERDOS number 2, and so on.)

Many of his articles deal less with an abstract mathematical theory than with concrete problems, which are often easy to understand but whose solutions are generally extremely difficult.

In March 1913, while the Jewish couple Lajos and Anna Erdos were looking forward to the birth of their third child, both of their daughters became ill with scarlet fever and died a few days later. So it is perhaps understandable that their newborn son, whom they gave the name Paul, was brought up with a great deal of attention and protection. When World War I broke out in the summer of 1914, the father was conscripted into military service and soon found himself in a Russian prisoner of war camp. The mother supported the family by working as a teacher of mathematics. Afraid that her son would take ill, she kept him at home and had him instructed by a German governess and private tutors. When Paul would become bored, he would browse in the mathematics books in his parents' library (the father was also a mathematician). Later he would say, "*And thus did numbers become my friends ...*"

It is reported that by the time he was four years old, he could calculate in his head, given a person's date of birth, how long that person had lived, in seconds.

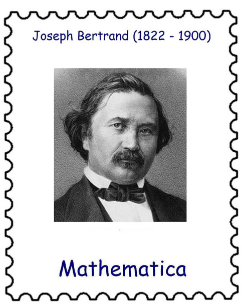
Following the end of the war—the victorious powers had reduced the territory of Hungary to one-third of its prewar size – Hungary was wracked by unrest. Romanian troops advanced as far as Budapest and overthrew the Communist regime of Béla Kun. The conservative Admiral Horthy seized power and instituted authoritarian rule over the country as regent (that is, on behalf of the Habsburg Austro-Hungarian Monarchy). Anna Erdos, who had been installed as a school headmistress by the Communists, lost her position and had even to fear for her life as Horthy's troops roamed the streets looking to root out and kill Jews and Communists. Since an especially large number of Jews had been active in the Communist regime, Horthy issued decrees that drastically restricted the conditions under which the Jewish population of Hungary could live.

In 1920, the father finally returned from Siberia and was able again to look after his family. During his years as a prisoner of war, he had taught himself English, which he now passed on to his son. However, since his father did not know how English words were pronounced, Paul Erdos adopted an accent that he was never able to get rid of.

From 1922 on, Paul attended a gymnasium. In 1930, despite his Jewish background, he was permitted to undertake studies in mathematics at the university, since he had taken first place in the traditional national mathematics competition.

Only a year later, when was just 18 years old, Paul Erdos discovered a new and elegant proof of a theorem on prime numbers that had been conjectured in 1845 by Joseph Bertrand and given a very complicated proof in 1850 by Pafnuty Chebyshev:

Between every natural number n and its double there is at least one prime number.



After completing his university examinations in 1934, he accepted a research fellowship at Manchester and Cambridge, since as a Jew, he had no possibility of a career in Hungary as a university professor. He visited his parents in Budapest three times a year.

But when the political situation in Central Europe reached a climax in 1938 with the *Sudeten crisis*, he left Hungary in a state of panic. From England, he travelled to the United States, where he accepted a visiting position at Princeton University.

The Polish-born mathematician STANISLAW MARCIN ULAM, a student of STEFAN BANACH, whom ERDOS had known since his time at Cambridge, arranged a permanent position for him at the University of Wisconsin, in Madison.

From this time forth, PAUL ERDOS never remained long in any one place. Till the end of his life, he constantly changed his place of "residence", travelling in the world from mathematician to mathematician with a suitcase containing everything he possessed, greeting every new host with the words, "My brain is open".

From 1941 until the liberation of Hungary, he received no news from his homeland. His father died in 1942, and a number of his relatives perished in concentration camps. His mother survived the period of terror as if by a miracle. It was not until 1948 that ERDOS was again able to visit the country of his birth.

In 1896, JACQUES SALOMON HADAMARD und CHARLES-JEAN DE LA VALLÉE POUSSIN gave the first proofs of the *prime number theorem*, which states that the function $\pi(x)$, which gives the number of primes less than the real number *x*, satisfies the relation $\lim_{x\to\infty}\left(\frac{\pi(x)}{x/\ln(x)}\right)=1$.

In 1949, ERDOS and ATLE SELBERG simultaneously discovered "elementary" proofs of the prime number theorem (here "elementary" means without the use of complex analysis). The two mathematicians could not agree on how their results should be published, since each had used work of the other in obtaining his result. ERDOS wanted publication of a joint paper, and SELBERG would not agree to that. In the end, separate papers appeared, and the following year, SELBERG was awarded the FIELDS Medal for his outstanding accomplishments, the highest honour in mathematics, comparable with the NOBEL Prize. ERDOS accepted this outcome with equanimity.

In 1952, he accepted a generous offer from the *University of Notre Dame* (in South Bend, Indiana) that gave him complete freedom regarding his teaching responsibilities. On his return from a visit to Amsterdam, he was subjected to an interrogation in which he was asked, among other things, his opinion of KARL MARX. His reply, that MARX was surely a man of importance, might have tipped the balance in denying him entry into the country. However, in an FBI file on him there is also a notation from the year 1941, when he had unintentionally (because he was deep in conversation on a mathematical problem) trespassed on a military reservation. As someone who was in regular contact with individuals living in Communist countries (for example, his mother and also a number theorist from Communist China), he was suspected of being a Communist spy.

During the following ten years, he spent most of his time in Israel (which he called "Is Real"). The *Technion* (Israel Institute of Technology) in Haifa named ERDOS a permanent visiting professor. Despite numerous invitations from American universities, it was not until 1963 that ERDOS was allowed to enter the United States. His fear that following his return to Hungary he would not be allowed to leave never materialized. However, in the 1970s, he refused for a long time to visit his homeland as a protest against its anti-Israeli policies.

He received numerous honours for his accomplishments, including at least fifteen honorary doctorates. All the prize money that he received he used to fund his own awards, offering cash prizes for the solutions of particular problems that he posed, whereby he himself judged the monetary value of a problem (between \$25 and \$5000). He also gave money to RAMANUJAN's widow.

Proofs of elementary conjectures posed by ERDOS:

- For all $n > 4$, there is at least one prime number $p < n$ such that p^2 is a divisor of $\binom{2n}{n}$. This conjecture was proved in 1996.

- The equation $\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ can be solved for every natural number $n \in IN$. This conjecture has yet to be proven for odd values of *n*.

In 1939, ERDOS proved that the number $\omega(n)$ of distinct prime factors of a number *n* randomly chosen from a sufficiently large set $\{1, 2, 3, ..., N\}$ is approximately normally distributed with mean $\mu = \ln(\ln(N))$ and standard deviation $\sigma = \sqrt{\ln(\ln(N))}$. With this result, he became one of the founders of *probabilistic number theory*, in which number-theoretic functions are investigated using the methods of probability theory.

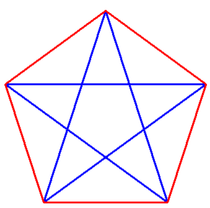
ERDOS was a man who “loved only numbers” (thus the title of a biography of ERDOS).

For example, an article in the *Journal of Recreational Mathematics* in which pairs of adjacent natural numbers $(n, n+1)$ were considered that had identical sums of their prime factors led him to ask how dense such pairs might be distributed in the natural numbers IN.
(For example, for $714 = 2 \cdot 3 \cdot 7 \cdot 17$, one has $2+3+7+17 = 29$, while for the adjacent number $715 = 5 \cdot 11 \cdot 13$, one has the identical sum $5+11+13 = 29$.)

Here is another example of this kind of problem: In the simple multiplication table of the natural numbers from 1 to 10 (that is, with $10 \cdot 10 = 100$ entries), there are $M(10) = 43$ different products. In 1960, he proved that the sequence $M(n)/n^2$ converges to 0!

Many of his publications deal with problems in *RAMSEY theory* (named for the brilliant British mathematician FRANK P. RAMSEY, who died in 1930 at the age of 26). Such problems consider the search for the minimal number of elements in sets for which particular properties hold with certainty. For example, ERDOS proved the following theorems:

- If one considers the edges of a complete graph with sufficiently many vertices (complete means that every vertex is joined to every other vertex by an edge) and colours these edges arbitrarily with two colours, then there exists a number $R(m,n)$ such that a complete subgraph on *m* edges is coloured with one colour, and a complete subgraph with *n* edges is coloured in the other colour. For the desired minimum number (RAMSEY number) $R(m,n)$ of points, ERDOS proved that $R(m,n) \le R(m-1,n) + R(m,n-1)$ as well as that $R(m,n) \le \binom{m+n-2}{m-1}$.



For example, $R(3,3) = 6$, since in a complete graph with only five points, one can colour the edges in such a way that there is no complete subgraph on three vertices of one colour or the other.

- In a sequence of $m \cdot n + 1$ arbitrarily arranged real numbers, there always exists an increasing subsequence of $m+1$ elements or a decreasing sequence of $n+1$ elements; or both. (numerical example: if one generates 50 random numbers, then one can remove 42 of them to leave an increasing or decreasing sequence of 8 numbers.)

ERDOS provided numerous ideas for problems to be included in mathematical Olympiads. For example:

- Two nonintersecting squares of side lengths a and b lie inside a square of side length 1. Prove that $a+b \leq 1$.
- For each of n integers one has $a_n < 1951$, and the greatest common multiple of each pair of these numbers is greater than 1951. Prove that $\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}<2$.

ERDOS made few demands on those around him. Money was just a burden for him. He worked twenty hours a day, keeping awake with coffee and amphetamines. (An original quotation: *A mathematician is a machine for turning coffee into theorems*.)

The death of his mother, who had accompanied him on his "travels" from 1964 to 1971, plunged him into a long depression. ERDOS never married. He died of cardiac arrest during a graph theory conference in Warsaw.

In his dealings with others he frequently engaged in a private vocabulary: children were *epsilons*, women *bosses*, men *slaves*, married people *prisoners*, the divorced *liberated*, a mathematical lecture a *sermon*. He considered God the *supreme fascist* (or SF for short), who caused socks and Hungarian passports to disappear and who kept the most beautiful mathematical theorems to himself. He held that a mathematician need not believe in the existence of God but that one should believe in THE BOOK, which contains complete proofs of all mathematical theorems.

When he was shown a particularly beautiful proof, he would express his approval by saying, "This one is straight from THE BOOK". And if he found a proof displeasing, he would say "Let's look for THE BOOK proof".

ERDOS eagerly took up the idea suggested by GÜNTER ZIEGLER and MARTIN AIGNER to collect proofs that likely appear in THE BOOK. He was able to give some suggestions for the presentation of the collection before his death. Today, the ever growing work *DAS BUCH der Beweise* (English *Proofs from THE BOOK*, French RAISONNEMENTS DIVINS) has reached its fourth edition and has been translated into at least fourteen languages.

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Translated by David Kramer

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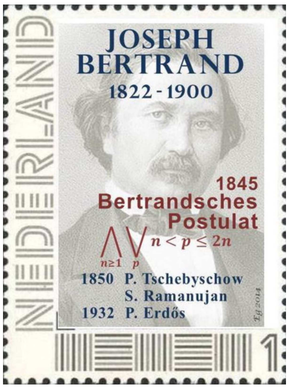
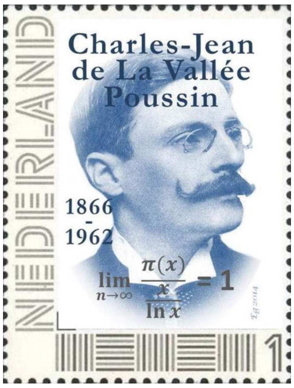
Here an important hint for philatelists who also like individual (not officially issued) stamps:





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