JOSEPH FOURIER (March 21, 1768 – May 16, 1830)

by HEINZ KLAUS STRICK, Germany

The name JOSEPH FOURIER can be found, written in golden letters, among the seventy-two names of famous Frenchmen on a frieze on the sides of the **Eiffel Tower**, in Paris. FOURIER’s grave occupies a prominent place in the cemetery **Père Lachaise** next to the graves of GASPARD MONGE and JEAN-FRANÇOIS CHAMPOILLION. But while the French postal service has dedicated several postage stamps to those two, it seems to have forgotten JOSEPH FOURIER. (drawing: © Andreas Strick)

JEAN-BAPTISTE JOSEPH FOURIER was the ninth of twelve children from the second marriage of a tailor in Auxerre (Burgundy). When he was ten years old, his parents died, one shortly after the other, and the boy was taken in by relatives. After attending a Latin school at the cathedral in Auxerre with great success, he transferred to the local military academy, where he quickly showed an exceptional talent for mathematics and physics. At the age of nineteen, he became a member of the Benedictine order, thinking that he would be able to work as an instructor in mathematics at the monastery school. During the following months, he developed misgivings about his decision and resumed contact with his mathematics teacher in Auxerre. He even wrote a paper on algebra during that time. By the time he turned twenty-one, he had decided that he would not accept such a fate, reasoning that at the age he had just attained, NEWTON and PASCAL had already joined the ranks of the immortals. He left the monastery, went to Paris, studied mathematics, and returned to his former school in Auxerre as a teacher of mathematics.

Still undecided about his future, the dramatic change in the political situation took the decision out of his hands: In 1793, he joined the local revolutionary committee. Thanks to his talents as a speaker, he soon wielded considerable influence over the development of the revolution in the region. His involvement in politics almost cost him his head: He became embroiled in an argument with the ROBESPIERRE faction, which managed to have him arrested. He avoided the guillotine only because the influence of his adversaries waned after the execution of ROBESPIERRE himself.

In 1794, FOURIER was accepted into the mathematics program of the newly established **École Normale** in Paris. His teachers were JOSEPH-LOUIS LAGRANGE, PIERRE-SIMON LAPLACE, and GASPARD MONGE. Simultaneously, as an outstanding student, he obtained a position as an instructor at the Collège de France and then moved on to the newly founded **École Polytechnique**. He was again arrested because of his revolutionary activities but was soon released. In 1797, he was appointed to a professorship in the chair for analysis and mechanics vacated by L LAGRANGE, and his fame as an outstanding teacher spread far and wide.

In 1798, he joined the group of scientists that accompanied NAPOLEON on his Egyptian campaign. When the French fleet was destroyed by the fleet of Admiral NELSON at the Battle of the Nile, the expedition was unable to return to France.
FOURIER became involved in reform of the educational system in Egypt. He was named secretary of the *Institut d’Égypte*, and he organized archaeological expeditions. NAPOLEON charged him with collecting valuable archaeological finds; NAPOLEON himself returned to France to seize power for himself.

In 1801, FOURIER was able to return to Paris, where he would have liked to resume his duties as a professor of mathematics. NAPOLEON, however, named him – against his wishes – prefect of the Département Isère. His new duties included draining the swamps around Lyon and building a road from Grenoble to Turin. He carried out these tasks to the great satisfaction of his superiors, though during this time, he also managed to complete two extensive monographs: *On the Propagation of Heat in Solid Bodies* and *Description de l’Égypte*.

The book about Egypt could not be published until NAPOLEON was satisfied that he had been sufficiently praised in every passage; in later editions of the work, FOURIER removed almost all the passages relating to NAPOLEON’s merits.

FOURIER’s enthusiasm for the culture of ancient Egypt was infectious. When in 1802, he showed the twelve-year-old JEAN-FRANÇOIS CHAMPOLLION a replica of the *Rosetta Stone*, the boy became obsessed with the idea of deciphering the hieroglyphics that appeared on the stone. In 1822, CHAMPOLLION presented his research to the *Académie des Inscriptions et Belles-Lettres* in Paris: he had deciphered the hieroglyphics!

FOURIER’s work on the propagation of heat in solid bodies was met with resistance, and not only because of the unusual physical modelling of how heat propagates. Namely, the mathematical treatment of the problem was initially rejected by LAGRANGE and LAPLACE, since they could not warm up to the idea of representing functions through the use of series developments of trigonometric functions.

Nevertheless, in 1811, FOURIER was awarded a prize of the *Académie des Sciences*. The prize committee, however, declined to publish the manuscript.

In 1814, NAPOLEON was deposed and banished to Elba. On his return, he marched with his newly mustered troops directly toward Grenoble. FOURIER attempted in vain to remind the citizens of their oath of loyalty to the king. When NAPOLEON entered the city, FOURIER fled. NAPOLEON magnanimously pardoned him for this “treachery” to his person and named him prefect of the Département Rhône, a position that was supplied with a handsome salary, which, however, was never paid.

After NAPOLEON’s final defeat, FOURIER returned to Paris, but he was unable to return to his previous position because of his cooperation with NAPOLEON.
In addition, the Bourbon king LOUIS XVIII declined to accept him into the Académie des Sciences, although FOUERI had obtained more votes than any other candidate. He was not accepted into membership until 1817. In 1822, the Académie named him permanent secretary. Four years later, FOUERI was elected to the Académie française, the society of the “forty immortals”.

In the last eight years of his life, FOUERI published a number of papers on problems in mathematics and physics, including an enlarged edition of the Théorie analytique de la chaleur, in which he made the following brilliant proposal: every periodic function can be represented as an infinite sum of sine and cosine functions (today, these are called FOUERI series). Specifically:

If \( f \) is a \( 2\pi \)-periodic function, then there exists an infinite series

\[
p_n(x) = \frac{1}{2} \cdot a_0 + \sum_{k=1}^{n} [ a_k \cdot \cos(kx) + b_k \cdot \sin(kx) ]
\]

that converges to \( f(x) \). For the (FOUERI) coefficients \( a_k \) and \( b_k \) one has

\[
a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx, \quad a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(kx) \, dx, \quad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(kx) \, dx \quad \text{if} \quad k \geq 1.
\]

If \( f \) is an even function, then \( b_k = 0 \) for all \( k \), while if \( f \) is an odd function, then \( a_k = 0 \) for all \( k \).

See the following examples for the interval \([-\pi, \pi]\):

\[\begin{align*}
\text{f(x) = x} & \quad \text{Approximation for } n = 10 \\
p_n(x) = 2 \sum_{k=1}^{n} (-1)^{k+1} \frac{\sin(kx)}{k} \\
\text{f(x) = |x|} & \quad \text{Approximation for } n = 6 \\
p_n(x) = \frac{\pi}{2} \sum_{k=1}^{n} \frac{\cos((2k-1)x)}{(2k-1)^2} \\
\text{f(x) = -1 if } -\pi < x < 0 \text{ and } f(x) = +1 \text{ if } 0 < x < \pi & \quad \text{Approximation for } n = 9 \\
p_n(x) = \frac{4}{\pi} \sum_{k=1}^{n} \frac{\sin((2k-1)x)}{2k-1}
\end{align*}\]
Here an important hint for philatelists who also like individual (not officially issued) stamps:

Enquiries at europablocks@web.de with the note: "Mathstamps"