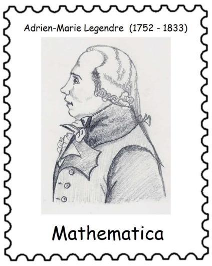
Evariste Galois (October 25, 1811–May 31, 1832)

by Heinz Klaus Strick, Germany

Many legends have arisen around the life of Évariste Galois, including speculations about his early death – he was not yet 21 years old. He is depicted on the French postage stamp from 1984 (part of a series on famous French citizens) as both a revolutionary and a geometer. One thing is certain: both politics and mathematics contributed to the brevity of his life.

Évariste Galois grew up in Bourg-la-Reine, a commune not far from Paris. His father, who served as mayor, kept his position even after the fall of Napoleon. His wife took charge of the education of their three children, teaching them Latin and Greek. At 12, Évariste attended the *Collège Louis-le-Grand*, in Paris, at first with great success. At 15, he showed a particular interest in mathematics, but instead of following the school's curriculum, he took to reading the works of Adrien Marie Legendre (1752–1833) and Joseph Louis Lagrange (1736–1813).



(drawing: © Andreas Strick)

Without taking the usual preparatory course, he enrolled at 17 for the entrance examination at the elite *École Polytechnique*. Galois failed the exam; returning to his old school, he paid even less attention to the instruction. At 18, Galois's first paper, on continued fractions, was accepted and printed by the journal *Annales de mathématique*. In early summer of 1829, he submitted two articles on the solvability of polynomial equations to the *Academy of Sciences*.

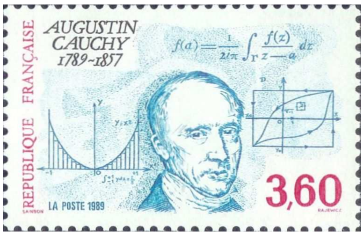
That same summer, a renewed power struggle arose between royalists and liberals; Évariste's father, who sided with the liberals, was driven by slander to suicide. Despite this shocking event, Galois enrolled shortly thereafter again for the entrance examination at the *École Polytechnique*, and again failed: he was unable to understand why he was asked to prove a proposition that in his eyes was fully obvious. Twenty years later, the following appeared in the *Nouvelles Annales Mathématiques*:

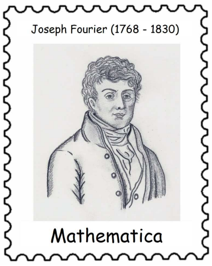
A candidate of superior intelligence was crushed by an examiner of inferior intelligence.

At the end of 1820, Galois completed the standard examinations for the baccalauréat degree; thereafter, he was permitted to study at the *École Normale Supérieure*.

Galois submitted a new article to Cauchy on the solvability of polynomial equations, only to learn that his work intersected in part with an article submitted by Niels Henrik Abel (1802–1829).

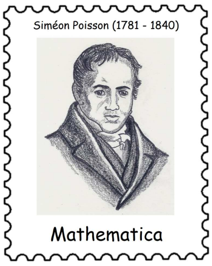
In February 1830, he completed another article, which was to be refereed by Joseph Fourier (1768–1830). But Fourier died suddenly in April of that year; Galois's submission was not found among his papers. That June, Galois learned that the *Academy*'s annual prize had been awarded posthumously to Abel and that his paper had not been read.





The political situation in France became worse in July 1830 following the liberal party's electoral victory. The king fled abroad. When the director of his school attempted to prevent his students from aligning themselves with the insurgents, he was publicly attacked by Galois in a letter to the *Gazette des Écoles*, which resulted in his expulsion from the *École Normale*.

Galois was now destitute and desperate, being unable to attend any university. He joined the *Republican National Guard*, which, however, by the end of 1830 was outlawed by the new king, Louis Phillipe (the so-called *citizen king*). Some of the officers were arrested for treason.

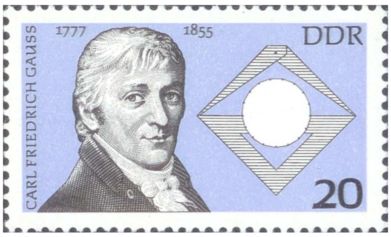


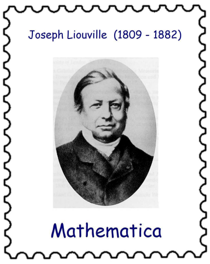
Siméon Denis Poisson (1781–1840) suggested to Galois that he resubmit his article (for the third time!) to the *Academy*, which he did at once.

At a celebration on the occasion of the release of the imprisoned officers in May 1831, Galois proposed a toast to the king with a dagger in his hand. He was arrested, and later released. On July 14, the anniversary of the storming of the Bastille, he was again arrested following his appearance in the streets heavily armed and wearing the uniform of the outlawed National Guard. While in prison, he received the news that Poisson had judged his article to suffer from an "insufficiently clear exposition", Galois attempted suicide, but was prevented from doing so by his fellow prisoners.

When the cholera epidemic reached the prison, he was transferred to hospital; there he fell in love with Stéphanie, the daughter of the attending physician.

It has never been satisfactorily explained how on May 30, 1832, he engaged in a duel with a fellow republican. During the night before the duel, Galois committed to paper everything that he wished to communicate to the world should he perish the following day. The name Stéphanie appears repeatedly in a farewell letter. Wounded in the duel, abandoned by his own seconds, he died the following day. A friend wrote up his mathematical legacy and sent the papers to several renowned mathematicians, among them Carl Friedrich Gauss, to no avail.

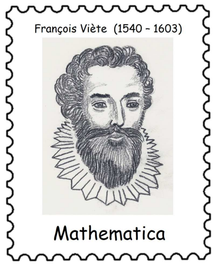




Recognition came only eleven years later, when Joseph Liouville (1809–1882) understood the significance of the theories developed by Galois and published them in his journal, in 1846.

In the mid-sixteenth century, mathematicians finally succeeded in finding mathematical expressions for the solutions of polynomial equations of the third and fourth degree; the solutions are expressed in terms of the coefficients of the equation together with the fundamental arithmetic operations and extraction of roots (*solution by radicals*).

François Viète (1540–1603) pointed out (*Viète laws*) certain relationships between the coefficients of polynomial equations and their solutions.



For example, for degree 2, if we take the equation $x^2-(x_1+x_2)\cdot x+x_1\cdot x_2=0$, then x_1 and x_2 are the two solutions. In degree 3, if $x^3-(x_1+x_2+x_3)\cdot x^2+(x_1\cdot x_2+x_1\cdot x_3+x_2\cdot x_3)\cdot x+x_1\cdot x_2\cdot x_3=0$, then x_1, x_2 and x_3 are the solutions.

The search for a solution procedure for equations of degree 5 proved fruitless. Gauss proved in his dissertation of 1799 that every equation of degree *n* has precisely *n* solutions in the set of complex numbers. This *fundamental theorem of algebra* guarantees the existence of solutions yet gives no hint as to how they should be expressed in terms of the coefficients.

In 1824, Abel proved that there can be no general procedure for expressing the solutions of polynomial equations of degree greater than 4 by showing that not all solutions can be expressed through the extraction of roots.

Galois's theory finally made it possible to determine, given a particular polynomial, whether there is a solution procedure for that polynomial based on its coefficients. In working with the so-called *permutation group*, Galois realized that there is a relationship between the structure of that group and the solvability of polynomial equations: Every such equation can be associated with a permutation group, and from the structure of that group, one can read off whether the equation can be solved by radicals.

A *group* (a term introduced by Galois) is a set on which a binary operation (usually called multiplication) is defined such that

(1) the product of two elements of the set is again an element of the set,

(2) there exists a *neutral* element such that multiplication of any element of the set by the neutral element leaves it unchanged, and

(3) for every element of the group there exists an *inverse element* in the group such that the product of an element and its inverse yields the neutral element.

A *permutation* is a mapping of a finite set onto itself.

Example: for a set of three objects, there exist 3! = 6 permutations:

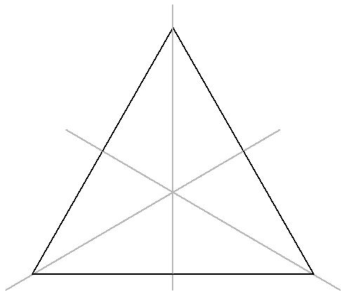
$$p_0=id=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\ p_1=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\ p_2=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},$$

$$p_3=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\ p_4=\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix},\ p_5=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}.$$

The permutation p_0 (the *identity mapping*) is the neutral element for the group.

This group of permutations can also be interpreted as the group of symmetries (reflections and rotations) of an equilateral triangle; it is called the *symmetric group* and denoted by S_3.

In the table, which shows the results of applying any two permutations sequentially, one can read off the following properties: the associative property is satisfied, and the group operation is *not* commutative.



S_3	p_0	p_1	p_2	p_3	p_4	p_5
p_0	p_0	p_1	p_2	p_3	p_4	p_5
p_1	p_1	p_0	p_3	p_2	p_5	p_4
p_2	p_2	p_4	p_0	p_5	p_1	p_3
p_3	p_3	p_5	p_1	p_4	p_0	p_2
p_4	p_4	p_2	p_5	p_0	p_3	p_1
p_5	p_5	p_3	p_4	p_1	p_2	p_0

This group possesses a number of subgroups (a *subgroup* is a subset of the group that is itself a group under the same operation).

These subgroups are $\{p_0\}$, $\{p_0, p_1\}$, $\{p_0, p_2\}$, $\{p_0, p_5\}$, $A_3 = \{p_0, p_3, p_4\}$, and the entire group S_3 itself.

The group A_3 is called the *alternating permutation group*; it contains the so-called *even* permutations. In an equilateral triangle, these are the rotations about 0°, 120°, and 240°.

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https://www.spektrum.de/wissen/evariste-galois-1811-1832/873210

Translated by David Kramer

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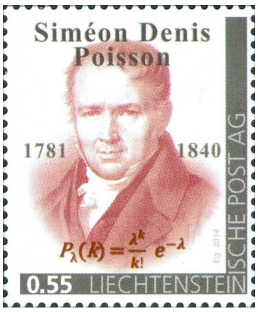
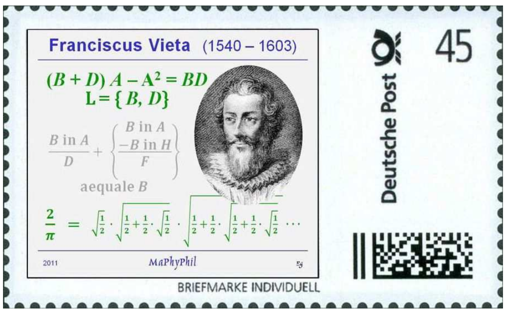
Here an important hint for philatelists who also like individual (not officially issued) stamps:





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