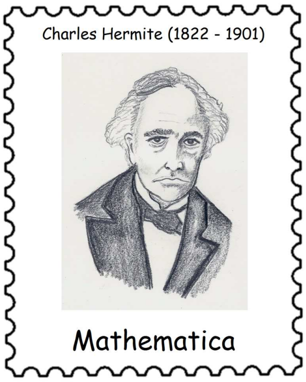
CHARLES HERMITE (December 24, 1822 – January 14, 1901)

by HEINZ KLAUS STRICK, Germany



At the young age of 33, he had already been inducted as a member of the *Académie des Sciences*, and in 1873, he succeeded in proving a theorem that brought him worldwide recognition; yet the French mathematician CHARLES HERMITE shared a fate similar to that of many well-known mathematicians whose life's work was never noticed by the general public.

In the mid nineteenth century, a number of mathematicians were doing research on the properties of numbers. In a way comparable to the discovery of the irrationality the numbers $\sqrt{2}$ and $\sqrt{5}$ in the fifth century BCE by HIPPASOS VON METAPONT, a mathematician from the school of the Pythagoreans, in 1844, JOSEPH LIOUVILLE obtained a proof that there exist transcendental numbers, that is, numbers that are not algebraic. *Algebraic* numbers are those that occur as zeros of polynomials, while all other real and complex numbers are called *transcendental*.

Solutions of linear equations $ax+b=0$ with integers *a*, *b* are always rational, while those that are solutions of quadratic equations $ax^2+bx+c=0$ with integers *a*, *b*, *c* can be irrational. GIROLAMO CARDANO made known in 1545 that all solutions of equations of degree 3 and 4 can be represented by radicals, that is, in terms of using the four basic operations of addition, subtraction, multiplication, and division and the extraction of roots.

NIELS HENRIK ABEL and ÉVARISTE GALOIS proved in 1824 and 1830, respectively, that there is no general solution formula for the zeros of polynomials of degree greater than 4 and indeed, that the solutions can in general not be represented by radicals.

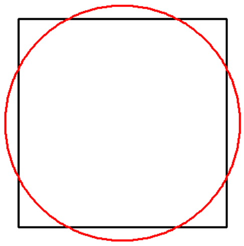
JOSEPH LIOUVILLE gave the first examples of transcendental numbers, among them the number 0.1 1000 100000000000000000 1000…, where there is a 1 at every $k!$ decimal place (that is, at the first, second, 6th, 24th, etc. places), and zeros otherwise.

LEONHARD EULER had proved in 1737 that the number $e=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\dots$, the base of the natural logarithm, is irrational, and in 1748 he conjectured that all numbers of the form $a^{\sqrt{b}}$ with rational a and natural *b*, such as, for example, $2^{\sqrt{2}}$, are not even irrational (as he put it), but a proof of this fact was obtained only in the 1930s, by KUZMIN, GELFOND und SCHNEIDER, who were able to give a positive answer to *HILBERT's seventh problem*, posed in 1900:

- If α is algebraic (with $\alpha \neq 0$, $\alpha \neq 1$) and β is irrational, then the power α^{β} is always transcendental.

LIOUVILLE was able to show that EULER's number *e* mentioned above could not be the solution of a quadratic equation, but that can be considered only a relatively small step in comparison to the proof that *e* is in fact a transcendental number, that is, that there is no polynomial equation of any degree to which *e* is a solution.

In 1873, CHARLES HERMITE was able to show precisely that. He gave a proof (which cannot be presented in an elementary way) in an article comprising thirty pages. It was then nine years later that FERDINAND VON LINDEMANN employed HERMITE's proof method to prove that the number π, the ratio of a circle's circumference to its diameter, is also transcendental.



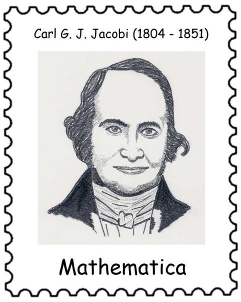
This result had as a corollary the solution to the millennia-old problem of squaring the circle (the question of the constructability – using only straightedge and compass – of a circle with area equal to that of a given square), namely that such a construction is impossible, since every construction with straightedge and compass leads to numbers in a limited class of algebraic numbers. After LINDEMANN had succeeded in proving the transcendence of π, HERMITE's star began to fade, though he was not bothered by the fact.

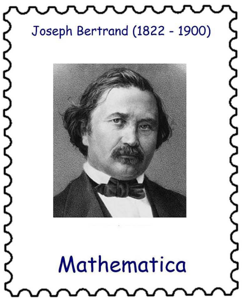
CHARLES HERMITE's father worked as an engineer in the salt mines of Lorraine. After marrying, he went to work for his parents-in-law, who were textile merchants. Interested more in art than in commercial trade, the father then opened a business in Nancy, in order to be able to take part in the cultural life of the provincial capital.

CHARLES was the sixth of his seven children. He was born with a congenital defect: a malformed right foot. His schooling began with attendance at a collège in Nancy, but then he moved to Paris, where in the years 1840-1841 he received mathematical instruction under LOUIS RICHARD, who had taught ÉVARISTE GALOIS, at the Lycée Louis-le-Grand.

Like his predecessor GALOIS, HERMITE was interested less in the material of the standard curriculum than in the writings of EULER, GAUSS, and LAGRANGE, which he read in his free time. While still a school pupil, he submitted two articles to the recently established journal *Nouvelles Annales des Mathématiques*, one of them a commentary on an article by LAGRANGE on the solvability of equations of the fifth degree, though he did so without knowledge of GALOIS's contributions to this subject (apparently, the editors of the journal were also at the time unaware of the writings of GALOIS).

Like GALOIS, he hoped to study at the *École Polytechnique*, but unlike GALOIS, he passed the entrance examination, though – to be sure – with a rather mediocre performance. After a year at the *École Polytechnique*, whose main task was to prepare a new generation for the military, he was not allowed to continue, the reason given that his physical infirmity made him unfit for military service. The decision to expel him was countermanded after the intervention of advocates on his behalf, but offended, HERMITE left that institution in order to complete his education elsewhere.





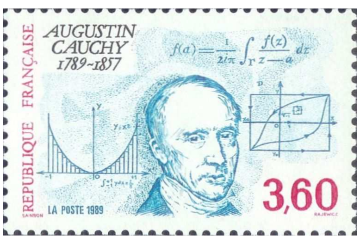
He corresponded with JACOBI on the subject of a special type of differential equations for which he had found a method of solution; this was an application of the methods of FOURIER that had not yet achieved general recognition. JACOBI expressed enthusiasm for the ideas that HERMITE proposed for dealing with elliptic functions; he expressly mentioned HERMITE's contributions in his collected works. HERMITE also became friends with JOSEPH BERTRAND, who at the time was still working as a teacher at the *École Polytechnique*; later, HERMITE married BERTRAND's sister.

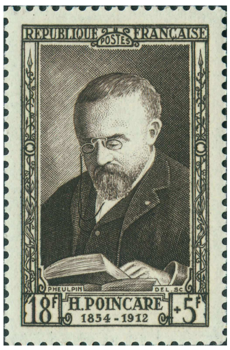
In 1847 he completed his studies and passed the requisite examinations, and a year later he was already working as a tutor at the *École Polytechnique*, the institution that just a few years previously had wanted to expel him. It must have been gratifying to him indeed when he was appointed to the very commission that decided on which candidates would be admitted!

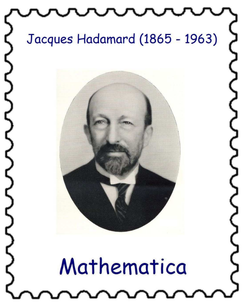
In 1856, the same year in which he was elected to the *Académie des Sciences*, he became infected with smallpox and his life was in danger. In this crisis, AUGUSTIN CAUCHY supported him with religious encouragement. Under the latter's influence, he converted to Catholicism, and in contrast to the radical republican GALOIS, became a convinced royalist. Having recovered from his illness, he made in 1859 a surprising discovery: although there is no general formula for the expression of the roots of a polynomial of degree 5, it is possible to represent them using elliptic functions.

From 1863 on, he worked as an examiner at the *École Polytechnique*. In 1868, he was appointed professor of analysis at the *École Polytechnique* as well as at the *Sorbonne*; he continued in this position until 1897.

Among his numerous students can be counted HENRI POINCARÉ, who was certainly the most significant and who admired and respected his teacher greatly. For POINCARÉ, who was particularly interested in such questions as how "discoveries" in mathematics arise, it was always a puzzle how HERMITE arrived at his brilliant ideas through such "logical" considerations.







HERMITE's student HADAMARD reported on the former's impressive and enthusiastically delivered lectures, which revealed the speaker's love for mathematics and its beauty almost in the sense of a religious belief.

In his lifetime, CHARLES HERMITE worked on a number of topics in a variety of branches of mathematics, including number theory, algebra, and complex analysis. He made decisive contributions in these areas. In mathematical lexicons one finds numerous terms connected with the name HERMITE, for example *HERMITE functions*, *HERMITE interpolation formula*, *HERMITE operators*, and *HERMITE matrices*, which also indicates the many-sided nature of his work.

After his death, his son-in-law ÉMILE PICARD, also an important mathematician, published an impressive collection of his publications and posthumous papers.

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https://www.spektrum.de/wissen/charles-hermite-1822-1901/1171317

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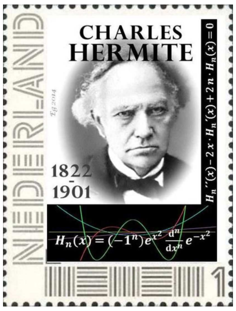

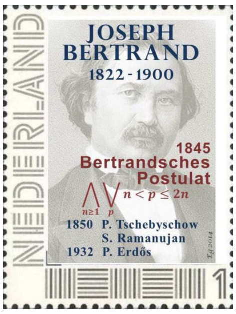

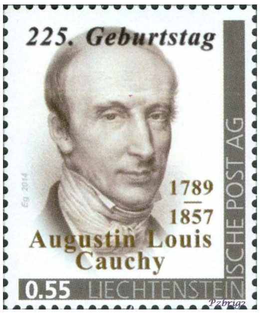

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