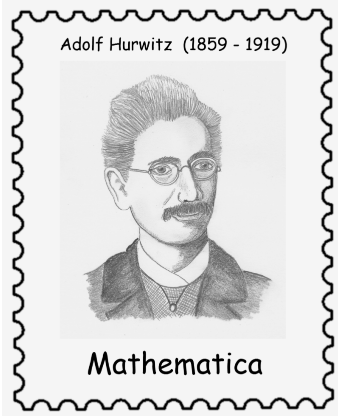
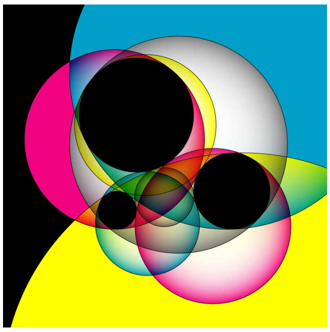
ADOLF HURWITZ (March 26, 1859 – November 18, 1919)

by HEINZ KLAUS STRICK, Germany



ADOLF HURWITZ had the great good fortune to have a mathematics teacher, DR. HERMANN CÄSAR HANNIBAL SCHUBERT, at the science high school Andreanum in Hildesheim, Lower Saxony, Germany, who at once recognized the extraordinary mathematical talent of his pupil and encouraged him in an unusual way: on Sundays, ADOLF was invited to visit his teacher at home, and there he was taught mathematical subjects that were not included in the curriculum. In 1876, when ADOLF HURWITZ was just 17 years old, a joint paper by pupil and teacher appeared in the journal *Mathematische Annalen* on a counting procedure in algebraic geometry involving the number of curves or surfaces in space possessing certain properties.

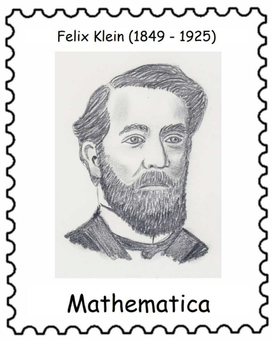


SCHUBERT had dealt in his dissertation with a generalization of a theorem of APOLLONIUS: that Greek mathematician had proved, 200 years before the Common Era, that three circles in the plane determine up to eight ($= 2^3$) circles that are tangent to the three given circles. SCHUBERT had shown that four spheres in three-dimensional space determine 16 ($= 2^4$) spheres that are tangent to the four given spheres. This geometric problem can be described algebraically by a system of quadratic equations. The question is then how many solutions the system has and how they can be determined. (figure: Wikipedia)

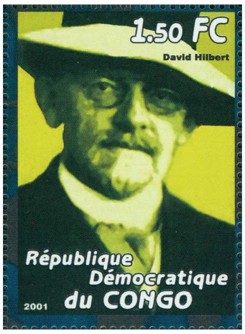
When it came time for ADOLF to graduate from high school, SCHUBERT convinced the young man's father to send his highly gifted son to university. The father, however, was a simple workman and could not afford such a luxury. Eventually, a friend of ADOLF's father undertook to finance the boy's education, and HURWITZ travelled to the Technical University (TU) in Munich with a very strong letter of recommendation from SCHUBERT to FELIX KLEIN recommending ADOLF for mathematical studies.

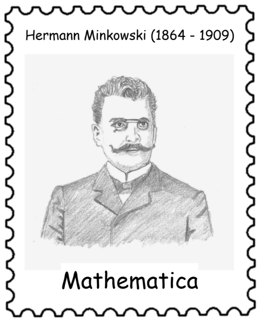
While HURWITZ was in Munich attending FELIX KLEIN's lectures, his former teacher moved to the prestigious Johanneum, in Hamburg. In 1874, SCHUBERT won a gold medal in a prize competition sponsored by the Danish Academy of Sciences, and in 1887, he was awarded the title "professor."

After only one semester, ADOLF HURWITZ transferred for a year to Berlin, where he attended lectures by WEIERSTRASS, KUMMER, and KRONECKER. In 1988, WEIERSTRASS's lectures on analysis were reconstructed, which would have been impossible were it not for the existence of HURWITZ's notes. During his Berlin period, HURWITZ remained in constant contact with FELIX KLEIN, collaborating with him on the publication of his *Elliptic Modular Forms*. He returned to Munich, and when in 1880, KLEIN was offered a professorship at the University of Leipzig, HURWITZ followed him, and there completed his doctoral studies.



Because of his insufficient knowledge of Greek (HURWITZ had attended a scientifically oriented high school, not one emphasizing the humanities), he would not have been able to complete his habilitation in Leipzig. He therefore went to Göttingen, where such linguistic hurdles did not exist. Because he was Jewish, HURWITZ had little chance of obtaining a permanent position. But then in 1884, FERDINAND VON LINDEMANN, who had become famous for his 1882 proof of the transcendence of π, offered HURWITZ a position as associate professor in Königsberg, and the 25-year-old HURWITZ accepted it gladly.





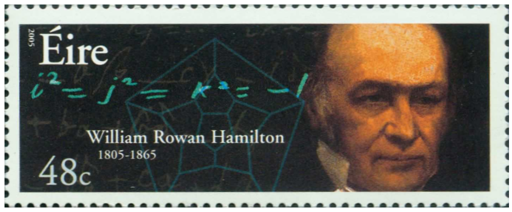
With two of his students, DAVID HILBERT and HERMANN MINKOWSKI, HURWITZ formed a lifelong friendship. In the following years, the three friends regularly spent their vacations together, even after MINKOWSKI had accepted a position in Bonn. In 1892, HURWITZ accepted a professorship at the *Eidgenössische Technische Hochschule* (today known simply as ETH) in Zurich.

He moved to Switzerland with his wife and three children, and there they lived for the remaining 27 years of HURWITZ's life. Only a few weeks following his move to Zurich, HURWITZ received an offer from the University of Göttingen for one of the most prestigious mathematical professorships in all of Germany. But out of loyalty to his new employer, he declined the offer.

During his student years in Munich, HURWITZ contracted typhus, and in 1886, he became infected a second time. With increasing age, he suffered from the aftereffects of the disease in the form of migraine attacks. After losing a kidney in 1905, his ability to work was greatly restricted for the remainder of his life.

During his professional career, HURWITZ published 96 substantial scientific papers, primarily on the analysis of complex-valued functions and the theory of RIEMANN surfaces. In 1897, he was honoured with an invitation to present a survey lecture at the first *International Congress of Mathematicians* in Zurich on the current state of mathematical research in this area. He was also recognized for his numerous contributions to number theory, algebraic structures, and geometry.

A number of theorems in a variety of mathematical areas are named for ADOLF HURWITZ. For example, certain polynomials with positive coefficients that play a role in stability theory are known as *HURWITZ polynomials* if all the zeros have negative real part (that is, they all lie to the left of the origin in the complex plane). HURWITZ provided a set of easily verified conditions that the coefficients must satisfy.



For complex numbers $a+b\cdot i$ with $a, b \in IR$, which can be notated in the form of an ordered pair $(a\,;b)$, two operations are defined, an addition and a multiplication. After many years of trying in vain to define analogous operations for ordered triples of numbers, WILLIAM ROWAN HAMILTON discovered in 1843 the possible operations on four-tuples, the *quaternions*.

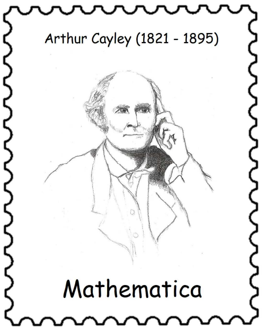
The four-tuples whose coefficients are either all integers or all half-integers (that is, half an odd number) are known as the HURWITZ quaternions. If one defines in particular $\varepsilon = \frac{1}{2}\cdot(1+i+j+k)$, then one has $\varepsilon^2 = \frac{1}{2}\cdot(-1+i+j+k)$ as well as $\varepsilon^3 = -1$. The set $Q_4 = \{0, 1, \varepsilon^4, \varepsilon^2\}$ is closed under addition and multiplication. It then follows that the set of HURWITZ quaternions is closed with respect to these operations (the HURWITZ semigroup).

+	0	1	ε^4	ε^2
0	0	1	ε^4	ε^2
1	1	0	ε^2	ε^4
ε^4	ε^4	ε^2	0	1
ε^2	ε^2	ε^4	1	0

·	0	1	ε^4	ε^2
0	0	0	0	0
1	0	1	ε^4	ε^2
ε^4	0	ε^4	ε^2	1
ε^2	0	ε^2	1	ε^4

The set of HURWITZ units, that is, the set of HURWITZ quaternions of modulus one, comprises 24 elements; they form the vertices of a regular figure in four-dimensional space. According to the four-squares theorem of LAGRANGE, every natural number can be expressed as the sum of four perfect squares. Therefore, for every natural number *n*, there exists a HURWITZ quaternion whose modulus is equal to *n*.

In 1898, HURWITZ proved that such operations are possible only for dimensions 1, 2, 4, and 8.



In 1845, ARTHUR CAYLEY published an article showing how addition and multiplication could be defined for the octonions (eight-tuples).

HURWITZ devoted particular attention to the subject of continued fractions. He proved a generalization of a theorem of DIRICHLET on the quality of the approximation of irrational numbers by rational numbers.

The HURWITZ approximation theorem is one of the most remarkable theorems of number theory. It states that for every irrational number *x*, there exist infinitely many pairs p, q of integers such that $\left|x-\frac{p}{q}\right| \le \frac{1}{\sqrt{5}\cdot q^2}$.

It follows that for $\Phi = \frac{1}{2}\cdot\left(1+\sqrt{5}\right) \approx 1.618$, the ratio known as the golden section, the inequality $\left|\Phi-\frac{p}{q}\right| < \frac{1}{q^2\cdot A}$ can possess at most finitely many solutions for every number $A > \sqrt{5}$ (or as DMITRY FUCHS and SERGE TABACHNIKOV put it, Φ *is the “most irrational of all irrational numbers, the number most averse to rational approximation”).*

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https://www.spektrum.de/wissen/adolf-hurwitz-1859-1919/1224328

Translated by David Kramer

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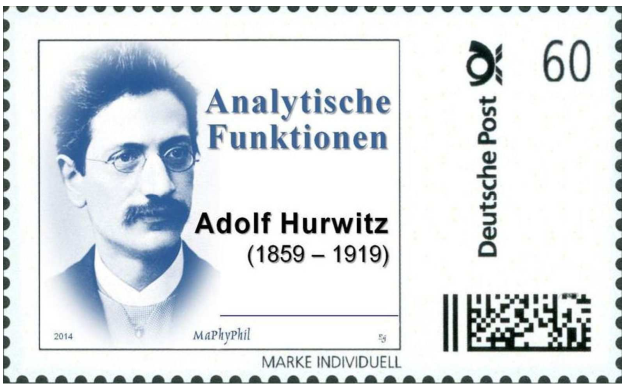

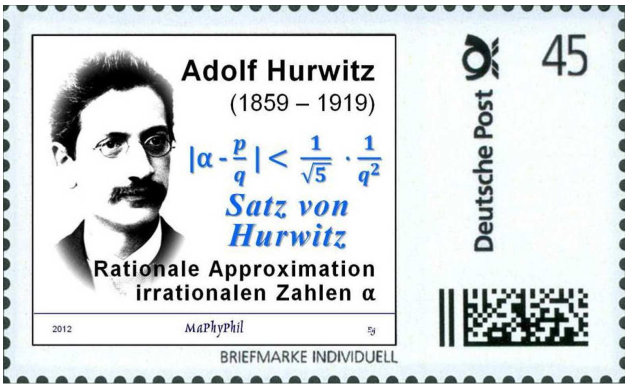

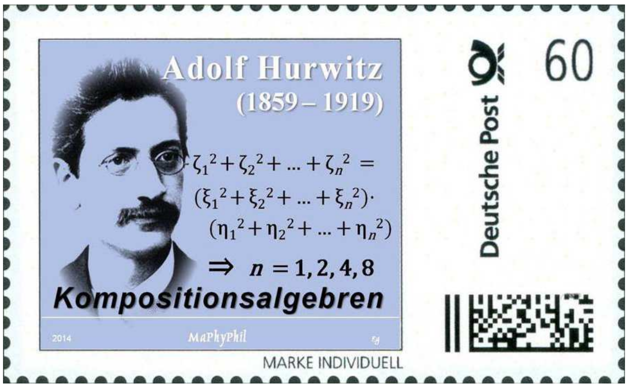

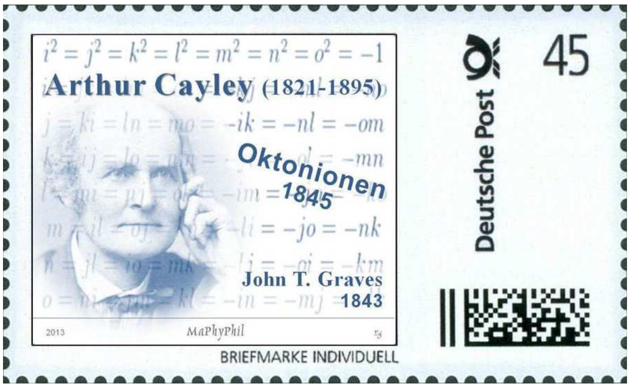

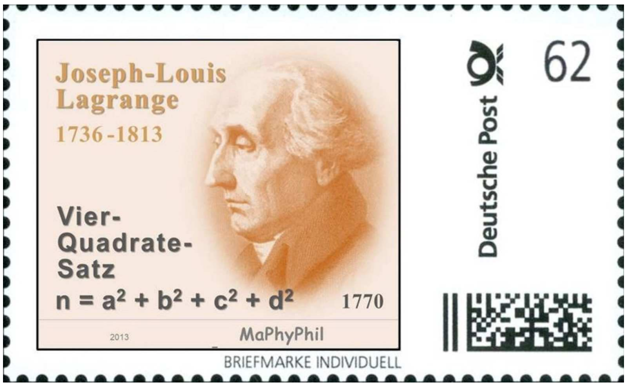

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