**Andrei Kolmogorov** (April 25, 1903 – October 20, 1987)

by Heinz Klaus Strick, Germany


One of Portugal's millennium postage stamps honors three important mathematicians of the twentieth century. Pictured from left to right are the French mathematician Jules Henri Poincaré (1854–1912), the Austrian (born in Brno, Moravia) logician Kurt Gödel (1906–1978), and the Russian Andrei Nikolaevich Kolmogorov.

Poincar´e is considered one of the last polymaths in mathematics as well as in physics; his numerous publications deal with an enormous variety of topics, beginning with number theory and encompassing the theory of relativity. Gödel, perhaps the most important logician of the twentieth century, shattered the foundations of mathematics with an article published in 1931. His renowned *incompleteness theorem* states that in principle, none of the possible axiom systems of arithmetic can be *complete* in the sense that it can be used to prove all the theorems of arithmetic. Indeed, there are theorems that can be neither derived nor refuted from the system of axioms.

Andrei Nikolaevich Kolmogorov never knew his parents. His mother died giving birth to him; his father lived in exile, due to his having joined a revolutionary group. In 1919, he perished in the Civil War. His mother's sister adopted him and took responsibility for his education. Andrei kept his grandfather's surname.

From 1910 on, he attended an advanced school in Moscow. On his graduation in 1920, he worked for a time as a railway conductor before commencing studies at *Moscow State University*. In addition to mathematics, he studied metallurgy and had a special interest in Russian history.

Later, Kolmogorov enjoyed relating an anecdote about a history seminar paper that he had written. His instructor criticized it, pointing out that while perhaps in mathematics it sufficed to justify an assertion simply by offering a proof, historians feel it incumbent upon themselves to support their theses with a number of arguments.


That he finally decided in favor of mathematics is due doubtless to his teacher Nikolai Nikolaevich Luzin (1883–1950), who recognized his student Kolmogorov's unusual talent. Already in early 1922, Kolmogorov published an internationally recognized paper on operators on sets.

In summer of that year, he surprised the experts with an example of an integrable function whose associated Fourier series diverges almost everywhere. (A Fourier series is a particular infinite sum whose terms are trigonometric functions.)

Before he had taken his examinations in 1925, Kolmogorov had published eight papers on a variety of topics, including – in collaboration with Aleksandr Yakovlevich Khinchin (1894–1959) – his first contribution to probability theory, dealing with the *weak law of large numbers*.

Around 1700, JACOB BERNOULLI (1654–1705) expounded a law that today is known as *BERNOULLI's law of large numbers*. It states that the probability that in a series of BERNOULLI trials, the difference between the relative frequency *X/n* and the underlying success probability *p* is equal to at most a given positive number $\varepsilon$ converges to 1 as *n* goes to infinity: $\lim_{n\to\infty} P\left(\left|\frac{X}{n}-p\right|\le\varepsilon\right)=1$.

This law is what appears on the Swiss postage stamp pictured: The mean value of the results of a number of trials tends to the expected value of the random variable.



In 1866, PAFNUTY CHEBYSHEV generalized BERNOULLI's result to sums of independent random variables and gave also an ingeniously simple proof (the *CHEBYSHEV inequality*).

KOLMOGOROV's 1925 contribution gives three conditions under which one has

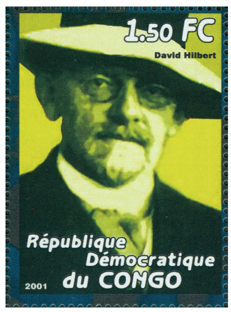
$$\lim_{n\to\infty} P\left(\left|\tfrac{1}{n}\cdot(X_1+\ldots+X_n)-\tfrac{1}{n}\cdot\left(E(X_1)+\ldots+E(X_n)\right)\right|\le\varepsilon\right)=1.$$

The above-mentioned conditions relate to the sequence of sums of the random variables, the sequence of associated expectations, and the variances; for this reason, the theorem is sometimes called the *three series theorem*.

In the following years, KOLMOGOROV published further papers on probability theory, but also in other areas of mathematics. With PAVEL SERGEEVICH ALEXANDROV (1896–1982) he traveled through Europe, visiting universities in Berlin, Göttingen, and Paris. In 1930, he accepted a chair in mathematics at *Moscow State University*.

As a university professor, KOLMOGOROV had an enormous effect on his students. He took a personal interest in them; on the regular hiking trips that he personally conducted, mathematics was the primary topic of conversation. Kolmogorov also authored a number of textbooks and encouraged mathematically talented students. With his 1933 work *Grundbegriffe der Wahrscheinlichkeits-rechnung* (Fundamentals of the Theory of Probability), published in German, KOLMOGOROV exercised a profound effect on the further development of the theory of probability.

In 1900, at the *International Congress of Mathematicians*, in Paris, DAVID HILBERT (1862–1943) named what were in his opinion the 23 most important outstanding unsolved problems in mathematics. The sixth of these problems asked how mechanics and probability theory (which at the time was considered part of physics because of the applied problems in the field) could be axiomatized. In an axiomatized formulation, one begins with a set of fundamental axioms, from which further laws are then derived; an analogy is the axiomatic system that EUCLID gave for geometry.




In vain, KOLMOGOROV encouraged various mathematicians to formulate a set of suitable axioms. The approach taken by RICHARD VON MISES (1883–1953) to define probabilities as limiting values of relative frequencies had, like other approaches, led to difficulties.


KOLMOGOROV's approach, on the other hand, is simpler. He restricted his formulation to determining what properties probabilities should have and made no attempt to define what a probability actually is.

Consider a set *S* of possible outcomes of a random experiment on which *U* and ∩ are defined (a so-called *sigma algebra*) together with a function *P* that assigns to subsets of *S* numbers from the interval [0, 1]. The function *P* is a *probability measure* if the following three properties hold (the *KOLMOGOROV axioms*):

- For every event $E \subset S$, the probability $P(E)$ is a real number between 0 and 1: $0 \le P(E) \le 1$.
- The certain event *S* has probability 1: $P(S) = 1$.
- The probability of the union of a countable number of independent probabilities is equal to the sum of the individual probabilities: $P(E_1 \cup E_2 \cup ...) = P(E_1) + P(E_2) + ...$, provided that $E_i \cap E_j = \{\}$ whenever $i \neq j$.

From these *fundamental principles* other properties can be derived, for example the complement rule and the general summation rule (inclusion and exclusion formulas).

In the ensuing years, KOLMOGOROV made further fundamental contributions to probability theory and statistics, in particular in the subject of *MARKOV chains*; he studied turbulence in the context of fluid mechanics, dynamical systems (application to planetary motion), and informatics and the theory of algorithms (the KOLMOGOROV complexity is a measure of the structure of character strings). He published in the areas of logic, analysis, and topology.

With VLADIMIR IVANOVICH SMIRNOV (1887–1974) he developed a widely applicable nonparametric goodness-of-fit test for studying the differ- ence between an empirical and hypothetical distribution function.


Based on his significant scientifi achievements, KOLMOGOROV received a great number of honors; he was one of the first scientists to receive the STALIN Prize, introduced in 1940. In 1962, he was awarded the BALZAN Prize (with a monetary award of one million Swiss francs), in 1965 the LENIN Prize, in 1987 the LOBACHEVSKY Prize, and in 1980 the WOLF Prize (with an award of 100,000 dollars). He was a member of the *Academy of Sciences* of the Soviet Union and of other scientific academies in numerous countries throughout the world.

First published 2007 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/andrei-n-kolmogorov-1903-1987/905709

Translated by David Kramer

English version first published by the *European Mathematical Society* 2011

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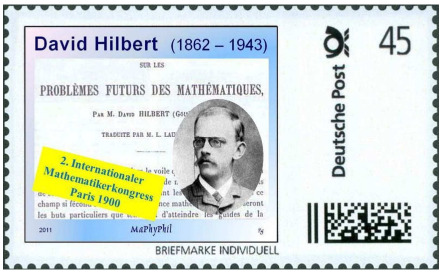



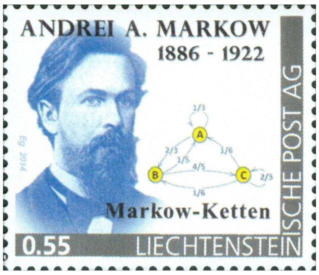



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