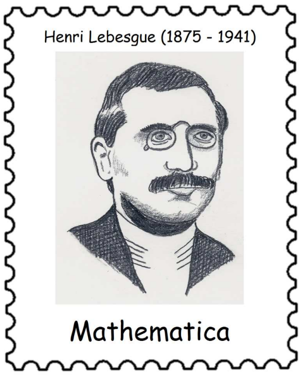
HENRI LEON LEBESGUE (June 28, 1875 – July 26, 1941)

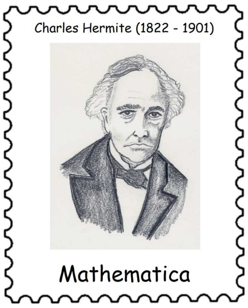
by HEINZ KLAUS STRICK, Germany



(drawings © Andreas Strick)

HENRI LÉON LEBESGUE was only three years old when his father and two sisters died of tuberculosis. His mother worked to support her family and give the boy the best education possible. While HENRI was still in primary school, his teachers noticed the lad's extraordinary talent.

A scholarship provided by his hometown of Beauvais (Picardie) allowed him to transfer to a lycée in Paris. After attending the *classe préparatoire* at the *Lycée LOUIS-LE-GRAND*, which had produced such eminent mathematicians as ÉVARISTE GALOIS, HENRI POINCARÉ and CHARLES HERMITE, he continued his studies in 1894 at the *École Normale Supérieure* (*ENS*).



In 1897, he passed the required examinations to become a teacher of students at the upper levels of the lycée (*agrégation*). He then worked for two years in the university library before serving three years in Nancy as a teacher at a lycée.

During this period, LEBESGUE composed the two works on which his fame rests:

- *Sur une généralisation de l'intégrale définie* (On a generalization of the definite integral)

and his doctoral dissertation,

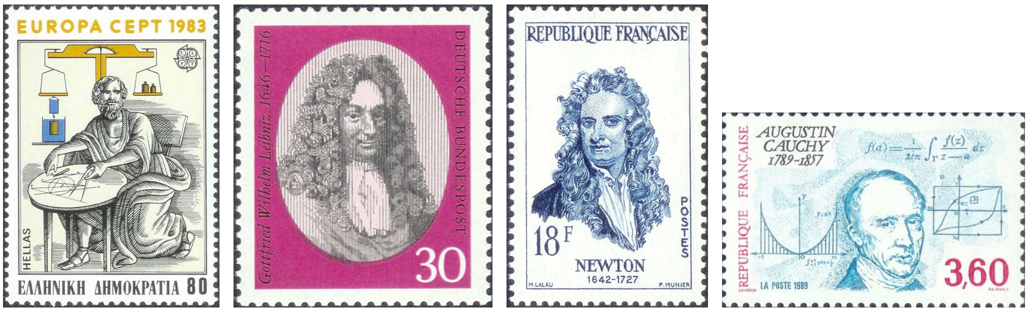
- *Intégrale, Longueur, Aire* (integral, length, area).

After completing his doctorate in 1902, he held a series of positions – as is usual in France – as a lecturer in provincial universities, in Rennes and Poitiers, before being called in 1910 to the Sorbonne.

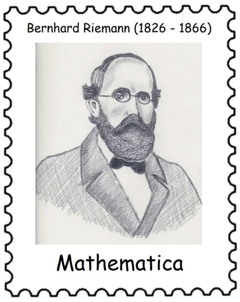
In 1921, he was named professor of mathematics at the *Collège de France*, an exceptional Parisian institution, which, known as the *Grand Établissement*, enjoys enormous prestige throughout France. The professors of this collège are charged with *teaching knowledge as it is being created* (*enseigner le savoir en train de se faire*), that is, to conduct basic research. In addition to his work at the *Collège de France*, LEBESGUE also lectured regularly at the *École Normale Supérieure*.

The problem of determining the area of surfaces bounded by particular curves goes back at least to ARCHIMEDES, who devised extremely clever methods to determine such areas as the area under a segment of a parabola.

Isaac Newton and Gottfried Wilhelm Leibniz discovered the connection between the *area problem* and the *problem of tangents* and formulated independently the *fundamental theorem of differential and integral calculus*, which states essentially that the integral function *F* of a continuous function *f* defined on a closed interval I is differentiable and that its derivative is equal to the function *f* being integrated.



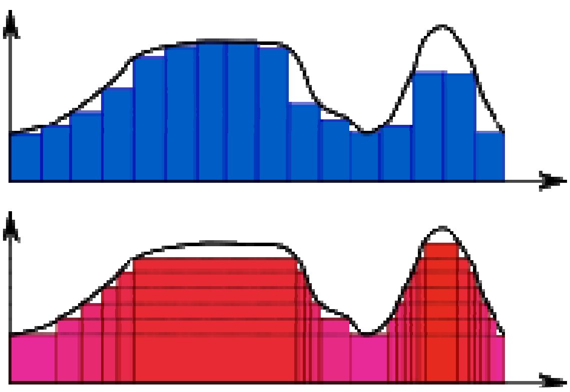
The precise conceptualization and proofs of this were given first by Augustin Cauchy. Bernhard Riemann later developed the concept of integrability of a function by dividing the interval I into subintervals and determining the areas of rectangles with base the length of the subinterval and height the value of the function at a point in the subinterval, that is, sums of the form $\sum_{i=1}^{n} f(t_i)\cdot(x_i - x_{i-1})$, where *i* is the index of the subinterval from x_{i-1} to x_i, and t_i is a point in the subinterval. If the sums converge to some fixed number for every choice of decomposition into subintervals and choice of points t_i, then that number is called the Riemann integral and is denoted by $\int_a^b f(x)dx$.



The generalisation of the Riemann integral to the calculation of the volumes of 3-dimensional or even *n*-dimensional bodies proved to be difficult.

Inspired by publications of Émile Borel, professor at the *École Normale Supérieure*, Lebesgue took on in his doctoral research the problem of measurability of sets and thereby arrived at a new definition of integrability.

While with Riemann, the domain of definition of a function stands in the foreground, Lebesgue began with the set of the function's values, which he divided into subintervals. An approximation of the Lebesgue integral is achieved by determining the areas of subsets of the plane whose height is taken from the length of a subinterval of the set of values and whose width is determined by the total length of the associated set of the domain of definition. More precisely: an approximation to the integral is the sum over the areas of the sets $f^{-1}([x_{i-1}, x_i]) \times [x_{i-1}, x_i]$.



(Wikipedia: Lebesgue integration CC0 – Svebert)

Lebesgue explains his approach in a vivid way thus:

Suppose that I have to pay a certain sum; I look through my pockets, and there I find coins and currency notes of various values. I give them to my creditor in the order in which I find them until I have reached the total amount of my debt. That is the Riemann integral. But I proceed otherwise. After taking all the money out of my pockets, I place all of the notes of the same value together, and I do the same with the coins, and I make my payment by handing over in sequence all the money of a given value. That is my integral.

The procedure of Lebesgue shall be illustrated by the example of the Dirichlet function D over the interval [0 ; 1], which is defined for all rational numbers by 1 and by 0 for for the irrational numbers. This function is discontinuous at every point of its domain. It is not Riemann integrable, since the limiting values of the upper sums and lower sums do not coincide. But with Lebesgue's method, the integral over the interval [0 ; 1] is equal to zero.

One could formulate the situation in a somewhat simplified fashion thus: if one ignores the countably many rational exceptions, the Dirichlet function is essentially the constant function with value zero. Therefore, the integral over the interval is also equal to zero.

The method introduced by Lebesgue is a generalization of that of Riemann:

- Every (bounded) Riemann integrable function on a bounded interval is Lebesgue integrable, but there are Lebesgue integrable functions that are not Riemann integrable.

A counterexample was found by the Italian mathematician Giuseppe Vitali in 1905.

Lebesgue's ideas about generalizing the notion of integral were not met with universal approval among mathematicians, particularly in France.

Charles Hermite, for example, expressed a fundamental aversion to having anything to do with functions that could be defined only by "complicated" rules: *I turn with horror and revulsion from this lamentable plague of functions that can have no derivative whatsoever.*

In the years that followed, however, more and more mathematicians worked on problems related to Lebesgue's ideas, even on finding a measure for "more complicated" sets. For example, Constantin Carathéodory made important developments in the theory.

Lebesgue's doctoral thesis can be seen as ushering in a new branch of mathematics, *measure theory*, whose development turned out to be of great significance for the theory of probability.

The Banach-Tarski paradox from the year 1924 indicates the sorts of problems that can arise in dealing with unmeasurable sets: a sphere can be divided into a finite number of unmeasurable sets in such a way that it is possible to put these pieces together to create two identical copies of the original; that is, the volume of the sphere can be "doubled."

By the time of his induction into the *Académie française* in 1922, Henri Lebesgue had written more than 90 works on measure and integration theory.

And many additional publications were to follow, including some in other branches of mathematics. Over the years, LEBESGUE received a number of honours from foreign universities. He was beloved and esteemed by his students. His principle of teaching was this:

For a teacher, there is, in my opinion, only one possible way of giving instruction, namely to stand before one's students and think (*… penser devant ses élèves*).

In an obituary, the French mathematician PAUL MONTEL wrote this:

He was a great scholar, an admirable teacher, a man of incomparable moral stature. His influence on the development of mathematics will last a long time – both through his own works and those that he inspired.

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https://www.spektrum.de/wissen/henri-lebesgue-1875-1941/1194477

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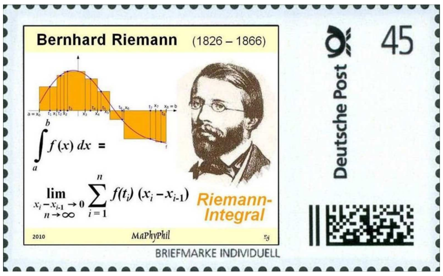

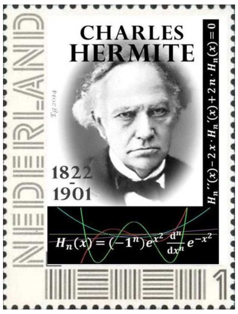

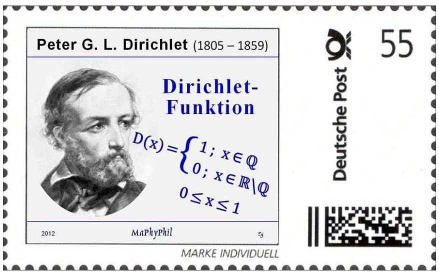

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